# A TEXTBOOK OF PHYSICS

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*From the painting by T. Phillips in the National Portrait Gallery, London*

# A TEXTBOOK OF PHYSICS

BY

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## Vol. III

## ELECTRICITY AND MAGNETISM

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# FROM THE PREFACE TO THE SEVENTH GERMAN EDITION

This volume deals chiefly with those phenomena usually designated as "*classical*" *electrodynamics* (including the theory of electrons), in the treatment of which the field and the quantities associated with it (with the exception of the elementary charges) may be regarded as continuous in character.

Considerably greater changes from the previous edition have been made in this volume than in Vol. I (Mechanics) and Vol. II (Heat and Sound). A large part has been completely rewritten. At the same time the number of illustrations has been very much increased, and some of the figures taken from the earlier edition have been redrawn.

One of the reasons for these extensive alterations is that all material relating to *Atomic Physics* has now been brought together so as to form a separate volume (Vol. V). Secondly, the older text has been compressed, in order to make space for the inclusion of those new advances with which it is to-day desirable that the student should be acquainted. This has made necessary a complete recasting of the sections concerned. Thus, for example, the treatment of discharges through gases, generators and motors, electric oscillations and waves, &c., has been considerably amplified and as far as possible co-ordinated.

Other sections have had to be rewritten because the point of view adopted has been altered in many respects. My own intentions coincided with the request, made to me from many sides, for a consistent use of the *volt-ampere system.* An alteration in this direction seemed to me particularly desirable since this book, in accordance with its conception and purpose, lays the principal stress upon the side of experimental physics. Now every experimental physicist employs only instruments calibrated according to the volt-ampere system. Moreover as a result of general usage many physicists have an intuitive appreciation of the significance of the quantities employed in this system, and before embarking upon an experimental investigation find it necessary to express formulæ in terms of these quantities. Also there is no longer any need to distinguish between two systems of measurement, the electrostatic and the electromagnetic, the simultaneous use of which would never be customary if the transformation factor did not happen to be a very convenient number. Anyone who is accustomed to seeing the formulæ written in other forms will find in the appendix a table giving the very simple transformations for the most important cases.

Another change is that the whole treatment has been based, with as great a degree of consistency as possible, upon the *field theory* of Faraday and Maxwell. Ever since Mie adopted this course with such distinguished success in his *Lehrbuch der Elektrizität und des Magnetismus*, which I found very stimulating as a student, the didactic treatment of the subject has developed more and more in this direction. The justification for a consistent adoption of this method of presentation will hardly be questioned to-day, in spite of the fact that changes, more or less violent, are gradually modifying its fundamental concepts and we no longer think as we used to do about the processes underlying electromagnetic phenomena.

The first chapter and the sections upon the *magnetic field of currents* and *electromagnetic induction* have been completely rewritten from this point of view. At the same time I have been at pains to preserve the character of the book: on the one hand to proceed as systematically as possible, on the other hand always to emphasize relationships. Thus for systematic reasons, and not for the sake of tradition alone, it appeared to me best to start with the behaviour of charges at rest, and to develop the other phenomena from this as basis.

I owe a debt of gratitude to all those who have assisted me in various ways with the composition of this book: especially Herr Professor F. Emde for many stimulating suggestions, fruitful criticisms, and references to relevant literature; also Herr Studienrat Dr. Schauff for suggesting many improvements in style and for his careful help in reading the proofs.

I should also like to express my thanks to the publishers for their sympathetic co-operation with regard to the new illustrations.

R. TOMASCHEK.

MARBURG ON THE LAHN,
*October, 1931.*

# PREFACE TO THE ENGLISH EDITION

I have taken the opportunity of the publication of this edition to make a few corrections and improvements. With a view to meeting the needs of English readers, some new matter has been added. I wish to thank the translator for the adequate manner in which he has carried out his task, and hope that in its new dress the work will greatly extend the circle of its friends.

R. TOMASCHEK.

MARBURG,
*24th June, 1933.*

## TRANSLATOR'S NOTE

I wish to express my thanks to Mr. A. J. Davies, B.Sc., of University College, Nottingham, for reading through the proofs of a large part of this volume.

L. A. W.

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# PART I

# THE STATICAL ELECTRIC AND MAGNETIC FIELD

## CHAPTER I

## The Electric Field of Charges at Rest

### 1. The Fundamental Electrical Phenomena

**Electrification by Friction.**—If a stick of sealing-wax or a dry vulcanite fountain-pen is rubbed with a piece of fur or of dry silk, it is observed that the rubbed bodies have acquired the property of attracting, over some distance, light bodies such as scraps of paper, small feathers, or particles of india-rubber (fig. 1). The region of space in the neighbourhood of the stick of sealing-wax or the fountain-pen is therefore in a peculiar condition, or state, in consequence of which a force acts on objects of a certain kind, causing them to move towards the rubbed bodies. This "state" of space is essentially distinct from other states which are associated in a similar way with manifestations of force; e.g. gravitation or magnetism; we call it the **electrical state.***

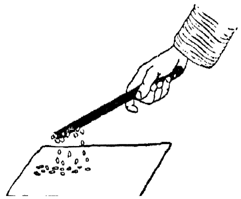

Fig. 1.—Attraction of light bodies by a rubbed stick of sealing-wax.

Specially effective methods for the production of the electrical state are the rubbing of glass with a woollen cloth smeared with a little zinc amalgam,† or of sulphur or sealing-wax with dry fur.

The forces concerned are most conveniently studied by observing their action on small light bodies, such as elder-pith balls or cigarette papers, attached to dry fine silk threads. For reasons which will appear later, it is best to have the pith balls coated with gold leaf, or some other metallic wrapping. In what follows we shall always suppose that the proof ball (i.e. small testing sphere) we are using is provided with a metallic covering of this sort (fig. 2). Such contrivances, by means of which the electrical state can be recognized, are called electroscopes.

---

* Gr., *ēlectron*, amber. The property possessed by amber when rubbed, of attracting light bodies, was known to the ancients; THALES (about B.C. 600) mentions that such phenomena had been observed. GILBERT (physician to Queen Elizabeth, 1540–1603) proved that many other bodies have the property; the word "electrical" is due to him.

† Arabian, *amalgám*, an alloy of a metal with mercury.

The body which exerts the forces is said to be charged with **electricity,** or to be the seat of an **electric charge.** The phenomenon is thus associated in our minds with the idea that a definite substance (Vol. I, p. 3)—the "electric charge"—is the source of these manifestations of force, the exciting cause of the electrical state in its neighbourhood. This concept of a substance has amply justified itself in experience, and we shall therefore make use of it in the following pages.

**Electric Field.**—Supposing we picture the electric charge as something substantial (but not *material*, see Vol. I, Introduction) we must not forget that the presence of this substance is always associated with a peculiar state of space, viz. the electrical state. A region of space throughout which a definite phenomenon is continuously displayed is said to be the *field* of the phenomenon. We may therefore say: electrically charged bodies are surrounded by an *electric field*.

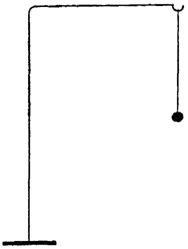

Fig. 2.—Proof ball

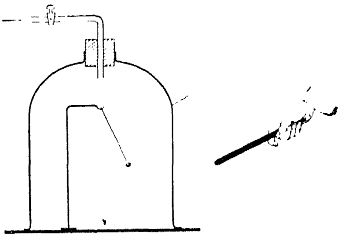

Fig. 3.—Force effect in a vacuum

**The Ether as Carrier of the Electric Field.**—A suitable place in which to fit up the little proof ball is under the dried glass bell of an air-pump (fig. 3). If an electrified body, e.g. a charged stick of sealing-wax, is brought near the glass on the outside, the little ball is attracted. If the bell is then as far as possible exhausted of air, the deflection is not found to be altered to any noticeable extent. The electrical state therefore exists even in space which is devoid of matter. Space must accordingly be filled with a substance which is capable of assuming different properties at different points of its volume. The carrier of the electrical state in space is called the **ether.**

It is important that from the very outset the idea which we form of the ether should be consonant with the facts (cf. Vol. I, Introduction). In any case, we must not ascribe to it the properties of a *material* substance, similar to the properties of a solid, a liquid, or a gas; and it is therefore meaningless to speak, for example, of the density or the temperature of the ether, or to expect that the laws of mechanics, i.e. the laws of the motion of matter, should be valid for the ether.

The ether is something entirely different from matter, and only advancing experience can reveal what are the properties which we may legitimately ascribe to it. Our task will be to make ourselves acquainted step by step with these properties, so far as they are at present known.

**Negative Effect of Air on the Electric Field.**—In the experiment just described, no change in the original deflection took place after the receiver was exhausted of air. We may therefore assume that the experiments in air which we shall describe would follow practically the same course in a vacuum. More exact measurements, to be cited later, prove that this assumption is actually correct (p. 85).

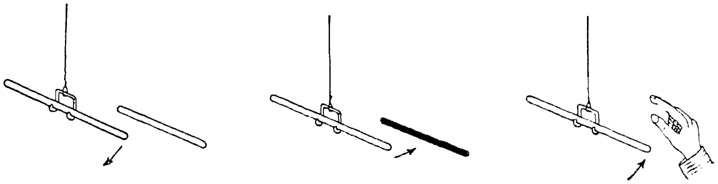

Fig. 4.—(*a*) Repulsion of two electrified glass rods; (*b*) attraction between rubbed glass and rubbed sealing-wax; (*c*) attraction between rubbed glass and a non-electrified body

**Two Kinds of Electricity.**—If we take a freely suspended glass rod (fig. 4), which has been rubbed with a woollen cloth and so electrically charged, and bring it near a second glass rod charged in the same way, the two rods repel each other. If we take a freely suspended stick of sealing-wax, charged by rubbing it with fur, and bring it near a second similarly charged stick of sealing-wax, the two sticks likewise repel each other. If, however, we bring a charged glass rod near a charged stick of sealing-wax, the two attract each other. The forces which act upon a stick of sealing-wax in the neighbourhood of the glass rod and of the second stick of sealing-wax are therefore in opposite directions. On account of this difference of behaviour we say that bodies may be either **positively** or **negatively electrified.** Those bodies are said to be positively electrified which behave like rubbed glass * (when the rubber is a woollen cloth smeared with zinc amalgam—with other rubbing materials, glass often behaves in the contrary way); and those bodies are said to be negatively electrified which behave like rubbed sulphur or sealing-wax (when these are rubbed with fur; vulcanite usually behaves like this, but frequently otherwise).

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* The choice of this kind of electrification as that which is to be called positive is perfectly arbitrary, and has been made on historical grounds. As we now know, the choice is unfortunate, particularly on account of its bearing on the selection of the direction to be considered positive in the case of an electric current in a metallic conductor.

The foregoing experiments prove that *bodies with like electrification repel each other, and those with unlike electrification attract each other.*

When we investigate in a similar way the *behaviour of non-electrified* (or *uncharged* or *neutral*) *bodies*, we find the following results. An electrified body, whether positively or negatively charged, is attracted by any uncharged body (fig. 4); and every electrified body likewise attracts every uncharged body. Should it happen, then, in any experiment that we observe an *attraction*, we can on the strength of this observation alone come to no conclusion on the question whether, or in which way, a body is electrified. We must always obtain a *repulsion*, before we can determine the kind of electrification we are dealing with.

The discrimination between the two kinds of electric charge by the use of the terms positive and negative is, apart from the opposite directions of the forces, to this extent justified, that + and − charges of equal amount when added together give the sum 0.

If in fact we take two small bodies, one of which is positively charged, and the other equally strongly negatively so that the fields excited by them are at corresponding points exactly equal and opposite (as may be tested by the strength of the attractions on a proof ball) and bring them as close to each other as possible, then their actions on the ether cancel each other; beyond a small distance from them no electric field is observable. An exact experimental proof on these lines will be given later (p. 105).

**Transference of the Electrical State.** If a little proof ball is brought into *contact* with a charged body, the ball, which before the contact was attracted, is now repelled. According to the preceding experiments, this can only mean that the testing body now carries a charge of the same sign as the charged body which touched it. A charge can therefore be communicated to other bodies. It is doubtless this property which has been chiefly responsible for the idea that electric charge has the character of a *substance.*

**Conductors and Insulators.**—The charged proof ball covered with foil, which has been referred to above, loses its electric charge altogether when it is touched by the finger; the electricity is conducted away through the finger. This total loss of electrification depends upon a perfectly general property of metals. A metal held in the hand does not become electrified on being rubbed; but if it is held by a handle of glass or vulcanite and then rubbed, it does become electrified. If it is then touched anywhere by the finger, it immediately becomes neutral again.

If, however, a vulcanite rod which has been electrified by friction is touched anywhere, only the place touched becomes non-electrified, the rest of the rod still remaining electrified. We can therefore separate bodies into two classes, *conductors* and *non-conductors* (insulators), according as the electrical state spreads (or is propagated) practically instantaneously over the whole body, or stays fast at the places where electrification has been produced by friction or by transference. A metal ball set up on an insulating rod of vulcanite or glass, or hung by a silk thread, retains its electrical state for a long time; consequently the air itself must be a non-conductor. (For further details see p. 23.)

From these experimental facts we infer that on, and in, conductors electricity is perfectly free to move; while on non-conductors no motion of electricity from place to place is possible.

*Note.*—There are no bodies on and in which there is *absolutely* no conduction of electricity. Even the air has a slight conducting power. Hence a body surrounded by air only likewise loses its electrified condition after some time. It would be more exact to speak only of *good* and *bad* conductors; however, for the present we shall distinguish only between perfect conductors and perfect non-conductors.

**Electrification merely a Separation of Charges of opposite Sign.**—When a stick of sealing-wax is rubbed with fur, the sealing-wax turns out to be negatively charged. But the fur also has become electrified, positively in fact, as may easily be shown by repulsion tests. If the sealing-wax, after being rubbed, is left wrapped up in the fur, and a proof ball is brought near the whole, no action on the ball is observed, although after the fur and sealing-wax are separated, both of them show strong action. The two fields neutralize each other. Electrification by friction is therefore only a separation of the two kinds of electricity.*

**Action at a Distance and Action through a Medium.**—At this point, the question may be asked: why do we assume that the region of space in the neighbourhood of charged bodies is in a state of a special kind? Is it not sufficient to regard a charged body as the source of forces acting at a distance upon other bodies, so that the "potential energy" of separated charges is the equivalent of the work done in separating them? It would certainly be possible to describe all the facts already mentioned, as well as nearly all that have still to be dealt with in the remainder of this chapter, on the basis of the assumption that forces between electrified bodies act at a distance *immediately.* This indeed was the view originally taken by physicists. It was suggested no doubt by the analogy between electrical and gravitational phenomena, the latter of which had already been thoroughly studied; and it continued to be the popular view up to the last quarter of the nineteenth century.

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* Otto von Guericke (1602–86) made the first electrical machine (1663). It consisted of a rotating ball of sulphur, and the rubbing was done by hand. He was also the first to observe electrical repulsion (1672). Stephen Gray (died in London in 1736) discovered the distinction between conductors and non-conductors (1729). Du Fay (1698–1739) discovered the difference between the two kinds of electricity (about 1734). Watson and Franklin, nearly at the same time (1747), announced the important law that in every electrical process *equal quantities* of positive and negative electricity are produced simultaneously.

Faraday,[*] however, was guided in his far-reaching discoveries by the entirely different idea that the *electrical state is carried by the surrounding space.* The electrical forces therefore reside in space, and are propagated in it from point to point. This is the theory of *action through a medium.* Faraday was led to it as the natural explanation of the results of the extensive and admirable series of experiments in which with extraordinary physical insight he put his questions to nature. The idea that electrical actions were influenced by a substance filling space (p. 84) was the dominating feature of Faraday's thought, but the results attained went far beyond this. It was not till nearly half a century later, however, that the experiments of H. Hertz supplied the complete verification of Faraday's theories. Two points were established by these experiments, viz. that electrical forces may exist in space devoid of matter and containing no electric charges, and that electrical processes are propagated with a finite velocity (p. 621). These experimental facts justify us in following Faraday from this point onwards in regarding the electrical state as something permeating space.

**Energy expended in Separation of Charges.** Since charges of opposite sign attract one another, work must be done to separate them. By the principle of energy (Vol. I, p. 90) this work must have its equivalent somewhere. In view of the considerations which precede, the natural assumption to make is that the equivalent is to be found in the excitation of the electrical state of the ether in the neighbourhood of the charges, so that the energy required to effect the separation is stored up as electrical energy in the space round the charges. This view will be gradually elaborated as we proceed, and in fact will be put into quantitative form.

## 2. The Electrical State of the Ether

**Field Strength.**—The electric field, like the gravitational field, is characterized by the property that at every point a force acts upon an object placed at the point (a mass in the case of the gravitational field, an electric charge in the case of the electric field). The electric field is a *field of force.*

---

[*] Michael Faraday was born on 22nd Sept., 1791, at Newington Butts near London. After some years (1804–12) spent as a bookbinder's apprentice and assistant, he was appointed by Sir Humphry Davy as laboratory assistant in the Royal Institution; Fellow (1824), Director of the Royal Institution Laboratory (1825), Professor from 1827; died at Hampton Court, 25th Aug., 1867. He is chiefly distinguished for his researches on electrolysis and electromagnetic induction, and for his recognition of the fact that the essential part of the action of a magnet or of an electrified body is to be found outside the body in its field of force. This view lies at the foundation of the modern theory of electromagnetic phenomena.

For the present (up to p. 110) we will confine ourselves to the consideration of the fields of charges *at rest.* **This branch of the science of electricity is called electrostatics.**

A field of force can be described quantitatively by specifying the magnitude and direction of the force acting on a test body placed at any point of the field. An example of such a method of description has been given in Vol. I, p. 181. If we indicate on a drawing the magnitude and direction of the force of gravity on a test mass of 1 gramme, for the various points of space, we obtain a graphical representation of the gravitational field of force. The force exerted upon unit of mass at any point is a measure of the strength of the field at that point. It is called the *intensity*, or *field strength*, of gravity at the point; we denote it by $\mathbf{F}$. The force $\mathbf{P}$ on an arbitrary mass, i.e. the *weight* of the mass, is obtained by multiplying the field strength by the mass, so that $\mathbf{P} = m\mathbf{F}$; for according to experiment,[*] weight is exactly proportional to mass (and also to "quantity of matter").

In exactly the same way, with every point of the region round an electrified body we can associate a vector, the *electrical intensity*, or *electric field strength* $\mathbf{E}$, this vector representing in magnitude and direction the force acting there upon a body electrified in a definite way, and chosen as unit.

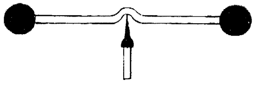

Fig. 5.—Electrical pointer

**Determination of the Direction of the Force.**—Just as the field of force due to gravity can be explored by means of a plumb line, or the velocity field of a moving liquid can be made visible by particles strewn over its surface (Vol. I, p. 364), so the electrical field can be traced out by means of an *electrical pointer*,[†] which can be set up as follows.

A fine quartz rod, supported (fig. 5) so that it can turn on a needle point, carries at its ends two cork balls covered with metal foil. If the two balls are oppositely charged to approximately equal strengths, they can be used to explore the horizontal field in the neighbourhood of a fairly large electrified body, since the pointer, wherever it is placed, will always point in the direction of the horizontal component of the force. To determine the directions in a vertical plane an exactly similar pointer is used, but in this case it is arranged so as to turn easily about a horizontal axis. If the direction of the horizontal component is determined first by means of the pointer set horizontally, the direction of the force itself will be indicated by the direction in which the pointer sets itself in the vertical plane determined by the first observation.

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[*] Cf. Vol. I, p. 194 (experiments by Eötvös).

[†] The method dates back to GILBERT (see p. 1).

**Determination of the Magnitude of the Force.**—A ball of elder pith, covered with aluminium foil and suspended by a fine thread, as in fig. 2, p. 2, is charged positively (say), and placed at various points of the field. The little pendulum is deflected from the vertical by the action of the field. We may suppose the force due to the field to be resolved into two components, one horizontal (P), the other along the line of the thread. We determine P by taking moments about the point of suspension H. When the direction of the force itself has been determined as above, its magnitude can then be calculated. If W (fig. 6) is the weight of the ball, $\delta$ the deflection, and $l$ the length of the thread, then P, the horizontal component of the force, is W $\tan \alpha$; also, for small deflections $\tan \alpha = \alpha = \delta/l$; hence $P = W\delta/l$. The value of $\delta$ can either be read off a protractor (fig. 7) or, better, it can be found by shadow projection as explained on p. 15.

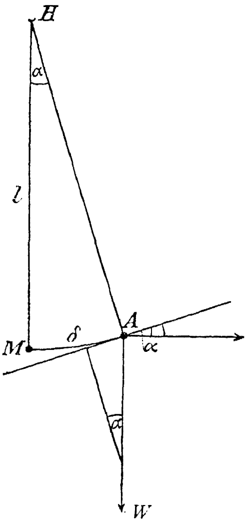


Fig. 6.—Forces acting upon an electrical pendulum

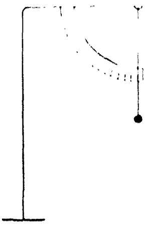


Fig. 7.—Electric pendulum

**Relation between Force and Field Strength.**—When we investigate the electric field in this way, we find the following results.

(*a*) The force on the test body turns out in general to vary in magnitude and direction from point to point of the field; the field strength assigned to a point must therefore vary as the point varies. In order to make a quantitative description attainable, we lay down the following definition.

***The intensity of the electric field, or field strength, E, at a point is taken as proportional to the force which acts on a test body charged in a definitely assigned way and placed at the point.***

**E** is a vector (Vol. I, p. 39) having both magnitude and direction.

(*b*) Further experiments show the following results. The force at any given point changes, when the electrical treatment of the little testing body is varied. If, for example, before bringing the charged ball into the field to be measured, we touch it with an *uncharged* ball of equal size and then remove the latter to a distance, we find that the force on the original ball at a given point of the field is now only half as great as it was before the contact. We find further that the magnitude of the force is reduced to half its former value at *all* points of the field, but that its direction is everywhere the same as before.

By collating the results of (*a*) and (*b*), we can therefore express the force on a test body by the equation

$$\mathbf{P} = Q\mathbf{E},$$

or: **the force is proportional to the field strength.**

With regard to the numerical values determined by the choice of units see pp. 50 and 66.

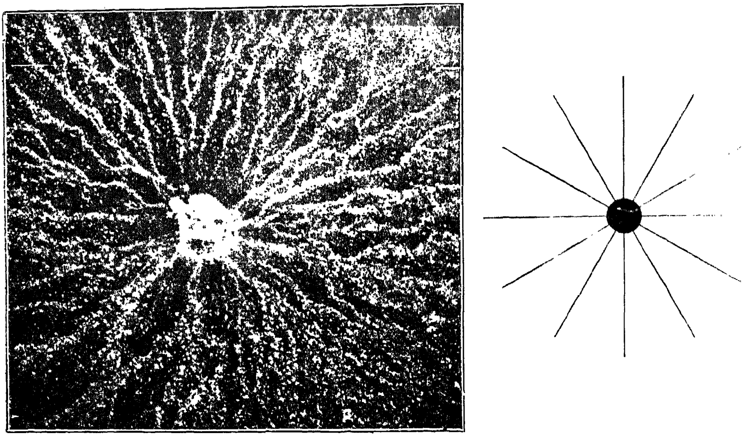
**The Charge.**—In order to obtain a clear idea of the significance of the factor of proportionality Q, we must understand definitely what has happened to the proof ball to make the forces in the second experiment—that is, after the contact with the uncharged ball—only half what they were originally. The ball which was touched by the charged ball, though previously uncharged, is now also found to be electrified. If it is now used in place of the original testing ball —the necessary precautions as to very good insulation being observed —it is found that the same forces act on it at a given place as on the original ball, after the latter has been touched. The two balls are therefore in the same electrical state. The cause of the diminished force is therefore the diminution of the electrical substance, half of which has been transferred to the other ball.

The result of the experiment described under (*b*) above compels us to regard the magnitude of the force as proportional to the " quantity of electricity " communicated to the test ball. The proportionality factor Q has therefore the significance of a " **quantity of electricity** " or " **amount of charge** ", or simply " **charge** ".

We see now that the reason why the forces on the ball become halved is that the charge on the ball has been halved. The experiment shows also that we are justified in speaking of quantities of electrical *substance* or charge, for the total quantity clearly remains unchanged after the contact. A rigorous proof that this is so is given on p. 53.

**Field Lines.**—If we proceed from point to point always in the direction of the force, and therefore of the field strength, at each point as it is reached, and then join the successive points by a line, we obtain a *field line*. We can imagine the whole region of space we are considering to be filled with such lines. They possess the property of never cutting each other, except, perhaps, at very special points, where the force is zero. This signifies that the direction of the force at each point of the field is uniquely determined, and can be obtained by drawing a tangent to the field line at the point in question.

Hence, if we also specify in some way the magnitude of the field strength at every point, we have a representation of the electrical state of the ether by means of the *vector field* of the field strengths.



a Fig. 8.—Field of a charged sphere

We shall definitely stipulate that a + charge is to be used as the test charge. *As a consequence of this convention, the field lines have a direction (or sense) pointing away from the + charges and towards the — charges.*

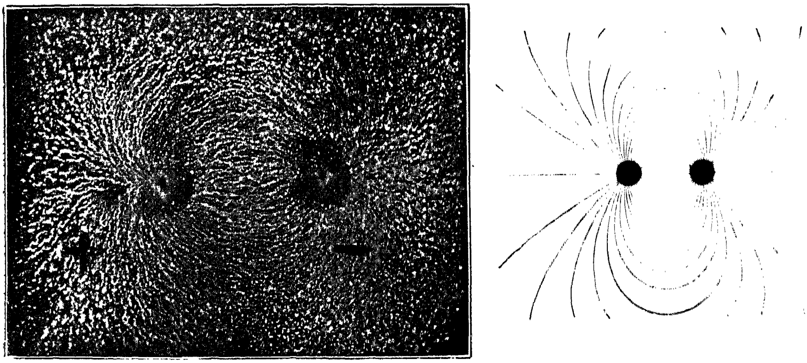


Fig. 9.—Field of two spheres with equal and opposite charges

When we investigate in this way the field of force surrounding charges distributed in any manner in space devoid of matter, and depict the result of the investigation graphically by means of field lines, we recognize as a fundamental property that these lines never end in empty space, but always go on until they reach an electric charge. Instead of following up the force from point to point, we may make the field lines visible in another way, by means of particles of powdered gypsum, freshly prepared from gypsum crystals; powdered rutile may be used in the same way, and crystals of quinine in vaseline oil are also suitable.

The illustrations (figs. 8–13) show the course of the field lines between various electrified bodies. They are obtained as follows. Sections of the bodies to be represented are cut out in tinfoil and stuck on a glass plate, and the plate is strewn with gypsum powder (see above). The tinfoil parts are charged, and the gypsum crystals, when the plate is tapped, take up the directions shown in the figures in the field round the various electrified parts. The corresponding complete figures in three dimensions are easily visualized. Alongside the figures so formed are the corresponding figures obtained by exact measurement with the help of an electric pointer, in the space around a similar arrangement on a sufficiently large scale. We see that the representations of the field lines obtained by means of the powder are only rough approximations. It must be specially emphasized that in the exact figures there is *no* crossing, and *no* running of the lines into one another. The branching which appears in the powder diagrams is caused by disturbing discharges.

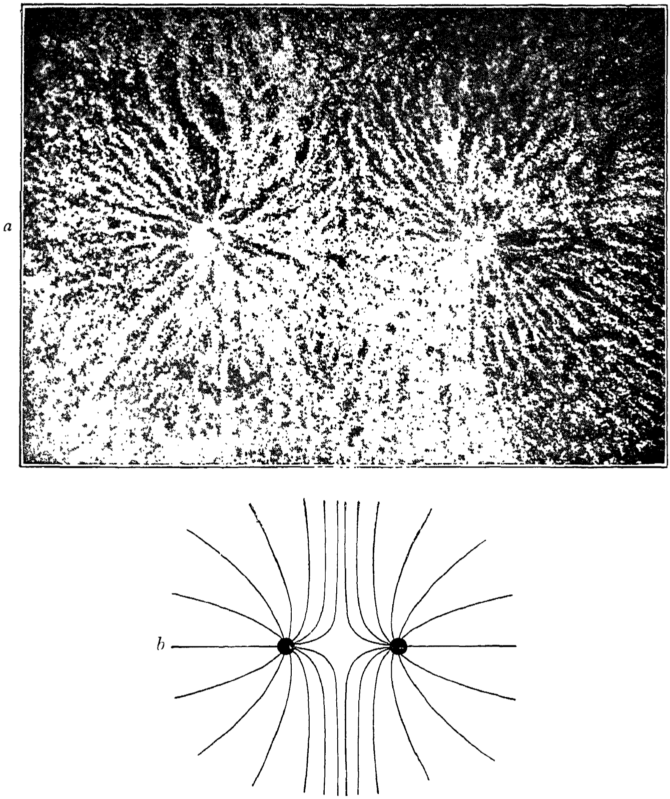


Fig. 10.—Field of two spheres with equal and like charges

Fig. 8 shows the plane field round a charged disc. It corresponds to a section through the field of a charged sphere which is at a great distance from other

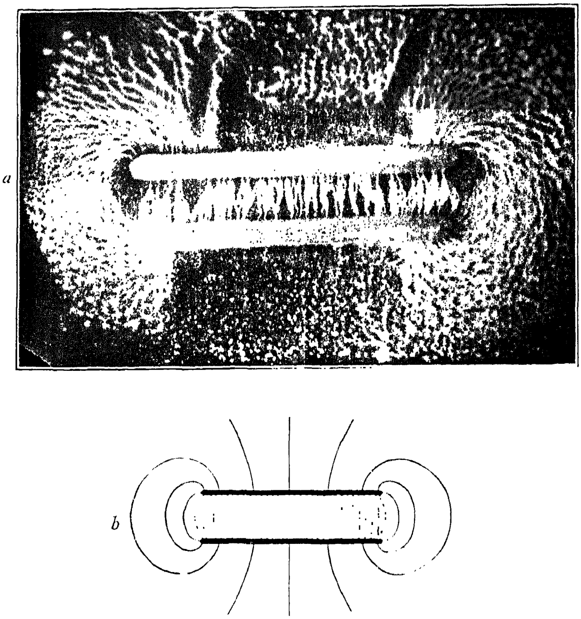


Fig. 11.—Field between two parallel planes (plate condenser)

bodies. The way in which the field lines run is exactly the same for a positively as for a negatively charged sphere; but the force on one and the same test charge has opposite directions in the two cases. Figs. 9 and 10 show the general course of the electric field of two small spheres carrying equal charges, these charges being unlike in fig. 9 and like in fig. 10. Fig. 11 shows the particularly important case of the field between two parallel strips of metal or, extended to three dimensions, between two parallel planes. In this case the field lines are all parallel, except near the edge. The nearer the two planes are to one another, the less the field lines bulge out at the edge, and the greater therefore is that part of the field in which the lines are straight and parallel. Fig. 12 shows an arrangement by means of which two insulated metal plates can be placed facing each other at any desired distance apart. An arrangement in which two parallel plates are set facing each other is called a **plate condenser.** Fig. 13 depicts the field lines when a charged body is present in the field of a plate condenser. By the introduction of the charge the field has become distorted.

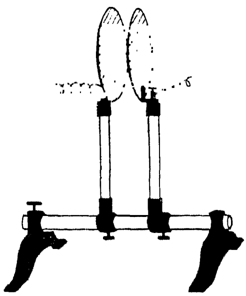


Fig. 12
Variable plate condenser

**Graphical Representation of the Numerical Value of the Field Strength.**—The illustrations of field lines given above, while they certainly indicate the *direction* of the intensity at every point of the field, do not, so far as we have yet explained, supply any means of determining the *magnitude* of the intensity at a given point. To attain this object, we shall make the convention that, where the intensity is greater, the field lines are to be drawn closer together, and conversely. By *density of the field lines* we shall understand the number of lines which pass through unit of area of a surface placed at right angles to the lines. Hence in this graphical method the value of the field strength (intensity) at any point of the field is represented by the number of field lines which cross unit of area of a surface drawn through the point perpendicular to the lines. In this way we characterize the continuous distribution of intensity by a discontinuous distribution of lines, i.e. by a finite number of lines drawn in the field. Clearer apprehension of the method—exemplified in figs. 8 to 13—will be promoted by the following discussion of a phenomenon which is more readily visualized, the streaming motion of a liquid.

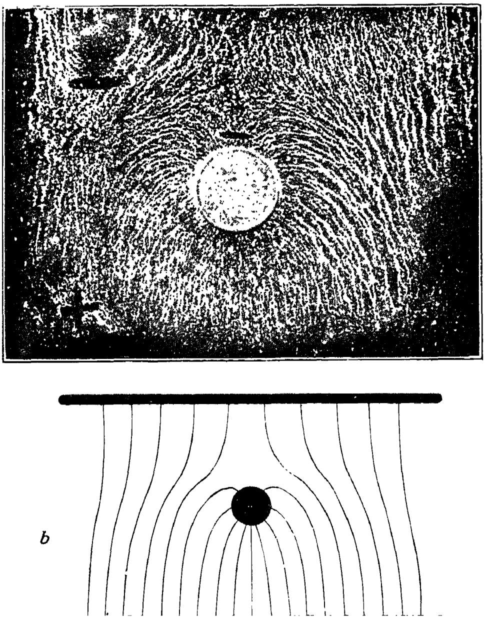


Fig. 13.—Charged body in a homogeneous field

**Flux of a Vector.**—Think of a tube through which a liquid is flowing (fig. 14). For simplicity we assume that the velocity of the liquid is uniform over the cross-section. The velocity of flow is characterized by the velocity vector (Vol. I, p. 384, Representation of Stream Lines). Since the walls of the tube are impenetrable and the water is incompressible, and since further we assume that there are no sources or sinks in the tube, it follows that exactly as much water flows in per second across section 1 as flows out across section 2. The quantity

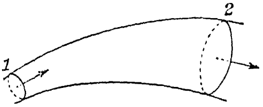


Fig. 14.—Liquid flow in a tube

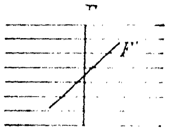


Fig. 15.—Flux through surface different inclination

of water which per unit time crosses a section drawn perpendicular to the velocity, is found by multiplying the velocity of the water by the area of the section. If the section F′ is not perpendicular to the velocity, then, as is shown by fig. 15, we have to take the area of the projection of the section upon a plane F perpendicular to the velocity. At all parts of the tube the product of the velocity and the cross-section perpendicular to it is constant.

The product of any vector by an area at right angles to it is called the *flux* of the vector across or through the area. The name is derived from the foregoing hydrodynamical example, in which the flux of the velocity vector is equivalent to the quantity of water flowing across the area in question per unit time. If we characterize the flow by stream-lines as explained above, then the number of these lines is constant at all sections of the tube. Thus the density of the stream lines does actually give the value of the velocity vector at any section.

In exactly the same way we can divide up the vector field of a field strength into tubes (*vector-tubes*), the walls of which are formed by lines of the field, so that no lines can enter or leave the tube through the walls. We assume that the space we are considering is free from charges, and therefore contains no sources of field lines. The flux $\mathbf{N}$ of the field strength in such a tube is therefore constant. Accordingly, the amount of the field strength can be characterized in this case also by the density of the field lines. In fact, for any section A of the tube we have $\mathbf{N} = \mathbf{E}A \cos\theta$, where $\theta$ is the angle between the plane of the section and that of a normal (orthogonal) section. Hence, for two normal sections we have $|\mathbf{E}_1| A_1 = |\mathbf{E}_2| A_2$, or $|\mathbf{E}_1| : |\mathbf{E}_2| = A_2/A_1$; i.e. the field strength at any point of the tube is inversely proportional to the area of the normal section at the point. We can now suppose the whole field to be divided up into tubes (called *unit tubes*), in such a way that unit flux ($|\mathbf{N}| = 1$) passes through each of them. A narrowing of the tube corresponds therefore to a greater field strength at the place in question, a widening to a smaller field strength. The field lines are therefore to be regarded as symbolizing the unit tubes, in the sense that each tube is replaced by a central line within it.

**Field Strength round a Point Charge. Coulomb's Law.**—For the purpose of tracing out the connexion between field strength and field line density in a particular case, we shall consider the field of a point charge, that is, a charged sphere which is very small in comparison with the dimensions of the field we are observing. From symmetry we see at once that the field is spherically symmetrical, as is also shown by fig. 8 (p. 10). The field strengths at various distances are measured with the help of an electrical pendulum of the type, for example, shown in fig. 16.

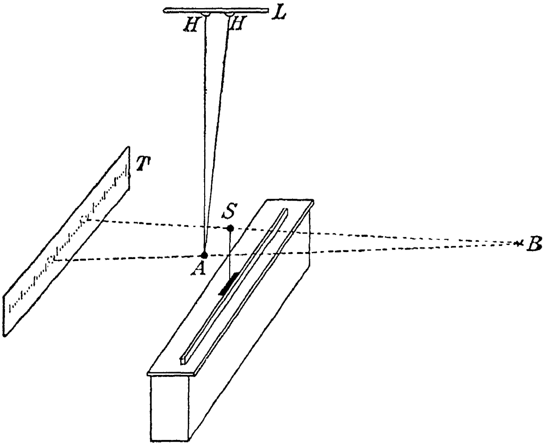


Fig. 16.—Measurement of the field strength near a charged sphere

A small proof sphere A covered with metal foil hangs from the ceiling of the room upon a bifilar suspension consisting of two fine silk fibres 2 metres in length. A little ball S of the same size as A is set up on a thin insulating holder at such a height above the table that S and A are at the same level. The foot of the holder is movable along a straight guiding strip of wood on which a scale is marked. The zero point of the scale is so placed that, when the foot of the stand is at the zero point, the ball on the stand is exactly at the place which the suspended ball occupies if hanging freely.

If a charge of arbitrary amount is communicated to the ball S, while it is in contact with A, the latter is repelled and takes up a new position, which changes when S is displaced along the guiding strip. The distance at which the balls rest under the repulsion is SA. But, in fig. 6 (p. 8), the magnitude of the repelling force can be determined from the deflection MA. The length SA (= MA) can be measured either immediately by the scale marked on the guiding strip, or, as in fig. 16, by the shadows of the balls thrown on a scale T when they are strongly illuminated by a distant point source of light B. The force can then be calculated by the method given on p. 8.

If we consider only such distances SA as are great relative to the dimensions of the balls and, working with a fixed charge, examine how the force depends on the distance, we find the result $P = k/r^2$; where the proportionality factor $k$ depends on the electrification of the balls. This factor $k$ can be further split up by writing P in the form $P = QE$ (p. 9). If $Q_1$ is the charge on the hanging ball, we may put $k = Q_1k'$, and then $P = Q_1k'/r^2$.

But we can just as well consider the ball on the fixed stand as the proof ball; and by the principle of action and reaction the force on it is equal and opposite to the force on the other ball (as we might prove at once by allowing the fixed ball to hang freely). Hence the factor $k'$ must be proportional to $Q_2$, the charge on the fixed ball. We therefore have the result: *the mutual force exerted by one point charge on another in empty space* (*p.* 2) *is proportional to the product of the charges, and inversely proportional to the square of the distance between them*. This is called **Coulomb's law.***

It has been shown that the value of P is proportional to $Q_1$ and $Q_2$, and inversely proportional to $r^2$. The actual value which is obtained for P depends on the system of measurement we employ (pp. 81, 98).

**Deductions from Coulomb's Law.**—The following important results follow from Coulomb's law.

1. *The field strength at a point in the region surrounding a charge is proportional to the charge by which it is caused.*
2. *The field strength at a point is proportional to the density of the field lines* (p. 9) *at the point, assuming that all the lines originate at the charge.*

For, since all the field lines† start from the charge, there being by hypothesis no other charge present, the number of lines passing through any surface surrounding the charge must be constant. But the distribution of the lines is spherically symmetrical. Hence the density of the lines must vary inversely as the square of the distance, since, if the distance is doubled, for example, the area of the spherical surface through which the lines—constant in number—have to go becomes four times as great. Generally, if N is the total number of the lines, and $r_1$, $r_2$ are two distances from the charge, then the two corresponding densities of the lines (i.e. number of lines ÷ area which they cross at right angles) are $N/4\pi r_1^2$ and $N/4\pi r_2^2$; the two densities of lines are therefore inversely as the squares of the distances, exactly like the forces or the field strengths.

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* This relation, discovered by COULOMB (Vol. I) in 1785, played a great part in the development of the theory of electricity, on account of its analogy to Newton's law of gravitation. It was the first quantitative relation found in electrical science.

† As explained above, the field lines are to be regarded as representations of the unit tubes.

The density of the field lines is therefore actually a suitable measure of the field strength (i.e. intensity).

The first of the two deductions stated above shows further that we have to take the field line density proportional to the point charge. Thus: *a given charge is the starting-point of a definite number of field lines.*

These statements are only intended to give a preliminary general idea of the facts. A thorough quantitative discussion of the relations involved will be given later (§ 11, p. 65).

**Summary.**—The pictorial representation of the statical electric field is therefore based on the principle that the number of field lines shown in the picture as issuing from any positive charge (their *source*) is proportional to the magnitude of that charge. Every line ends on a negative charge. The negative charges act therefore as *sinks* for the lines. The field lines show at every point of the field the direction of the force on a positive charge; their density indicates the magnitude of the field strength at the point in question.

This magnitude, it is true, we cannot yet define numerically, since we have not yet defined the unit of field strength. If, however, we suppose the whole field to be surveyed by means of one and the same testing charge, the field strength everywhere becomes perfectly definite, and with the help of a proportionality factor can be reduced at any time to the unit eventually chosen.

For a field of definite type, the force at a given point will increase in proportion to the charge by which the field is excited, and so also therefore will the density of the field lines.

**Diagram of the Field Lines in a Plate Condenser.**—It has already been proved (p. 12) that the field lines between the faces of a plate condenser (except very near the edges) are parallel straight lines. If we investigate the field between the plates by means of an electric pendulum, exactly in the way described above, we find the field strength to be everywhere the same (again leaving out the edges). From this it follows that the density of the field lines between the faces is everywhere constant (except at the edges). The field between the faces, having everywhere the same magnitude and direction, is therefore **homogeneous** (Vol. I, p. 191).

**Hydrodynamical Analogy.**—As already mentioned on p. 14, the representation of the current field of a moving incompressible fluid by means of the stream lines was discussed in Vol. I, pp. 375, 383. That discussion is exactly—even quantitatively—analogous to the foregoing treatment of the electric field, as will frequently appear in later pages.

In this analogy, the electrical field strength corresponds to the velocity of the liquid. In motion under a velocity potential (Vol. I, p. 375), a stream line, on account of the incompressibility of the liquid, can only begin or end at a source or sink. The following method (due to Hele-Shaw) makes the stream illuminated by a distant point source of light B. The force can then be calculated by the method given on p. 8.

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## 3. Electrostatic Induction. Lines of Force

We go on now to consider a second property which may be used to describe the electric field. An electric charge calls forth in its neighbourhood a certain condition of space—the electrical state—which we recognize by the forces which act on objects in the neighbourhood and which we can measure by the field strength. But the electrical state of the ether in the region surrounding the charge is accessible to observation in another way also.

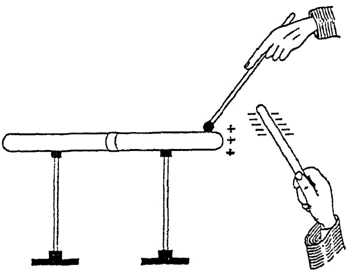

Fig. 17.—Electrostatic induction in the field of a charged vulcanite rod

If a metallic body is brought up near a charge, say an electrified vulcanite rod (as in fig. 17, for example), then the conductor becomes electrified and continues in this state so long as it remains in the field of the charged vulcanite. To prove this, we touch the conducting body with a proof sphere of metal, held by an insulating handle. The sphere, after the contact, turns out to be charged, as may be shown by its action on a sensitive electric pendulum.

The following phenomena are observed. The charge on the conductor, on the side next the vulcanite, is positive, i.e. of opposite sign to the charge on the vulcanite; on the remote side the charge is negative, i.e. of the same sign as the charge on the vulcanite. In the intermediate region there is a zone where no sensible charge is found. When the conductor is removed to a distance, out of the field, it is found to be uncharged as at the beginning.

The phenomenon is called *electrostatic induction*, or, when there is no danger of ambiguity, simply *induction*;* and the conductor is said to be *charged by induction*. The facts may also be stated in the following form.

*The charged vulcanite induces on the side of the insulated conductor next to it an unlike charge, and on the side remote from it a like charge.*

Keeping the conductor in the field, we now make an important experiment. We separate the adjacent and the remote parts from one another, without disturbing the insulation; and we then remove the two parts out of the field, and test them for electrification. They are found to be equally and oppositely charged.

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* The name "*influence*" is also used.

We shall next investigate how the charges produced by induction on the conductor depend on its position and distance from the influencing charge. For this purpose we employ a double plate of the form shown in fig. 18.

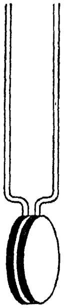

Fig. 18
Double plate for investigating electrostatic induction.

It consists of two small plates of the same size, fastened to long handles of good insulating material. The plates, while touching each other internally, are brought up to the part of the field we wish to investigate, set in any desired direction, and then separated. The charges on the two halves are examined outside the field by means of a sensitive electrical pendulum. This can be done more or less quantitatively with the help of the piece of apparatus already mentioned and shown in fig. 16. The charge used to excite the field is a well insulated, strongly charged metal ball.

A. We now bring the double disc up to a point of the field, and examine the induced charges obtained when the plates are separated at one and the same point in the field, but with different orientations of their plane. It is found that the direction of the plates at the moment of separation is a very important circumstance. The greatest charge is obtained when the plates are set at right angles to the direction of the field. If the plates are oblique to this direction, the charge is smaller; if they are parallel to it, there is no charge.

B. We bring the double plate up to various points in the field, setting the plane of the plates perpendicular to the field lines at each of the points; or, what comes to the same thing, setting the plane so that the induced charge obtained is as great as possible. It is found that where the field strength, as determined by our earlier methods, is smaller, there also the induced charge is smaller in the same proportion, and conversely.

C. We vary the charge on the metal ball by which the field is generated. Just as the field strength at all points of the field is thus altered proportionally, so also is the induced charge altered proportionally everywhere.

These experimental results may be stated in the following way. The experiments A show that the observed induced charge can be regarded as the flux of a vector through the double plate (p. 14). The direction of this vector, in the experiments in space devoid of matter, is the same as the direction of the field strength at the point considered. The experiments B and C show that the magnitude of this vector is proportional to that of the field strength.

**This vector is called the displacement or electrostatic induction. It is denoted by the symbol D.**

The magnitude of the vector **D** is measured by the method already explained for the field strength, viz. by the flux through a unit of area set at right angles to the direction of the vector. Hence the *displacement is measured* by the charge induced on an area of 1 sq. cm. set at right angles to its direction.

The preceding experiments prove that *the electrical intensity* (*or field strength*) *is proportional to the displacement.*

We can therefore write

$$\mathbf{D} = K_0\mathbf{E},$$

where $K_0$ is a factor of proportionality, the value of which depends on the units employed (p. 66).

The vector field of the displacement can be symbolized by lines, in the same way as the field of the intensity. The remarks made at p. 13 apply here also.

**The lines which characterize the vector field of the displacement are called the lines of electrical force.**

They are not identical with the field lines, since the measure of the displacement is different from that of the intensity; but the *form* of the lines of force, in empty space, is identical with the form of the field lines. We can therefore regard the picture of the one field as also representing the other field to a certain scale.

From diagrams of field lines the following theorem can be seen to be true.

*The lines of force in an electrostatic field begin and end at charges.*

In an electrostatic field a line of force can always be continually produced until it reaches a charge.

It is important to understand clearly a remarkable view which connects the electrical state with a certain excited condition of the ether, a view which was originated and elaborated by the genius of Faraday, but was misunderstood by his contemporaries. According to Faraday's ideas, at any point of space in the neighbourhood of an electric charge not only is there a hypothetical field strength, which can be determined by bringing a proof charge up to the point, but, even when there is no proof charge there, the ether is in a condition of electrical excitation due to the main charge.

The state of excitation of the ether is in Faraday's view something real. The charges are accordingly not mere centres of force in the same sense as, for example, the masses referred to in Newton's law of gravitation but, as Faraday believed, they are the originating centres of changes of state in the surrounding medium. On this theory, the view which regards the charges as the ends of lines of force receives a definite interpretation.

From the above explanations we see that the electric field can be represented in two ways, by means of either the field strength or the displacement. This appears at first sight to be a needless complication.

The advantage of this duplicate possibility in the picturing of an electric field will come out clearly when we reach the discussion of fields occupied by matter (§ 15, p. 82).

**Mechanical Analogy.**—Similar relations exist in an elastic medium which is in a state of deformation (or strain) under any applied forces. The strains in the interior of a medium (in an analogous way to the electric displacement) show their presence, with the help of some indicating mechanism, by forces of tension and pressure across sections through the medium. Just as we can represent the elastic state of a medium by means of either stresses or strains, so we can represent the electrical state of the ether either by the intensity or by the displacement. In an elastic medium strain and stress are connected by Hooke's law (Vol. I, p. 197), which states that the two are proportional to one another; this law may therefore be said to correspond to the relation given above, $\mathbf{D} = K_0\mathbf{E}$.

**Lines of Force and Stress in the Medium.**—It is possible, in the light of Faraday's ideas, to regard the lines of force as more than mere symbols of unit tubes of flux of the vector "displacement" (tubes chosen arbitrarily, i.e. according to the system of measurement employed). We can, in fact, regard the lines of force as having a real existence, in much the same sense as a vortex filament or a vortex ring (Vol. I, p. 381) has a real existence as a particular state of space. Although this view departs from the strict conception of a continuum, it is nevertheless of very great value as an aid to the imagination, and we shall go in some detail into its applications.

If we consider the diagram of the lines of force of two oppositely charged spheres, we can easily imagine that the attraction between the two spheres is brought about by a tension in the lines of force, and that therefore—to employ a mechanical analogy—the ether must be in some sense in a state of stress.

Since the charges situated at the two ends of a line of force attract each other, the force in the direction of the lines must be a tension. Clearly, however, there must be some mutual action between the lines themselves; for otherwise they would necessarily, like stretched rubber bands, always follow a straight path between the charges, so as to be as short as possible. A glance at fig. 9 (p. 10) shows, however, that this is not the case, the lines in this example bulging outwards in the middle. There is therefore a transverse force of repulsion between the individual lines of force; *the lines behave like stretched elastic threads which mutually repel each other.*

**In the electric field a tension exists in the direction of the lines of force, a pressure in all directions perpendicular to them.**

Since the charges are bound, as the preceding experiments show, to the bodies which carry them, the forces which are observed are forces between material bodies, e.g. between the glass rods and the sticks of sealing-wax in the examples given above. Fig. 13 (p. 13) shows very clearly how the force arises. The figure shows the force to be composed of two parts. The charged body's own field strengthens the homogeneous field on the side next the positive plate, but weakens it on the other side. The charged body is on the one side pulled towards the positive plate by the *tension* of the lines of force, and on the other side pushed towards it by their *pressure.*

## 4. Persistency of the Electric Field

**Insulators and Conductors.**—We can now consider a little more fully the difference (already mentioned at p. 4) between insulators and conductors. The question to be examined may be introduced in this way. The experiments with which we began show that bodies which are electrically charged obey the forces of tension or pressure in the lines of force (§ 3, p. 21). We now ask: can the electrical substance move in obedience to the tension of the lines independently of the body with which it is associated?

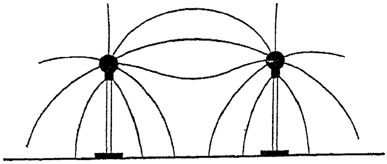

Fig. 19.—Diagram of the form of the lines of force between two oppositely charged balls upon insulating stands.

To settle this question, we proceed as follows. We set up at a suitable distance from one another two metal balls, attached to glass or, better, amber handles (fig. 19). We give the two balls charges of opposite sign, say by touching the one ball with a rubbed glass rod, the other with the material used to rub it; the field thus produced is approximately represented by the lines shown in fig. 19. This state persists without change for a long time. It follows that the charges cannot obey the forces along the lines. We now repeat the experiment, after we have connected the two balls by a metal wire. We can then prove that, after the connexion has been made, either the two balls are both *uncharged* or the charges they carry are of the *same sign*.

This result can only mean that the two charges of opposite sign have yielded to the tension of the lines of force, and have united with each other. If the positive charge was exactly equal to the negative one, the whole system is now free from electrification; if one of the charges was in excess, the excess charge is now distributed over the system, so that the two balls are similarly charged. If we connect the two balls by a silk thread, no equalization of the charges takes place. If we use straw (a *semi-conductor*) the equalization takes a long time. Conductors are therefore characterized by the property that charges can move freely in them, in obedience to the tension in the lines of force; in semi-conductors this only occurs to a certain extent, in insulators scarcely at all.*

The following is a list of solids classified according to their conducting power.† Liquids and gases will be dealt with later (p. 23).

*Conductors.*—All metals, charcoal, graphite, metallic lustrous minerals such as iron glance and lead glance.

*Semi-conductors.*—Glass, most stony minerals, linen, leather, straw, wood.

*Insulators.*—Quartz, mica, resins, silk, paraffin, vulcanite, bakelite.

The best insulators for fine electrical apparatus are amber and quartz. Strictly speaking, there are no perfect insulators in which a charge can be maintained indefinitely, except a vacuum, that is, pure ether. The perfect insulator is therefore an abstraction, like an absolutely rigid body.

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* Here the question is left open, whether charges of *both signs* are freely movable. It is known, however, that only *negative* charges are movable without simultaneous motion of matter. See, however, p. 345.

† Quantitative statements are given at p. 223, where conduction is treated more thoroughly. At present the main point is to be able to recognize under what circumstances a statical field is possible at all.

**Earthing.**—A large fraction of the total number of lines of force proceeding from a charged system may terminate in the neighbourhood, as happens, for example, with the system in fig. 19. If therefore, in the experiment described on p. 22 for example, we connect one ball by a wire with the wooden table, then the ball loses its charge, for the electricity flows away through the wire, and becomes united, through the wood of the table and the walls or ground, somewhere in the neighbourhood with the opposite charge. We say that the charge is *conducted to earth,* or that the ball is *earthed.* In the same way, we can in general discharge the ball by touching it with the hand, for clothing and the human body conduct to some extent, and so set up the connexion to earth. To break the earth connexion we may wear dry rubber shoes or stand on some non-conducting material, contact with the hand being then no longer sufficient to cause earthing.

In all electrical experiments, therefore, care must be taken that the bodies to which we wish to communicate electrical charges are insulated from the earth. Also, we can now see why in the experiments described in the introductory sections it was necessary to use silk threads, glass handles, pieces of amber, and so on.

**Conductivity of Liquids and Gases.**—If again in the experiments of p. 22, we connect the balls first by a dry silk thread, and then by a wet one, we find in the first case very good insulation by the thread, but in the second case very good conduction. It follows that water is a conductor, especially, as we shall see later, if it contains salts in solution; but very carefully purified water is a poor conductor. Since in damp air a film of water is formed on the surface of many solid substances, particularly glass, and since such a film conducts fairly well on account of the salts dissolved in it, these substances are not reliable insulators in damp air. For other liquids see p. 248.

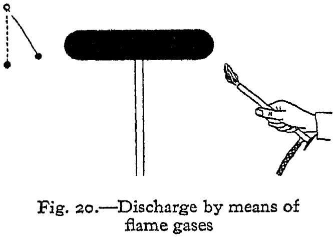
Fig. 20.—Discharge by means of flame gases

Air, and gases generally, are very good insulators. There are conditions, however, in which they become conductors, e.g. very high temperature or very low pressure.

Conductivity resulting from high temperature is easily shown by bringing a charged body into contact with a flame (fig. 20). The body also loses its charge very rapidly if the gases, after leaving the flame, are made to pass along its surface; the gases therefore retain the conductivity, which they have acquired by the high temperature, for some time after they have cooled down. If good insulation is desired in a room, flames must be carefully excluded, for the gases can retain their conductivity for a long time.

**Insulation and Field Strength.**—No insulator maintains its insulating power beyond a certain intensity of field. When the intensity exceeds a definite value, the material can no longer prevent the charge from giving way to the tension of the lines of force; the insulator becomes ruptured, usually with formation of a spark and damage to the material, if that is a solid. Quantitative results are given in an Appendix.

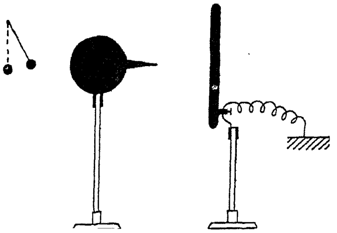

Fig. 21.—Point action

Even air becomes a conductor under the influence of strong electric fields. A ball, even if well insulated, loses its charge very quickly if fitted with a projecting point (fig. 21). The general course of the lines of force for this case is shown in fig. 22. It will be seen that the lines are crowded together near the point, showing that the intensity there is very great. Under the action of this high intensity the charges can get away from the point and yield to the pull of the lines of force (*action of points*). The charges also carry with them the air near the point, so producing an *electric wind*, which can be demonstrated well by means of a flame (fig. 23). To make the phenomenon easily visible, it is necessary to charge the ball continuously, e.g. by the electrical machine described below.

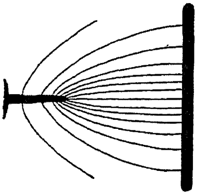

Fig. 22.—Form of the lines of force between a point and plate

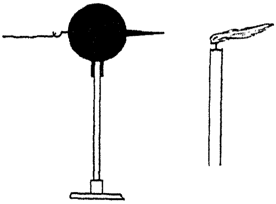

Fig. 23.—Electric wind

When the oppositely charged conductors are brought nearer each other the density of the lines of force increases so much that the air becomes conducting, and finally a spark leaps across. The spark may be regarded as a filament of air of high conducting power, which effects the connexion between the bodies. The value of the rupturing intensity in gases falls rapidly with diminishing pressure. At low pressures air is a bad insulator (p. 298).

*Applications of the Action of Points.*—Point action is probably responsible for a part of the effectiveness of a *lightning conductor*. The air in the neighbourhood of the point, in the presence of a highly charged rain cloud, becomes continuously conducting on account of the great density of the lines of force, so that the discharge of lightning is deflected in the direction of the lightning conductor (p. 351).

Point action is frequently used as a means of withdrawing a charge from charged conductors, without having to touch them, though the discharge is only partial, since with diminishing intensity the point action finally ceases.

One such application of point action is made in the *frictional electrical machine*, an apparatus in which electricity is continuously produced by friction, applied to a rotating insulator.

The *plate electrical machine* (fig. 24) consists of a glass plate S, the rubber R (two leather pads treated with zinc amalgam, pressing against the plate S by means of a spring), a metal ball C (the "conductor"), and the collecting contrivance H. Two silk aprons L are also attached to the rubber, and lie against the glass. They prevent (by the formation of a double layer, p. 106) the spreading of the charge over the glass, which always conducts a little. The collecting device H consists of two wooden or metal rings, carrying a row of fine points on the side facing the plate; by means of these a part of the charge of the glass plate is transferred to the ball C. The rubber and the conductor stand on insulating supports.

When the plate S is turned, it becomes positively charged by the friction of the rubber, and the rubber becomes negatively charged. Negative electricity is collected on the conductor K.

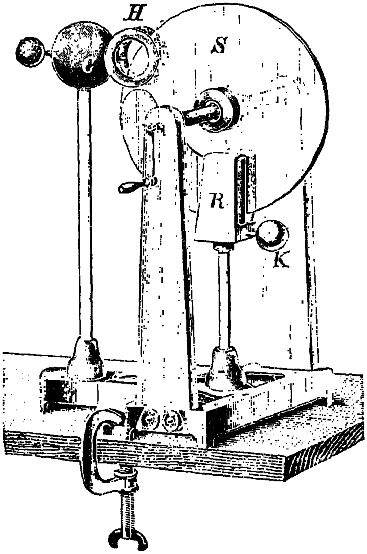

Fig. 24.—Plate electrical machine

**The Intensity in Conductors.**—When a conductor is charged, the lines of force arrange themselves in such a way that they connect the opposing charges by as short a path as is consistent with the equilibrium of tension and pressure in the lines. If for example we charge an insulated metal ball, its lines of force end on the surrounding bodies. The charges on the ball are therefore pulled outwards, and in the state of equilibrium must be situated on the outside surface of the ball. Clearly it makes no difference in this respect if the ball is hollow.

It can be verified that the charges do actually behave in the way which on the ground of their free mobility in conductors we are led to expect.

An insulated metal ball of 5 cm. radius is electrically charged; the lines of force due to the charge end on the surroundings. If we touch the ball at two opposite points simultaneously with two proof spheres of the same size, one of which is solid and the other hollow, and test the electrical state of the two testing spheres in succession by means of the same little electric pendulum, we find exactly the same throw of the instrument in the two cases. It follows, in accordance with the equation given at p. 9, that the solid sphere has taken up the same quantity of electricity as the hollow one. The conclusion is therefore justified that in electrification only the surface of the body matters, and that the charge distributes itself on the surface only.

To investigate the matter further, we employ a hollow metallic body, e.g.

a hollow sphere (fig. 25), in which there is a hole large enough to admit a proof plane or a proof sphere, without touching the edges of the opening. If we charge the hollow metallic body, and then touch it on the outside with the proof sphere, the latter when tested with the electric pendulum is found to be electrified. If, after discharging the test sphere with the finger, we insert it within the hollow body, touch the inside surface with it and then withdraw it without touching the edge of the opening, we find that it is absolutely free from electrification. The same result is found when we investigate the internal surface of any conductor, no matter of what shape, provided the lines of force end on its external surroundings.

*The charge of a metallic conductor therefore resides on its surface.*

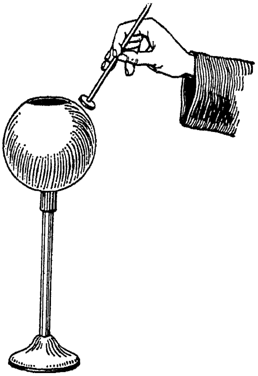

Fig. 25.—Proof that there is no charge on the internal surface of a hollow conductor

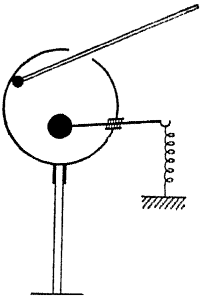

Fig. 26.—Field in the interior of a hollow metal sphere

The opposing charges may of course be so situated that the surface spoken of here is the *inner* surface. This is easily proved as follows. Into a hollow sphere of metal, provided with an opening as in the experiment just described, is introduced a small ball; the latter is insulated from the hollow sphere and connected with the earth (fig. 26). The hollow sphere is then charged. In this case only a portion of the system of lines of force goes from the outside surface to the external surroundings. The greater portion goes from the inner surface to the small ball, which on account of its connexion to earth also belongs to the "surroundings". Fig. 27 shows the lines of force. If we now introduce through the opening a little testing sphere fitted on an insulating handle, taking care that it does not touch either the edge of the opening or the small ball inside, and if we bring it into contact with the inner surface of the hollow sphere and then carefully withdraw it, we shall find, in accordance with our expectation and in contrast to the preceding case, that it is *charged.*

*Faraday's Cage Experiment.*—We see from these experiments that an electric field can never penetrate into a hollow space completely enclosed by a conductor. The lines of force coming from outside always end on the external surface. We can convince ourselves of this by an experiment similar to the one illustrated in fig. 3 (p. 2).

An uncharged electric pendulum is set up on an earthed metallic plate. If we bring a charged body near it, we get a throw of the pendulum. We now set a wire cage (fig. 28) over the pendulum. If we now bring the charged body near, *no* throw follows, provided that the wire netting is sufficiently fine. The field does not penetrate into the interior of the space shut off by a conductor. If the cage is insulated, we can also charge it as strongly as we please, but still no field can be detected inside it.

In 1836 Faraday carried out this experiment on a large scale. A box, covered externally with conducting material, was made of such a size that he could go inside it himself. When the box had been insulated from the earth, he went into it with a sensitive electroscope. By means of powerful discharges, intense sparks were then made to play over the outside of the box. No throw of the electroscope was observed, and Faraday himself felt no effects from the sparking.

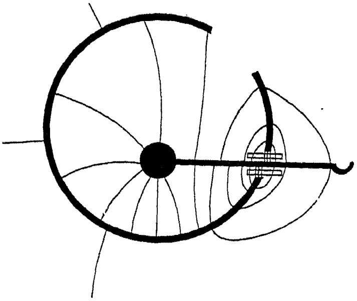

Fig. 27.—Diagram showing the lines of force in fig. 26

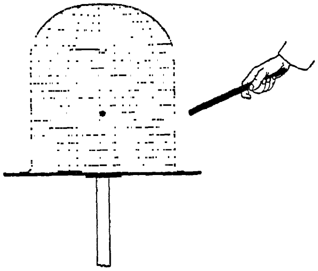

Fig. 28.—Faraday's cage

The final result of the above experiments can be summarized in the statement:

**The electric intensity in the interior of a conductor is zero.**

*Electrostatic Screening Effect.*—These phenomena are made use of in practice for the purpose of protecting sensitive electrical apparatus from the influence of external electrostatic fields. The apparatus is enclosed as completely as possible within a metallic envelope, connected throughout to the earth.

*Form of the Lines of Force at the Surface of a Conductor.*—Since the charges are free to move in a conductor, the lines of force which end on the conductor must do so at right angles to the surface. For any tangential component would cause a displacement of charge, which would continue so long as any tangential component was left. For a rigorous proof of this * see p. 92.

## 5. Electric Pressure

**The Condition of Equilibrium of the Field Lines.**—Up to this point we have characterized the electrical state of space round two opposite charges by the diagram of the field lines. In most cases, however, it is unnecessary to possess so detailed a knowledge of the field as a whole, and the question arises whether there is not some single magnitude by which the field may be characterized more summarily.

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* Frequently there are particular places where this is not the case. The special feature distinguishing these places (usually points or closed curves) is that the direction of the force is indeterminate. See e.g., in fig. 63, p. 56, the great circle at right angles to the direction of the field.

Taking the diagram of field lines (fig. 9, p. 10) for two opposite charges, we remark that in the state of equilibrium under the tension and transverse pressure of the lines of force, the line density, as also therefore the field intensity, is greater in those parts of the field where the field lines between the charges are shorter. The conjecture at once suggests itself that in any given case a simple relation must exist between these two quantities, viz. the density of the field lines and their length. This is in fact true. We can find out what the relation is by the following considerations.

**Work in the Electric Field.**—The work done over an element of path $ds$ is (Vol. I, p. 85) equal to the length of the element multiplied by the projection of the force on the direction of the element. If the magnitude of the moving charge is Q, and the intensity at the place in question **E**, then (p. 9) the force is $\mathbf{P} = \mathrm{Q}\mathbf{E}$; thus **P** has the same direction as **E**. The work done on the charge Q when it moves over $ds$ is therefore

$$d\mathrm{W} = \mathrm{P}\cos(\mathbf{P}, \mathbf{s})\,ds = \mathrm{QE}\cos(\mathbf{E}, \mathbf{s})\,ds.$$

The expression is positive when a positive charge moves in the direction of **E**, so that in this case work is gained. It is negative when work has to be expended. The work gained when a charge moves from a point M to a point N is therefore, since **E** varies in general from one place to another, given by the integral of this expression over the whole path, i.e. we have

$$\mathrm{W} = \mathrm{Q}\int_{\mathrm{M}}^{\mathrm{N}} \mathrm{E}\cos(\mathbf{E}, \mathbf{s})ds.$$

The value of the work depends therefore on the product (at each point) of E and the component of the path in the direction of **E**. This component, viz. $ds\cos(\mathbf{E}, \mathbf{s})$ we shall denote by $ds_{\mathrm{E}}$. The above integral is therefore what has been called (Vol. I, p. 87) the line-integral of the electric intensity.

**The line-integral of the electric intensity between the points M and N may be called the electric pressure or tension * between M and N.**

The total pressure between the two ends of the path is therefore given by

$$\mathrm{U} = \int_{\mathrm{M}}^{\mathrm{N}} \mathrm{E}\cos(\mathbf{E}, \mathbf{s})\,ds = \int_{\mathrm{M}}^{\mathrm{N}} \mathrm{E}\,ds_{\mathrm{E}}.$$

If the path is traversed in the reverse direction, the value of the pressure clearly changes sign. The notion of pressure allows us to express work done in the electric field in a simple way. We have

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* Not analogous to mechanical pressure or tension. An equivalent expression is "potential difference"; see p. 231.

$$W = QU_M^N.$$

*The work done when a charge* Q *moves from one to the other of two points, between which there exists a pressure or potential difference* U, *is equal to the product* QU *of the potential difference and the charge.*

The numerical value of the work done depends on the units adopted for the three quantities W, Q, U.

We therefore write

$$W = fQU,$$

where $f$ is a factor depending on the units used. As to the value of this factor, cf. p. 50.

**In a Homogeneous Field Pressure is Independent of the Path.**—That the pressure between two points in a *homogeneous* field has always the same value, whatever path be taken between the points, may be seen as follows. To determine the pressure between the points A and B (fig. 29), we have to evaluate the line-integral along any curve from A to B. We can approximate to the curve as closely as we please by a staircase-like series of short straight lines, alternately parallel and perpendicular to the field. The little lines perpendicular to the field contribute nothing to the line-integral, since for these $\cos(\mathbf{E}, \mathbf{s}) = 0$. The line-integral for any curve is therefore given by the product of the intensity and the projection AP of this curve on the direction of the field. This projection is the same for all curves from A to B, and the pressure is therefore independent of the path.

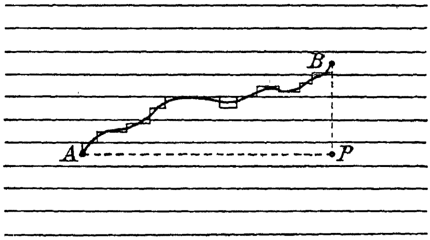

Fig. 29.—Pressure in a homogeneous field

**Work over a Closed Circuit in a Homogeneous Field.**—It follows from what precedes that for any closed curve in a homogeneous field the line-integral is equal to 0; for the integral from B to A is equal and of opposite sign to that from A to B (fig. 30). It therefore follows from the expression for the work obtained above that the total work done, when a charge moves round a closed circuit in a homogeneous field, is equal to 0.

**In a Statical Field the Work over a Closed Circuit is Zero.**—By means of the principle of energy, the result just obtained may be extended to an arbitrary field. If in fact the work over a closed circuit were not zero in a given field, or, in other words, if in the motion of a charge between two points the work gained on the outward journey, for example, were greater than the work lost on the return journey, the field remaining unchanged, then we could gain an indefinite amount of work by taking the charge round the complete circuit, without any change taking place in the field; for after the circuit is traversed the original conditions are exactly restored. This, however, would be a violation of the principle of energy, which denies the possibility of "perpetual motion".

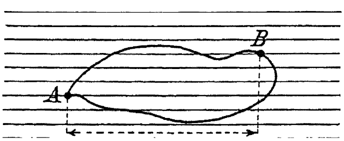

Fig. 30.—Closed path in a homogeneous field

Hence the principle of energy gives the result:

*In a statical field the work over any closed circuit is zero.*

If follows that the line-integral of the intensity round any closed curve, in the statical field, is zero, i.e.

$$\oint \mathrm{E}\, ds_{\mathrm{E}} = 0;$$

where the small circle attached to the sign of integration indicates that the path is closed.

This may also be expressed in the form:

*The pressure in the electrostatic field is independent of the path.*

It follows that $\int_{\mathrm{A}}^{\mathrm{B}} \mathrm{E}\, ds_{\mathrm{E}} = \int_{\mathrm{A}}^{\mathrm{B}} \mathrm{E}_s\, ds$; which justifies our procedure in specifying the electric field between a fixed and a variable point by a single variable number.

If the line-integral round a closed path has a value other than 0, say a positive value, then work is gained when a charge is taken round the circuit. This work can only come from the electric field, i.e. the lines of force must suffer some change in the process, and the energy of the field must diminish. If we take the charge round the circuit again and again, the value of the line-integral must finally fall to zero, since it is impossible to extract an indefinitely great amount of energy from the field. But this value zero corresponds, as we have just seen, to the state of equilibrium.

*The electrostatic field, with its field lines, is therefore so constituted that the energy of the field, subject to the conditions given in the case considered, has a minimum value.*

Once it has been established that the field lines in a plate condenser run parallel to each other, it can be seen from the preceding results, without measurement of the field strength, that the field must be homogeneous. In fact, if this were not the case, that is, if the density of the field lines were variable, we could choose a closed path in the field, round which the line integral of the intensity would not vanish (fig. 31). Such a distribution of the field lines cannot therefore correspond to a state of equilibrium. Fields of this sort will come before us later. They are exactly analogous to those types of fluid motion called laminar (Vol. I, p. 371). Such a field would therefore be a *vortex field* (see below).

The introduction of the idea of pressure as the line-integral of the electric intensity is of very great advantage in the treatment of all electrical problems, for in this way a fundamental property of the field lines is expressed in a quantitative form which is very easy to apply.

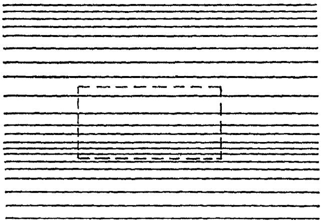

Fig. 31.—Field without definable pressure (vortex field)

The notion of electric pressure also derives special importance from the practical considerations (1) that measurements in the electrostatic field are almost exclusively—on account of their great convenience—measurements of pressure (p. 33), and (2) that pressures can easily be quantitatively reproduced (p. 44).

**The Electrostatic Field is Irrotational.**—From the condition of equilibrium defined by means of the notion of pressure we can deduce another fundamental property of the electrostatic field, viz. that in equilibrium no field line (and hence no line of force) can be closed, or return into itself. In fact, if such a line existed in the field considered, we could take a closed circuit round which the line-integral of the intensity would not vanish (cf. e.g. Vol. I, p. 379). Such an arrangement is therefore inconsistent with a state of equilibrium. A field line, or line of force, which is closed is called a " vortex ", from its analogy to a closed stream line in vortex motion of a fluid (Vol. I, p. 381). We can therefore say:

*The electrostatic field is irrotational, i.e. free from vortices. Its field lines begin at positive charges and end at negative charges. In the state of equilibrium closed field lines do not occur.*

Fields of force also exist, as has just been mentioned (p. 30), in which no closed lines of force occur, and which nevertheless are vortex fields; the case when closed lines of force occur is only a special case of a vortex field. The relations involved have already been fully discussed under fluid motion (Vol. I, p. 377). The essential condition that a field should be irrotational is that at any part of the field the line-integral round a small closed curve must vanish.

**Intensity and Pressure.**—If we take the notion of pressure as our starting-point, we can regard the *pressure* between two points of a field line, whose distance from one another is equal to the unit of length, as giving the mean *field strength,* or *intensity,* between the two points.

We can therefore determine the intensity by measuring the pressure between two points 1 cm. apart and upon a line lying in the direction of the field (cf. p. 65).

Since the pressure is the same for all field lines of the same system, i.e. for all field lines which begin and end at the same points (or on the same conductors), it follows that the *mean* intensity over a field line is inversely proportional to its length.

**Potential.**—To state the value of a pressure means nothing, unless the two points or conductors are specified, the pressure between which is referred to. Many investigations are simplified if for one of these places of reference we take a fixed conductor which is always easily accessible. A very convenient conductor for the purpose is the earth itself (stones, masonry, wood are in fact satisfactory conductors if sufficiently damp).

**The electric pressure at a point relative to the earth is called the electric potential at the point.**

The value of this definition lies in the fact that the pressure between two points can be found at once when their potentials are known.

We can in fact take the line, along which we are to calculate the line-integral from A to B, in such a way that at least one point of it, say E, lies on the earth's surface (the ground). We have then (fig. 32)

$$U_{AB} = U_{AE} + U_{EB}.$$

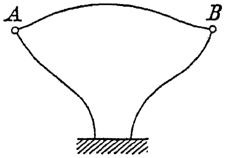

Fig. 32.—Pressure = potential difference

But $U_{AE}$ is the potential $V_A$ at the point A; and $U_{EB}$, with sign changed, is the potential $V_B$ at the point B. Hence

$$U_{AB} = V_A - V_B.$$

*The pressure between two points is equal to the difference of the potentials at the points (or briefly, their potential difference).*

Clearly pressure can only be specified as a potential difference in those cases where it is independent of the path, i.e. where the field is irrotational. It is only in such cases that the idea of potential has any meaning.

The property of being irrotational is also possessed by the gravitational field (Vol. I, p. 182). The property was found as the condition for the existence of a gravitational potential, or the condition that work in the field should be independent of the path, as we know from experience that it is. Starting from this experimental fact, we can introduce into electrostatics also all those analytical methods which are founded on the existence of a potential (cf. e.g. Vol. I, p. 181 et seq.).

**Potential Distribution in Conductors.**—Since there are no lines of force in the interior of a conductor in equilibrium (p. 26); since again one and the same line of force cannot begin and end on the surface of a single conductor (for then the two charges, being free to move in the conductor, would obey the tension of the line of force and at once neutralize each other); and since further the lines of force are at right angles to the surface of the conductor, because the tangential components which would otherwise be present would produce a displacement of the charges along the surface of the conductor; it follows that the line-integral of the electric intensity between two points of a conductor must be nil. Otherwise put:

**Between two points of one and the same conductor there exists in the state of equilibrium no potential difference.**

It also follows that the potential difference between two different charged conductors has a constant value, no matter what points on their surfaces we consider.

**Value of the Potential Difference in Special Cases.**—

(*a*) The field between two parallel planes has been found above (p. 17) to be homogeneous; the intensity has therefore the same value everywhere. Since the distance between the plates is also constant, equal to $s$ say, the line-integral becomes simply

$$U = Es.$$

(*b*) We consider the system formed by two spheres carrying equal and like charges. The arrangement of the field lines is (as is shown by fig. 10, p. 11, and as indeed is to be expected, since the charges are interchangeable) symmetrical with respect to the plane bisecting at right angles the line joining the charges. If we take the line-integral along this joining line we see at once that the parts contributed to it by the two halves of the line are equal but of opposite sign; hence the value of the whole line-integral is 0.

The same is also true for any line joining the spheres; for the value of the line-integral is independent of the path (see p. 30).

(*c*) Another important case is that of two concentric spheres, carrying equal but opposite charges; for the field due to these is very similar to the field due to a charged insulated conductor whose surroundings are at a considerable distance. For one sphere, the field lines are distributed uniformly, and run radially, so that if produced into the interior of the sphere they would pass through the centre. Hence the intensity is the same as if the charge on the sphere were concentrated at its centre, or (p. 16) $E_r = fQ/r^2$. If another conducting sphere is present, outside the first and concentric with it, the field lines are exactly as before. Hence the potential difference between two concentric spheres of radii $r$ and $R$ (*spherical condenser*) is given by

$$U = fQ\int_r^R \frac{dr}{r^2} = fQ\left(\frac{1}{r} - \frac{1}{R}\right).$$

The effect of the term $1/R$ diminishes rapidly as $R$ increases. If $R$ is very great compared to $r$, we can put $1/R = 0$. We see then that the potential of an insulated charged sphere relative to its surroundings is practically independent of the position of the surroundings, provided they are at a sufficient distance from the charged body.

## 6. Measurement of Potential Difference

**Principle of the Method of Measurement.**—To measure a potential difference we require first of all an instrument capable of indicating the presence of this difference and of determining its magnitude. The magnitude or numerical value can, in the first instance, only be specified in terms of an arbitrary unit. Hence, in order that we may be able to repeat a measurement at any time and anywhere, we have next to define a unit, which can be reproduced correctly at any time, and without too great difficulty. We shall first deal with the *measuring instruments*.

The principle underlying the measurement of potential differences rests on the fact that the potential difference between two conductors is the same for all of their points (p. 33). If then the two points A and B, the potential difference between which is to be determined, are put in conducting connexion with two metallic bodies C and D insulated from one another, the potential difference between C and D is the same as that between A and B. The metallic bodies C and D can be set up in such a way that the field between them exerts, on easily movable parts, mechanical forces which are definite and measurable. An instrument of this type is called an **electrometer** or **electrostatic voltmeter.***

The forces exerted on the movable parts by the tension and pressure of the lines of force are balanced by gravity or an elastic force.

**Simple Electrometer.**—In this the tension or pressure of the lines of force, corresponding to the potential difference to be measured in the electrometer, is used directly.

The most primitive instrument, suitable only for rough measurements, is the "gold-leaf electroscope" (fig. 33). It consists of a metal rod, from the lower end of which are suspended two mobile leaves or strips of metal foil (gold or aluminium); these are enclosed in a metal case, in which there are glass windows.

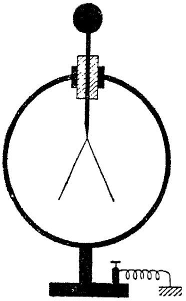

Fig. 33.—Gold-leaf electroscope

The metal rod carrying the leaves is let into the metal case through a stopper made of some insulating material, such as vulcanite or paraffin-wax. If one point of the system to be examined is put in conducting communication with the upper end of the metal rod, and the other point of the system with the metal case, then the potential difference to be measured will now exist between the leaves and the sides of the case. Fig. 34 shows that the leaves are partly pulled apart by the lines of force ending on the sides of the case, and partly pushed apart by the transverse pressure of the lines of force in the space between them.

An improved form of the gold-leaf electroscope is Exner's *electrometer* (fig. 35), which is often used for measurements in the open air. By means of cheeks with a plane surface, which can be moved sideways, and which represent the second pole of the instrument, the field is better defined; and a fixed rod situated between the movable leaves, which makes their deflections symmetrical, also helps towards this end. The deflections are read off with the help of a magnifying glass. For transport, the cheeks are pushed together and thus protect the leaves.

The tension on the leaves can be still further strengthened, as in Grimsehl's form of the instrument (fig. 36), by the insertion, near the movable aluminium

* For the unit of pressure called the *volt* see p. 43.

leaf F, of an arm B, which is in conducting connexion with the casing. When this is done, nearly the whole of the lines of force which issue from the movable leaf F end on this arm; this result is also partly due to the fact that the fixed bar A, attached to the rod CS insulated from the case by the vulcanite E, has a charge of the same sign as F. The movable leaves are therefore attracted towards B. The more favourable distribution of the lines of force makes the electroscope more sensitive.

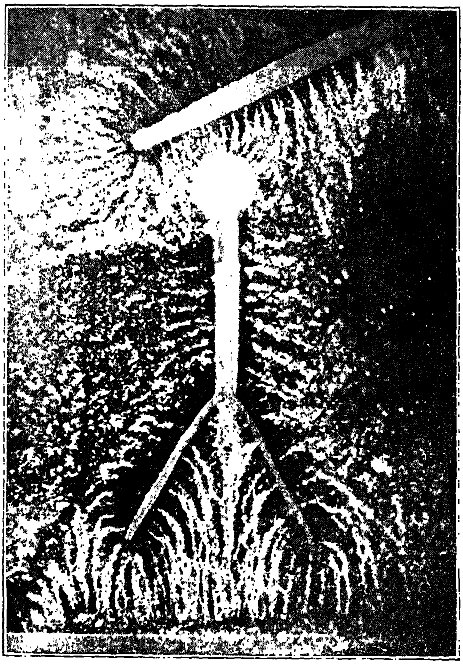

Fig. 34.—Form of the lines of force in a gold-leaf electroscope

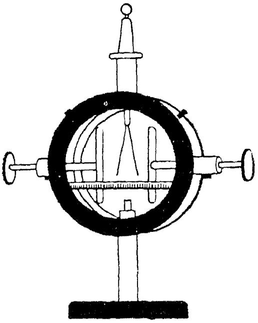

Fig. 35.—Exner's electrometer

For higher potential differences a form of the instrument due to K. F. Braun is often employed (fig. 37). This electrometer consists of a fixed rod, and a double pointer movable in a bearing situated a little above its centre of gravity; the rod and pointer are in conducting connexion. The metal case forms the other pole. The mode of action is in principle the same as that of the instruments already described.

It should be specially noted that the panes of glass, which in most types of electroscope are much too large, should be as far as possible covered by a conducting metal plate, or by tinfoil, connected to the casing; otherwise, because of the charges on the badly conducting glass, the field is not perfectly definite, and the readings obtained may be quite wrong.

A practical form of electrometer constructed on the foregoing lines has the following features. A movable plate in the middle is in conducting connexion with a fixed plate on the left and with one pole of the potential difference to be measured; a fixed plate on the right is connected with the other pole. The motion of the middle plate is communicated to a pointer. A magnetic damping device is provided.

**Electrometer with Auxiliary Field.**—In the preceding types of instrument, to which a large number constructed on similar principles might be added, the potential difference which is to be measured is caused to act between a movable and a fixed part, and the mechanical forces in the field so formed are employed. It is possible, however, to obtain a decided increase of sensitiveness by means of an auxiliary field. For the force on a *given* charge is proportional to the electric intensity; hence, by producing a very great intensity we can considerably increase the force on the moving charged part.

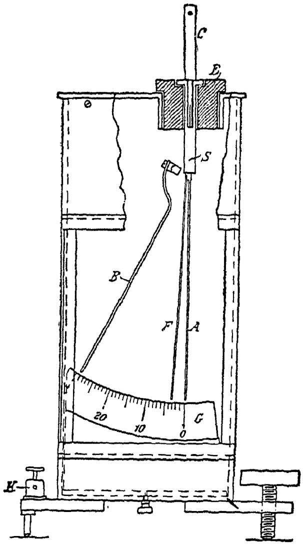

Fig. 36.—Grimsehl's electrometer

Since, further, the force on a movable part in a field of *given* intensity increases in proportion to its charge, we can also proceed by another method; viz. we can insulate the two fixed parts, between which the potential difference exists which we wish to measure, from the movable part, and we can communicate to the latter from outside an independent charge, taken as great as possible.

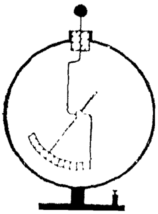

Fig. 37.—Braun electrometer

Almost all the more recent electrometers are of the auxiliary field class. The following are some of the types most commonly used.

One type of construction goes back to a form due to Bohnenberger (fig. 39). The two plane condenser plates, $S_1$ and $S_2$, are set up, insulated from one another, at a distance *d* apart, in a glass case; they are connected with the two poles of a source of electricity, and so charged to the potential difference U. Midway between the two condenser plates hangs the conducting leaf B, which consists of a piece of aluminium foil, or may be replaced by a quartz fibre, silvered or otherwise made conducting. The leaf is suspended by a metal pin let into the case, insulated from it, and terminating at its upper end in a screw terminal K. The pressure to be measured is the same as that between the metal pin on the one hand, and the case on the other. To secure a symmetrical distribution of the lines of force, and to exclude the influence of the surroundings, the second pole of the potential difference under examination, viz. the case, is earthed. Care is also taken that the two plates shall have equal and opposite potentials with respect to the earth. The deflection of the leaf or fibre which occurs when the pin is charged is usually read off by a microscope. Its amount is more or less exactly proportional to the pressure. It will be seen that in this type of instrument it is the force of gravity which is used as the counter-force.

In the Lutz single-string electrometer a very fine flexible platinum fibre is stretched between two knife-edges (fig. 40). The fibre is about 6 cm. long and has a thickness of about $\frac{1}{2}\mu$. On account of the elastic stress in the fibre, the instrument is in large measure insensitive to movements due to shaking or tilting.

Moreover, the electrical sensitiveness can be regulated very easily by means of the tension in the fibre. In accordance with the principles referred to above, the connexions with the instrument can be arranged in such a way that the potential difference to be measured is that between the two knife-edges, and the fibre is given an auxiliary charge; or else an arbitrary auxiliary potential difference is applied between the two knife-edges, and the thread is connected with one pole of the potential difference to be determined, while the other pole and the casing are earthed. In the latter case, care must be taken that the two knife-edges are at equal and opposite potentials with respect to the earth. Alternatively, we may connect the thread and one knife-edge with one pole of the potential difference to be measured, and the other knife-edge with the second pole, so that we can dispense with the auxiliary charge (idiostatic * connexion). This gives an arrangement in which the action of the lines of force is similar to

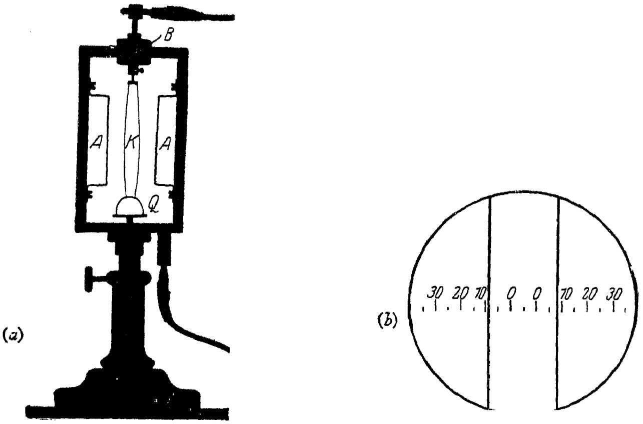

Fig. 38.—(*a*) Model of a "bifilar electrometer" or "bifilar voltmeter". Range from about 30 to 400 volts. (*b*) Field of view of a bifilar voltmeter.

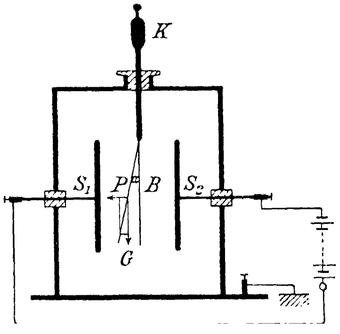

Fig. 39.—Bohnenberger's electrometer

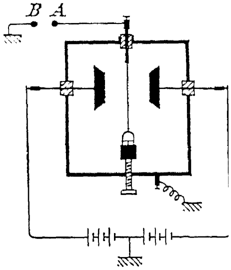

Fig. 40.—Principle of the single-string electrometer

that in the Grimsehl electrometer, viz. a deflection on the one hand by the transverse pressure of the lines of force issuing from the knife-edge with like charge, and on the other hand by the tension of the lines of force coming from the knife-edge with unlike charge.

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* From Gr., *idios*, own, i.e. without auxiliary charge.

A very sensitive instrument is the Wulf bifilar, or two-string, electrometer. In this two quartz fibres carrying like charges, and stretched by a small weight or a bent spring, are attracted by two side-pieces charged with the opposite sign, and the attraction is observed microscopically or by projection (fig. 38).

*Quadrant Electrometer.**—A flat metal box (fig. 41) of about 5 cm. diameter and height 1 cm. is cut into four quadrants $S_1$, $S_2$, $S_3$, $S_4$ ($S_4$ is not shown in the figure). Inside the box a thin plate F (called the "needle") of aluminium, or of mica which has been made conducting, is suspended so that it can turn freely about a perpendicular axis A (a thin aluminium rod). The whole needle system hangs by a very thin fibre, ·01 to ·001 mm. in thickness, which is either of metal, or of quartz which has been rendered conducting by silvering or by covering it with some hygroscopic substance.

A form of the needle (Dolezalek's) which on account of its stability and damping is much used consists of two sheets of silver paper, pasted together at the edges but pressed apart in the middle. The opposite pair of quadrants $S_1$ and $S_3$ are in conducting connexion with one another, as also are $S_2$ and $S_4$. In the position of equilibrium, the needle is set, say, as shown in fig. 41, so that one part of it is situated in the pair of quadrants $S_1$, $S_3$, and the other part in $S_2$, $S_4$. If the two pairs of quadrants are now oppositely charged, the needle will be drawn into the pair having a charge of the opposite sign to its own. This instrument also can be connected up in the three ways mentioned in a preceding case.

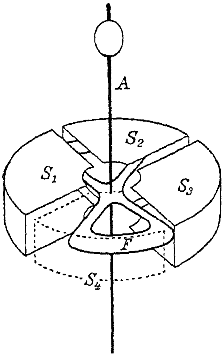

Fig. 41.—Quadrant electrometer

1. An auxiliary potential difference is applied between the needle and one pair of quadrants (the pole connected with that pair of quadrants being then usually earthed). The potential difference to be determined is applied between the two pairs of quadrants (quadrants connexion). With a constant auxiliary potential difference the deflection is proportional to the potential difference whose value is to be measured.

2. The two pairs of quadrants are charged to equal and opposite potentials from two sources. The needle is connected up with the potential to be measured, the opposite pole being therefore earthed (needle connexion). In this case also the deflection is very nearly proportional to the potential difference under examination.

3. The needle and one pair of quadrants are connected with one another (and usually earthed). The potential difference to be measured is applied to the two pairs of quadrants. The deflection is approximately proportional to the square of the potential difference applied (double connexion, or idiostatic connexion).

Fig. 42 shows the construction of a practical instrument of this type. In use, the whole instrument is enclosed in a tightly fitting brass case with a window through which to observe the mirror on the needle.

**Condensing Electrometer.**—For the measurement of very small potential differences, sensitive electrometers would be necessary; but these are not easy to use, on account of the insulation difficulties which often occur. We may, however, magnify the given potential difference to practically any extent by making use of an artifice due to Volta (1782). This depends on the fact that the potential difference between the plates of a plate condenser, viz. $U = Es$ (p. 33), increases in proportion to the distance $s$ between the plates. In fact, at all distances small compared to the diameter of the plates, the density of the lines of forces remains the same, so that the intensity is also constant.

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* Invented by Lord Kelvin in 1860.

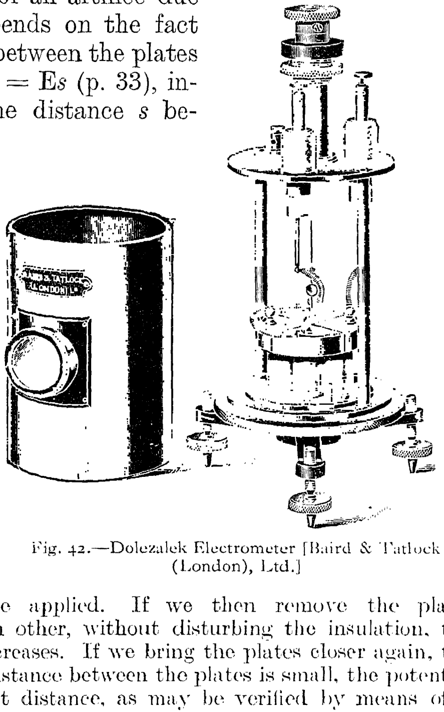

Fig. 42.—Dolezalek Electrometer [Baird & Tatlock (London), Ltd.]

The following experiment may be made. The two plates of the condenser shown in fig. 12 (p. 12) are connected respectively with the two poles of an electrometer (fig. 43). The distance between the plates may be 5 cm. When we charge one of the plates, the electrometer shows a definite deflection, corresponding to the potential difference applied. If we then remove the plates to a greater distance from each other, without disturbing the insulation, the deflection of the electrometer increases. If we bring the plates closer again, the deflection falls. So long as the distance between the plates is small, the potential difference is proportional to that distance, as may be verified by means of a calibrated electrometer (p. 45). If, however, the distance between the plates becomes comparable with their diameter, the potential difference increases more slowly, since the lines of force spread out more and the intensity consequently falls.

If we have a small potential difference to measure, which on direct trial with the electrometer gives no deflection, we first of all connect the system containing the potential difference in question to the condensing arrangement, thus charging the latter. We then break the connexion with the original system, and move the plates farther apart. If the plates are sufficiently large, and the original distance between them is small, we may easily get a magnification up to a hundredfold. For the purpose of making the original distance as small as possible, one of the plates is given a thin coating of varnish, so that the two can be allowed actually to touch one another. The plates are often brought directly up to the electrometer, as shown in fig. 44. With the help of a source of known potential difference

(p. 44), this so-called *condensing electrometer,* as also any other electrometer, can be calibrated.

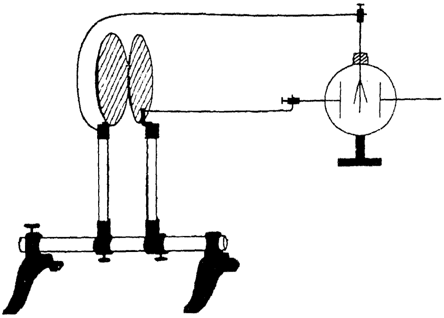

Fig. 43.—Increase of the potential difference by elongation of the lines of force.

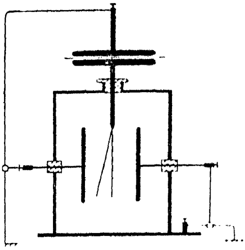

Fig. 44.—Condensing electrometer

## 7. Production of Constant Potential Differences. Unit of Potential Difference

In view of the methods so far discussed for producing electric fields, the possibility might occur to us of setting up a definite potential difference, which might be used as a unit, by a strictly defined process of friction between two strictly defined bodies. The idea, however, is scarcely practicable; besides, much better methods are available, to the discussion of which we now proceed.

**Voltaic Cells.**—If we dip two plates made of two different metals, or materials conducting like metals (such as carbon, copper oxide, lead oxide, lead peroxide), in water or in an aqueous solution of some salt or acid or base, we can establish a potential difference (the so-called *electromotive force* or E.M.F.) between the metal plates. It is very minute, so that to show it a sensitive electrometer is needed; or else Volta's artifice of increasing the length of the lines of force must be employed (p. 39). The potential difference or E.M.F. is found to be independent of the size of the metal plates; on the other hand it depends on the material of which they are made. It is possible to draw up a list of the metals in order, such that every metal becomes charged positively with respect to one farther down the list when both are immersed in water, but negatively with respect to one which appears earlier in the list. This so-called electromotive series, for the most important materials, runs as follows:

**+ Carbon, platinum, silver, copper, iron, tin, lead, zinc, aluminium, magnesium, sodium −.**

Slight impurities in a metal may alter its position in the electromotive series very considerably. Further, the electromotive series is not independent of the liquid in which the metals are placed. For these reasons the information obtainable from the list is only approximate.

Two different metals, set up in liquid which is a conductor of electricity, form a *voltaic* (or *galvanic*) *cell* (or *element* or *couple*). Liquids which do not conduct electricity, e.g. petroleum, paraffin oil, &c., do not serve the purpose—neither would mercury. The number of possible voltaic elements is very large, but only a few of them have proved of practical value.

The following nomenclature is used. The two plates of the cell are called **electrodes** *; the liquid is called the **electrolyte** †. The tops of the electrodes, which project out of the electrolyte, are called the poles of the cell. When the two electrodes are connected by a conducting wire, we say that *the circuit is closed*; and when no such connexion is present, that the *circuit is open*. The potential difference or E.M.F. between the poles is different in these two cases (p. 236). In the present chapter we shall consider open circuits only.

**Historical Note.**—ALOISIO GALVANI (1737–98), Professor of Anatomy at Bologna, observed in 1789 that the freshly prepared leg of a frog, lying near an electrical machine, began to twitch when electric sparks were drawn from the conductor of the machine. The twitching became more vigorous when Galvani touched the nerves with a metallic body. He then touched the nerves with a wire, while the frog's leg lay on a metal plate, and observed that the muscles contracted when plate and wire were brought into contact. Galvani attached no special importance to the fact that the metals were different. To explain the phenomenon he suggested that the nerve and muscles might be regarded as the coatings of a Leyden jar charged by the processes of the animal, and discharged by contact with the metals. The electricity produced was therefore considered by Galvani to have its source in these vital processes.

ALESSANDRO VOLTA (born at Como in 1745, Professor of Physics in Como 1774–79, Lecturer in the University of Pavia 1779–1804, Director of the Faculty of Philosophy in Padua 1815, died at Como 1827) followed up Galvani's experiments, and established the fact that the application of two different metals is essential for their success. He proved that two different metals become oppositely electrified when they are put in contact with each other. To prove this, he took a zinc plate and a copper plate, both clean, and connected the lower one with the knob of a gold-leaf electroscope, and the upper one with the case. When he put the two plates in contact, and then removed the upper one, the electroscope showed a deflection, indicating a positive charge when the zinc plate was below. The deflection indicated a negative charge when the copper plate was put under the zinc (*Volta's fundamental experiment*). He arranged the metals and some other materials of similar behaviour in a series:‡

+ Zinc, lead, tin, iron, copper, silver, gold, carbon,
graphite, pyrolusite —.

The order of the metals in this list is the reverse of that given in the previous electromotive series, and it looks at first sight as if Volta had discovered a new phenomenon, an electrical excitation of metals relative to each other without the assistance of a liquid. In reality it is not so. In fact, in Volta's experiment, unless special precautions have been taken, there is always a film of moisture on the surfaces of the metals, and it is to the presence of this film that the potential difference is due. If the film of moisture is removed by thorough drying, the potential difference becomes distinctly smaller. What is measured in Volta's experiment is essentially the potential difference between the water films on the two metals.

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* Gr., *hodós*, path; electrode, path of the electricity.

† Gr., *lysis*, loosing.

‡ See p. 105 (the law of A. COEHN).

Of the many possible combinations, by means of which a galvanic cell can be constructed, we may mention two which are in extensive use at the present day, the so-called *Leclanché** *cell* and the lead accumulator. In earlier times the *Daniell cell*† filled an important place, for it represented the first fairly serviceable unit of potential difference, to which the unit now in ordinary use was kept nearly equal when it was adopted. The following data are merely descriptive of the composition of certain cells; the origin of the E.M.F. is considered on p. 110; the mode of action of the cells, and the reasons for adopting a particular composition, on p. 279.

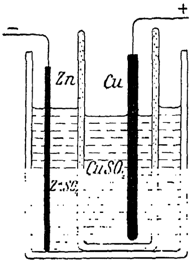

Fig. 45.—Daniell cell

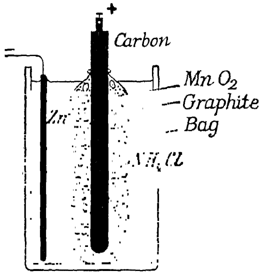

Fig. 46.—Leclanché cell

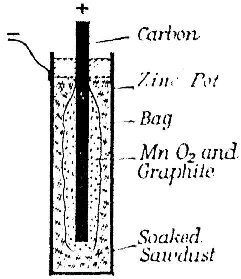

Fig. 47.—Dry cell

**The Daniell Cell.**—This (fig. 45) consists of a copper electrode, which dips into a solution of copper sulphate in water, and a zinc electrode, which stands in a solution of zinc sulphate in water. The two solutions are separated from each other by a porous partition of unglazed earthenware. Zinc forms the negative, copper the positive pole.

**The Leclanché Cell.**—This cell (fig. 46) contains: amalgamated zinc, ammonium chloride (10 to 20 per cent solution in water), manganese dioxide, and carbon. Usually the manganese dioxide is mixed with graphite, and packed round the carbon rod in a bag. An extremely popular form of this cell is the dry cell (fig. 47), in which the ammonium chloride solution is mixed with water-attracting (hygroscopic) salts, and thickened by the addition of meal, sawdust, &c. The volume may be filled up with pitch or paraffin. An actual *dry* cell without liquid or fused electrolyte does not exist. Usually the beaker-shaped zinc electrode forms the external container.

**The Lead Accumulator.**—The positive pole of this cell is lead peroxide ($PbO_2$), and the negative pole lead. The electrolyte is dilute sulphuric acid. This is at present probably the most popular cell. (For further details, and explanation of the name, see p. 288.)

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* G. Leclanché (1839–82). The cell was known in 1868.

† J. F. Daniell (1790–1845): cell, 1836.

All the cells described, while showing quite good constancy of potential difference, are yet dependent far too much on temperature, purity of the materials, and other circumstances to be suitable as standards. Combinations have therefore been sought after, which are readily reproducible, and therefore composed of materials easy to manufacture in pure form, and in particular dependent as little as possible on temperature.

The best form of cell, which has been accepted internationally, is the **cadmium standard cell** (also called the *Weston cell*).

It consists of an H-shaped glass vessel (fig. 48), having platinum wires sealed into the lower ends of its two limbs. One limb contains pure mercury, over which is laid a paste made by mixing mercurous sulphate ($Hg_2SO_4$) with some mercury and a saturated solution of cadmium sulphate. The lower part of the other limb contains a layer of cadmium amalgam (12·5 per cent Cd). The remaining parts of the vessel are filled with crystals of cadmium sulphate ($3CdSO_4 + 8H_2O$) and with saturated solution of cadmium sulphate. The limbs are closed airtight with paraffin. The mercury is the positive, the cadmium amalgam the negative pole. The E.M.F. of this cell is almost independent of temperature (see below).

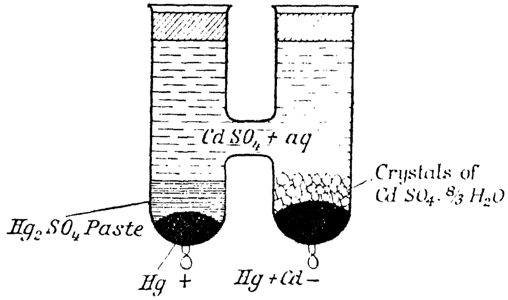

Fig. 48.—Cadmium standard cell

**Unit of Potential Difference.**—In earlier times the E.M.F. of the Daniell cell was employed as the unit, and called a Daniell. When requirements grew more exacting, this unit became inadequate, and another unit was defined internationally, the volt.* This unit is derived from the properties of current electricity (p. 377), and has been so chosen that it is approximately equal to the old unit, the Daniell.

**It was internationally agreed (Washington, 1911) that the potential difference of the cadmium standard cell at 20° C. shall be taken as equal to 1·0183 volt.†**

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* In honour of Volta (p. 41).

† The unit of potential difference so defined by means of the standard cell is called the **international volt** (p. 221). The value for the cadmium standard cell is therefore 1·0183 int. volt.

For varying temperature $t^\circ$ C., the potential difference of the cell is given by the formula

$$E_t = 1{\cdot}0183 - 0{\cdot}0000406\,(t - 20) - 0{\cdot}00000095\,(t - 20)^2$$

in international volts.

We have therefore available, in the E.M.F. between the terminals of the standard cell, a potential difference which is readily reproducible and expressible in terms of the volt.

The E.M.F.s of the elements referred to above are: Daniell cell, 1·09 volt; Leclanché cell, 1·5 volt; accumulator, 2·0 volts. For laboratory purposes it is sufficient in many cases to make use of the E.M.F. of a (well charged, p. 289) accumulator as a reference potential difference of 2 volts.

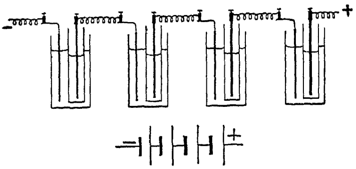

Fig. 49.—Cells connected in series

The **unit of electric intensity** (or **field strength**) is 1 volt/cm. (see p. 31).

**Production of Potential Differences of any required Value.** A multiple of the potential difference of a cell can be obtained by connecting up, as in fig. 49, any number of cells in such a way that the positive pole of each cell is connected (by a conductor) to the negative pole of the next cell.* The cells are then said to be arranged or connected *in series*. A combination of a number of cells is called a **battery.**

*The electrostatic potential difference between the open ends of a battery composed of several cells connected in series is equal to the sum of the E.M.F.s of the individual cells.*

This result, which can be easily verified experimentally, may be deduced as follows. Let the potential of 1+ relative to 1− be $U_1$; 1+ is connected to 2−, so that the 2− has also the potential $U_1$ relative to 1−. In fact, since the constant E.M.F. between the terminals of a cell on open circuit is set up spontaneously by the electrical forces in the materials, the potential $U_1$ is not changed at all when 2− is attached. But 2+ has relative to 2− the potential $U_2$, hence 2+ has relative to 1− the potential $U_1 + U_2$; and similarly for any number of cells.

This arrangement of cells, viz. in series, is very common in practice. In the ordinary pocket flash-lamp battery there are usually three cells, which in series yield a potential difference of 4 or 5 volts. In so-called high-tension batteries, dry cells, up to 100 in number, are arranged in series, so that potential differences up to 150 volts are available. Provision is usually made for obtaining lower differences if they are required. When cells are connected in series, it should be borne in mind that the potential difference of any cell relative to the first cell is directly proportional to the number of cells between these two. It is therefore well to take care, especially when differences of several hundred volts are in question, that the individual cells are well insulated.

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* In electrical diagrams a battery is represented, as in fig. 49, by alternations of thin and thick lines standing respectively for the carbon or positive plates (thin lines) and the zinc or negative plates (thick lines) of the cells.

**Historical Note.**—Volta obtained large differences of potential by piling up plates of zinc and copper, separated by pieces of felt moistened with sulphuric acid, in the order copper, felt, zinc, then copper again, and so on. This **Volta's pile** (1800) was the only means, a hundred years ago, of producing strong electric currents. A variant of this is *Zamboni's pile,* which consists of alternate layers of gold and silver paper, touching each other on the metallic sides. A pile of this sort under good conditions usually gives about 100 volts.

It is not customary to use voltaic cells for potential differences over 1000 volts or so at most; and for pressures even so high as this, they are used only when special uniformity of pressure is required. For higher pressures there are more advantageous methods; e.g. besides the historic frictional machine there is the influence machine (p. 60) up to 100,000 volts; and we have also transformers, depending on the properties of current electricity, with which theoretically any pressure can be reached, and potential differences up to two million volts have already actually been obtained. Such differences of potential are of the same order of magnitude as those which occur in thunder clouds. The potential differences which we produced in our first frictional experiments usually amount to several thousand volts.

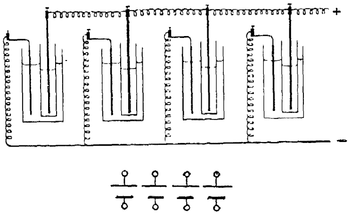

Fig. 50.—Cells connected in parallel

*Grouping of Cells in Parallel.*—The E.M.F. of a cell is a property of matter, i.e. it is determined by the electrical forces residing in the materials of which the cell is composed (§ 18, p. 105). It is consequently independent of the size of the cell. If then we connect the like poles in a set of cells all of the same kind, i.e. connect the cells *in parallel* (fig. 50), the potential difference of the battery so formed is equal to that of a single cell. If the cells are not all alike, but have different E.M.F.s between their terminals on open circuit, clearly when like poles are connected each cell will strive to set up its own characteristic E.M.F., and equilibrium is impossible. To deal with the processes which then take place in the cells would take us beyond the limits of electrostatics. A case of the kind is considered on p. 233.

*Production of Small Potential Differences.*—A theoretically possible method of obtaining small potential differences is to communicate part of the charge of an insulated body to an uncharged body (as e.g. in the experiment described on p. 8). In practice, however, other methods are employed; these will be discussed later.

**Calibration of Electrometers.**—In order to be able to measure any potential difference in volts, all we now need is to have the scales of the foregoing instruments graduated in volts. For this purpose we connect the terminals of the standard cell with the binding screws of the electrometer. The deflection which then occurs corresponds to 1·0183 volt. By connecting cells in series we can in theory set up any required higher potential difference (p. 44). Usually we employ sources of current which are easier to handle, and the E.M.F. of which can be referred to that of the standard cell (e.g. lead accumulators giving 2 volts per cell, p. 42).

For any instrument we can in this way draw up a table, in which the scale values are expressed in volts. The results are usually recorded graphically, abscissæ representing scale values, and ordinates the corresponding values in volts. If the deflection is proportional to the potential difference, we obtain a straight line. Fig. 51 shows as an example the calibration curve of a single-string electrometer. Pointer instruments are always provided with ready-made scales graduated in volts.

**Range of Electrometers.**—Simple electrometers without auxiliary charge are unsuitable for potential differences under 100 volts. It is possible, however, by using Volta's artifice (p. 39) to demonstrate differences as low as 1 volt, though not to measure them very exactly. With an auxiliary charge, electrometers can easily measure to 0·1 volt, and even under suitable conditions to 0·01 volt. The most sensitive instrument, but at the same time the most inconvenient one to handle, is the quadrant electrometer: the ordinary forms range down to 1/1000 volt, and very small instruments may reach a lower limit of $10^{-6}$ volt. Reliable measurements of potential differences under 0·1 volt are only possible when special precautions are taken, as they are extremely apt to be affected by disturbing influences (see p. 41, contact potentials; p. 261, thermal potentials).

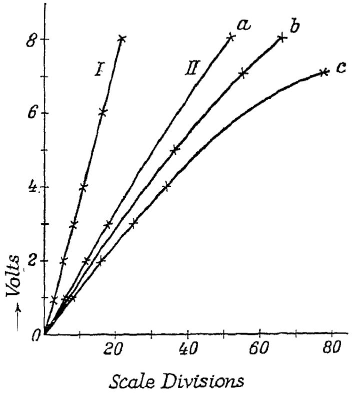

Fig. 51.—Calibration curve of a single-string electrometer. I. Fibre tightly stretched; knife-edge E.M.F., 88 volts. II. Fibre loosely stretched; knife-edge E.M.F.s, (*a*) 84, (*b*) 88, (*c*) 92 volts.

To measure *very high potential differences*, Braun's electrometer can be used up to about 10,000 volts. For still higher pressures, special instruments have been constructed; but these do not differ in principle from the simple electrometers, and we shall not enter here on the details of their construction.

**Measurement of Charge with the Electrometer.**—Now that we are able to measure potential differences, we can also compare *charges* with one another by means of the forces exerted on them in a homogeneous field, in accordance with the fundamental equation (p. 9)

$$\mathbf{P} = \mathbf{Q}\mathbf{E}.$$

If we introduce a charge into the homogeneous field between two plane plates the distance between which is small compared to their area, the charge being so small that it does not sensibly disturb the homogeneity of the field (p. 57), then a force $\mathbf{P} = \mathbf{Q}\mathbf{E}$ acts on the charge, where (p. 33) E is equal to U/$s$, U being the difference of potential between the plates, and $s$ their distance apart. If the force can be measured, the charge is found from the equation $Q = Ps/U$.

If the various quantities are measured in one of the customary systems of units, a factor of proportionality $f$ is also required, the value of which will depend on the units adopted; we have then $Q = fPs/U$.

The measurement of charges by this method can only be carried out conveniently in a few special cases (cf. § 8 following). In general charges are measured by other methods, which will be discussed in detail later (p. 79).

## 8. The Elementary Electrical Charge. The Unit of Charge

**Atomic Structure of Electrical Charge.**—In the course of our discussion of properties of matter (Vol. I, p. 259) the question arose whether the divisibility of matter is unlimited, or whether in the nature of the case there are limits beyond which subdivision cannot proceed. We were thus led to an explanation of the molecular and atomic structure of matter and to the definition of a minimum mass, namely that of the hydrogen atom, the unit by repetition of which, so far as is known at present, all matter may be regarded as built up.

Various electrical phenomena, which will be treated in detail in later sections (pp. 158, 319), suggest that electricity also has an atomic structure, so that it would not be possible to diminish an electric charge indefinitely, there being a certain limit at which the process would necessarily come to an end. We go on to consider this fundamental property of electric charge.

The procedure is essentially the same as that already illustrated in the case of matter, i.e. we try to obtain the smallest possible particles of an electrically charged body, and in this way we come to a certain limit, say the limit of microscopic visibility. As it turns out, we are then actually in a position to answer the foregoing question.

**Balanced Drop Method.**—The investigation of the charge on the smallest possible particles was first proposed and carried out by EHRENHAFT* (1910). He found that the method indicated at the end of § 7 above was very suitable. The direct measurement of the force can be evaded very neatly as follows.

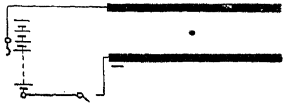

Fig. 52.—Measurement of charge by the balanced drop method

The charged particle of mass $m$ and charge Q is suspended in a homogeneous field (fig. 52), the field strength being so adjusted that the electric force on the particle is directed upwards, and is just sufficient to balance its weight. We thus have the relation $mg = QU/s$, so that $Q = mgs/U$, or (if the units are varied) $Q = fmgs/U$; where U is the potential difference between the charged plates producing the field, $s$ the distance between the plates, $g$ the acceleration of gravity, and $f$ a conversion factor. Here $g$ is known, U can be determined by an electrometer, and the length $s$ can be measured by a suitable instrument. The mass $m$ is found from the radius and specific gravity.

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* [R. A. MILLIKAN, Professor of Physics in the University of Chicago (1910–21), states in his book *The Electron* that the method used by EHRENHAFT had been published by De Broglie in Paris a year earlier. Millikan also remarks of this method that it only gives a mean charge, whereas his own method (illustrated in fig. 53) determines the charge on an individual particle.]

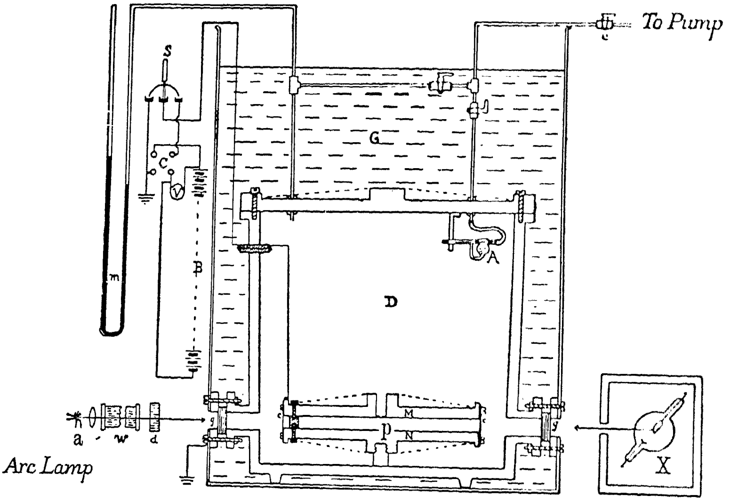

Fig. 53.—Millikan's determination of the elementary charge

A, Atomizer through which the stream of oil is blown into the cylinder D. G, Oil-bath to keep the temperature in D constant. M and N, Copper plates of the air condenser. $p$, Small hole in the upper plate M, through which the oil droplets enter. B, 10,000 volt battery. $a$, Arc lamp. $w$ and $d$, Vessels containing liquid to absorb heat rays. X, X-ray tube to ionize the air in the condenser and thus modify the charge on the droplet. (From *The Electron*, by R. A. Millikan.)

All the quantities being measured, we insert their values in the formula just given—the potential difference in volts, the mechanical quantities in C.G.S. units—and thus obtain a definite numerical value for Q.

Since the particles dealt with are usually very small, so that we cannot make measurements on them even with the help of the microscope, the radius is often determined with the help of Stokes's law of resistance (Vol. I, p. 372), by measuring the uniform velocity with which the particle falls in air.

In carrying out the experiment (fig. 53), paraffin oil (e.g.) is converted into spray by an atomizer, and thus at the same time electrified by the friction at the nozzle. The paraffin cloud formed by the smallest droplets is introduced into the field, and a particle is followed with the microscope. By regulating the potential difference (inserting more or fewer cells in series in the circuit) we can arrive at a point when the droplet is just balanced.* We then cut out the field, and observe the velocity with which the droplet falls under gravity alone. We have then, as explained above, if U, $s$ and the specific gravity of the droplet are known, all the data needed to calculate Q.

**Elementary Charge.**—Observations of this type made on very small particles, and especially the investigations of Millikan, have shown that the observed charges are always whole number multiples of a definite minimum charge. We have therefore the result:

**Electrical charge occurs only in multiples of an elementary charge.**

If in the fundamental formula given above (p. 9) we express the force in dynes and the field strength in volts/cm., the value of the elementary charge is found to be $1{\cdot}59 \times 10^{-12}$ dynes ÷ (volts/cm.). This charge is always denoted by the symbol $e$.

We have before us here an entirely new fundamental property of electricity, exactly analogous to the atomic structure of matter. We shall find (p. 266, and Vol. V) that the atomic structure of matter and the discontinuous structure of electric charge are probably only different aspects of one and the same fundamental process. The elementary charge thus ascertained forms the *natural unit of charge*, and from a physical point of view it would be most logical to express all charges as multiples of this charge. Just as matter only occurs in masses which are multiples of the hydrogen atom (except for small divergences, which are discussed in Vol. V), so electricity only occurs in multiples of the elementary charge. The latter statement is absolutely exact; up to the present no sign of any exception to it has been found.† The existence of an elementary charge of the magnitude stated above has been established not only by the method already described, but also by many other methods, which will be referred to as they naturally come up later in this book.

The *negative* elementary charge has been given the name of *electron*, for reasons which will be explained later (p. 322).

**Elementary Lines of Force.**—The electrical excitation in the ether has its end-points at the charges. These are the places where the electrical state of the ether originates.

The most natural course to follow, and one which would be consistent with all known facts, would be (as suggested by LENARD) to associate every elementary charge with *one* line of force. From this point of view the line of force is regarded as a real entity or process in the ether, which begins and ends at the two charges. The ends of the line of force have special properties. The negative end is capable of independent existence as an *electron*, without being necessarily attached to a material body; and is therefore observable as *pure electrical substance*. The positive end, on the contrary, is of material character; or at least it is always associated with the mass corresponding to a hydrogen atom. What the nature of the combination is—whether the hydrogen atom or other similar part of an atom is *merely* bound to the positive charge, or whether the mass of the hydrogen atom represents, purely and simply, the end of the line of force *itself*—these are questions which have not yet been answered. The apparently continuous structure of the field is therefore built up statistically by the tension and transverse pressure of the lines of force corresponding to the elementary charges which are always present in enormous numbers, and through which the charges which occur in ordinary experience develop their action. Hence, when we define the state of the field at a given place by the number of lines of force there, we are not merely using a convenient rule; from the present point of view something physically real actually exists at the place, this something being composed of separate parts equal in number to the lines of force (if the elementary charge is taken as the unit of charge). This view, according to which a real existence is ascribed, exactly as was done by Faraday, to the individual lines of force—in the sense that they exist in a certain measure as separate entities—has not yet, it is true, been established on an experimental basis, the difficulty being that a single line of force cannot be followed as an individual; the tendency of the most recent experiments, however, is to make the view increasingly plausible. Moreover, there are no known experimental facts with which it is inconsistent, it lends itself well to the formation of a mental picture, and in the most natural way transfers the undoubted discontinuity of the electric charges to the field which these charges generate.

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* In consequence of the *Brownian movement*, very small particles are continually moving to and fro, so that the state of exact balance cannot be determined at once. What is done is to make the E.M.F. a little higher or lower than is necessary, so as to include the correct value between limits.

† EHRENHAFT thought he had found small particles (subelectrons) carrying charges smaller than the elementary charge $e$. His results appear, however, to be incorrect, and to be explainable by the difficulty of determining the radius and specific gravity of these small particles satisfactorily. We cannot here go into the very interesting details. (See R. A. MILLIKAN, *The Electron.*)

**Hydrodynamical Analogy.**—It may also be mentioned in this connexion that the behaviour of the lines of force is in many respects (cf. p. 164) similar to that of vortex filaments (Vol. I, p. 381). This idea, brought forward by LENARD and, for the negative electron, by STARK, afterwards received remarkable support from the theory of the spinning electron (Vol. V).

**Practical Unit of Charge.**—When a unit is being defined, two considerations must be attended to, if a useful result is to be obtained. The unit must be easy to produce, and it must be possible to reproduce it exactly at any time. Now although the elementary charge has always precisely the same value, yet its exact measurement is one of the most difficult problems in physics. It is therefore unsuitable as the foundation of a practical system of measurement. When we were defining the unit of mass, we did not start from the hydrogen atom; in the same way, the practical measurement of charge has been based on a fairly large unit, chosen, like the unit of mass, on historical grounds. The unit of charge was originally defined in such a way that the number giving the force (or work) in an electric field (p. 81) was to be a multiple, by a power of 10, of the number expressing the force (or work) in the C.G.S. system. The unit of charge, called the **coulomb**,* was so chosen that in the equation for the work, $W = fQU$ (p. 29), the factor of proportionality $f$ has the value $f = 10^7$, when W is measured in ergs, U in volts, and Q in coulombs. This unit, however, is very difficult to reproduce. For practical purposes, therefore, the coulomb has been defined in terms of that quantity of silver which carries a charge of 1 coulomb, if every silver atom carries the elementary charge. This quantity is 1·118 mgm. Ag/coulomb, and can be very easily determined by weighing.* Since 1·118 mgm. of silver contains $6{\cdot}28 \times 10^{18}$ atoms (viz. $6{\cdot}06 \times 10^{23} \times 0{\cdot}001118/107{\cdot}88$), it follows that the coulomb contains $6{\cdot}28 \times 10^{18}$ times the elementary charge.

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* In honour of COULOMB (p. 16, and Vol. I, p. 222).

Hence the elementary charge is $e = 1{\cdot}59 \times 10^{-19}$ coulomb.

1 coulomb is therefore equivalent to

$$10^7 \frac{\text{dynes}}{\text{volt/cm.}} = 10{\cdot}2 \frac{\text{kilograms weight}}{\text{volt/cm.}}.$$

A charge of $10^{-7}$ coulomb is therefore that charge the force on which in a field of 1 volt/cm. is 1 dyne.

The method of producing a charge exactly equal to 1 coulomb, or to some fractional part thereof, will not be discussed till later. For the relative measurements in the following sections we use the following method of preparing a definite charge. We charge a large, well insulated metal sphere of 20 cm. diameter to 1000 volts relative to the earth, and touch it with a small proof ball of 2 cm. diameter, provided with a long insulating handle. The charge on the proof ball charged in this way is almost exactly $10^{-9}$ coulomb. We may use this charge in the meantime as our comparison charge.

## 9. The Distribution of the Charge on Conductors

Since the field strength inside a conductor is zero, it follows that the field cannot penetrate into a hollow space enclosed by metal (Faraday's cage experiment, p. 27). Any lines of force coming from the outside must therefore terminate on the outer surface of the conductor. There must therefore be charges there, for a line of force can only end at a charge—charge, and end of a line of force, being practically equivalent expressions. We shall now examine this phenomenon more closely.

**Faraday's Ice-pail Experiment.**—For this purpose we employ the opposite arrangement from that used in the cage experiment, and consider the field which proceeds from a charge situated in the *interior* of a closed conductor.

We charge a small proof ball fixed on a long insulating handle, and determine the potential or the field strength at any point in the neighbourhood (fig. 54), say by means of an electric pendulum, or more exactly by one of the methods cited in § 6 (p. 33).

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* This definition of the coulomb is not perfectly exact; the factor $f$ differs from $10^7$ very slightly, just as the metre is not exactly the $10^7$th part of a quadrant of the earth's meridian. Hence, as in the case of the metre, an international unit has been defined, viz. 1 coulomb = charge carried by 1.118 mgm. of silver (p. 153).

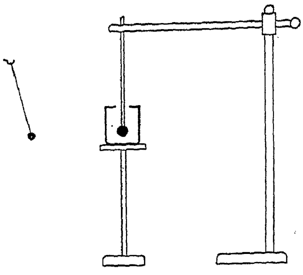


Fig. 54.—Field outside a hollow metal vessel produced by a charged body inside the vessel.

We now surround the proof ball by a nearly closed insulated metallic vessel, taking care that the ball and vessel do not come into contact. The potential, or the field strength, at the same external point as before, remains unchanged. Hence the total field-exciting influence proceeding from the surface of the enveloping vessel is the same as that from the ball inside.

If we take the ball out, preventing it from touching the vessel, and remove it to a distance, the pendulum goes back to zero; an electrometer connected to the vessel also shows no deflection. If we reintroduce the ball into the vessel, the original state of things is restored (fig. 55).

We now bring the proof ball into contact with the inside of the vessel. At the moment of contact neither the pendulum nor even a sensitive electrometer shows any change.

If we then remove the ball from inside, we find that the deflection of the pendulum or electrometer this time remains unchanged; the vessel is therefore charged. The ball, however, which we have taken out is found to be uncharged (fig. 55).

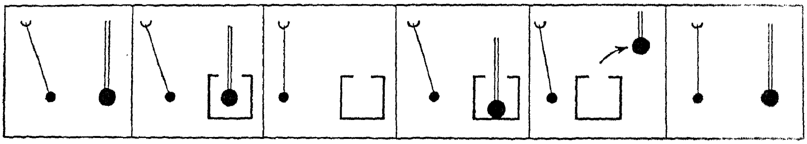


Fig. 55.—Faraday's ice-pail experiment

The diagrams of lines of force in the various cases are shown in fig. 56. Fig. 56*a* shows the field of the proof ball; the field strength at any point is indicated by the density of the lines of force, to which it is proportional. When we enclose the test charge within the hollow vessel (which strictly should be a sphere), the external field is found to remain unchanged (fig. 56*b*). In this case, we may easily satisfy ourselves by trial that the exact position of the proof ball within the conductor is of no consequence so far as the external action is concerned (fig. 56*c*). We know, however, that no lines of force exist in the interior of the metal; it follows that they must come to an end on its inner surface, and begin again in equal numbers on the outer surface. This theoretical result is verified experimentally when the hollow vessel is touched by the proof ball (figs. 56, *d*, *e*, *f*). The equal and opposite charges inside the vessel unite with each other and therefore disappear. The lines of force in the space outside are not affected (p. 52).

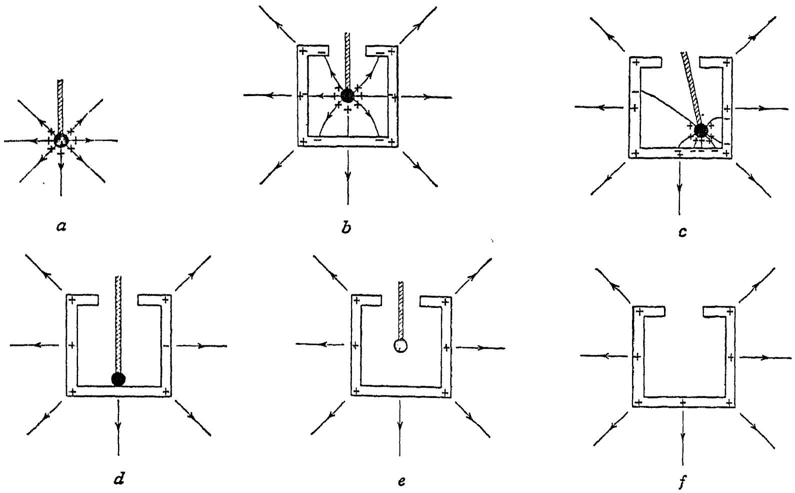


Fig. 56.—Diagrams showing the lines of force in fig. 55

This may be shown even more convincingly by the following experiment (fig. 57). We touch the vessel with the finger, so as to discharge it, and we then allow it to remain insulated. We now lower the charged ball into the conductor. While the ball is inside (but not in contact with) the vessel, we connect the latter to earth by a touch with the finger. We then insulate the conductor again, and withdraw the ball. The vessel is now charged with a charge which is equal and *opposite* to that on the sphere. This may be shown by re-introducing the ball into the vessel and bringing the two into contact; both are found to be completely discharged. The way in which the lines of force run in the different cases will be gathered at once from fig. 58. We have therefore the result:

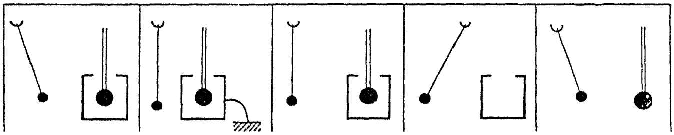


Fig. 57.—Charging by earthing while under the influence of another charged body

*The total charge of a system of bodies remains unchanged, even if* *charges pass from one body of the system to another, provided that no charges enter the system from outside, or are conducted away from it.*

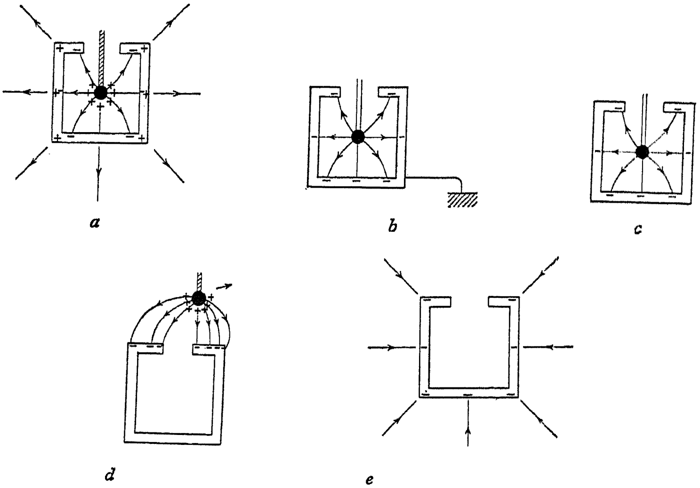


Fig. 58.—Diagrams showing the lines of force in fig. 57

This last condition is broken in the preceding experiment when the vessel is touched by the finger.

**Measurement of Charges by the Electrometer.**—The properties just described may be used for the purpose of transferring the *whole* charge of a body to an electrometer, and thus deducing the value of the charge in a satisfactory way from the magnitude of the deflection (p. 79 ). In fact, if the charged body is merely made to touch an electrode of the electrometer, then both the body and the electroscope are still charged after their separation. The quantity of electricity which has been communicated to the electrometer depends on the relative sizes of the body and the instrument, and on the nature of the surroundings.

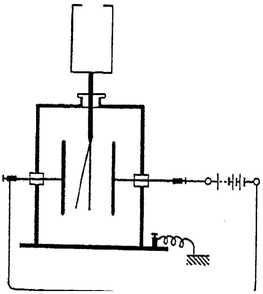


Fig. 59.—Cup electrometer

In order to transfer the *whole* charge, a metal vessel or " cup " is fitted to the upper part of the electroscope (fig. 59). The body whose charge is to be measured is introduced into the cup, and made to touch its inside surface; having been thus discharged, it is withdrawn, without having touched the electroscope. The latter now carries the whole charge which formerly belonged to the body, and therefore shows a deflection corresponding to this whole charge. Figs. 60, *a* to *d*, show how the lines of force run; the details will become quite clear after the explanations of § 10 (p. 56).

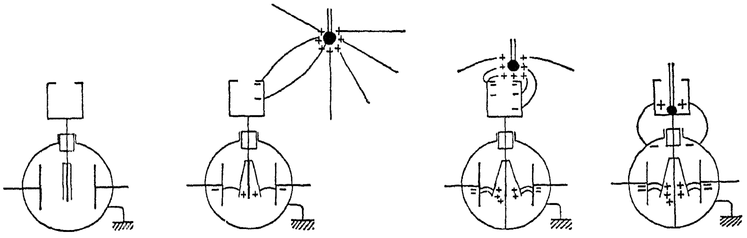


Fig. 60.—Measurement of charge with the cup electrometer

*Note.*—We may obviously deal with the charge of the electrometer in another way, viz. keep the body in the cup, connect for a moment the two poles of the electrometer, whose second pole (the case) is earthed; and, after breaking the connexion, lift the body out of the cup so that it does not touch the sides. The electrometer then shows a deflection of the same absolute amount as in the case of direct contact (fig. 58), but in the opposite direction.

**Distribution of the Charge.**—The knowledge we have acquired will now be used for the purpose of investigating the distribution of the charge on the surface of a conductor. We touch the conductor, at the spot we wish to examine for charge, with a metallic plate (*proof plane*) the dimensions of which are small compared with those of the conductor (fig. 61). The lines of force which originally ended at this part of the surface of the conductor now end on the proof plane—assuming that the field remains the same as before, a condition which may be ensured by the use of a long insulating handle. Since the ends of the lines of force carry the charges, the proof plane will also take over the charge corresponding to the part of the surface in question; and this charge will be the greater, the more lines of force ended on this portion of the surface.

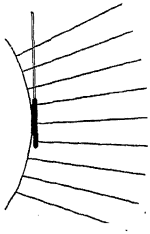


Fig. 61.—Measurement of the surface density of charge on a conductor by means of a proof plane.

Hence this method enables us to determine the charge situated on a definite area of the surface of the conductor.

**The amount of the charge per unit area is called the density of charge, or surface density, and is denoted by $\sigma$.**

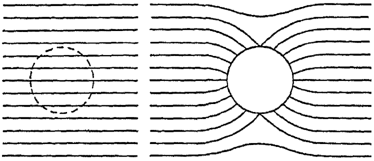
Surface density has the dimensions coulomb/cm.$^2$.

On the surface of a *spherical conductor*, situated at a great distance from any other body, the distribution of the electric charge is perfectly uniform. If in fact the sphere is touched with the proof plane, and the charge on the latter is examined by the electrometer, the deflection obtained is independent of the particular point where the sphere was touched. The surface density on an insulated sphere which is remote from other bodies is therefore the same at all points. The result also follows immediately from the diagram of lines of force (fig. 8, p. 10). For conducting surfaces which are *not* spherical, the distribution of charge depends on the form of the surface. An insulated metal box, of cubical shape, e.g. a biscuit-box, is more highly charged at the edges, and particularly at the corners, than at the middle of the faces. This also can be demonstrated with the help of the proof plane.

The diagrams of lines of force will show features corresponding to these facts. In particular, at points on the surface where the radius of curvature is small, the density of the lines, and therefore the charge density also, may become very great, as has already been explained (p. 24) for the case of points. The intense concentration of charge at the point is easily verified by the above method with the help of a very small proof sphere (see further p. 76).

## 10. Electrostatic Induction

**Behaviour of Uncharged Conductors in a Static Electric Field.**—We have already (p. 18) encountered the phenomenon of *induction*, i.e. the occurrence of electrical charges on conductors, originally uncharged, when these were introduced into an electric field. We shall now use the additional knowledge which we have since gained, to make a more exact study of this phenomenon. We take a charged body, say a plate condenser, and into its field we introduce a metal ball (fig. 62). The original distribution of lines of force will clearly no longer represent statical conditions, for the lines must end normally on a conductor (p. 27). Hence a motion of electricity will take place on the sphere, in such a direction that the lines of force meeting its surface become shorter, as we may see at once by considering tangential components. Indeed, it is only to be expected that the lines of force should draw nearer each other on account of the disappearance of the transverse pressure of the parts of the lines of force within the sphere. We therefore obtain the distribution of fig. 63.



Figs. 62 and 63.—Conducting sphere in a homogeneous field

Fig. 64 shows the electrification of a spherical conductor, produced by induction when a charged rod is brought near the sphere.

These experiments bring to light an entirely new property, viz.:

*Conductors contain charges which can move freely, and which, in the normal state, i.e. when the conductor is not electrified, must be so* *thoroughly intermixed that they produce no external effect or, in other words, they neutralize each other.*

They can be pulled apart from each other, however, by the tension of the lines of force of an applied electric field. The introduction of a conductor into the electric field has the effect of cutting some of the lines of force in two. The new ends of lines of force which thus arise are occupied by the charges and, as a further consequence, the lines of force are displaced from their original positions (see above).

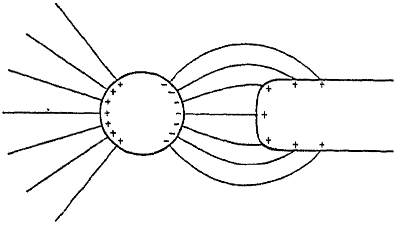


Fig. 64.—Induced charges upon a neutral sphere under the influence of a charged rod

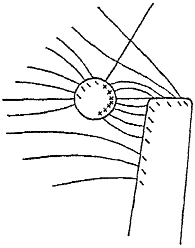


Fig. 65.—Attraction of an uncharged conductor as a result of induction.

A similar phenomenon occurs when a charged body is brought near an electroscope (fig. 34, p. 35). Lines of force impinge on the leaves, and pull them apart. When the body is removed to a distance, the lines of force disappear, and the leaves fall back.

*Note.*—Thus the introduction of a conductor changes the form of the field. If the dimensions of the conductor are small compared to those of the field as a whole, the change is small in proportion. It is for this reason that a body used for testing purposes, such as a proof plane, should be as small as possible. The testing body used to measure the field strength must be so small that the effect produced on it by induction is practically nil, so that, if it is *uncharged*, the force acting upon it in the electric field is negligible (see below). Conversely, its charge must be small, in order that the inductive effect of its own field may not alter the distribution of charge on the body which is exciting the field under examination.

**Attraction of an uncharged Conductor by a charged Body.**—We can now understand also why a conductor which is not itself electrified is attracted by a charged body. When a charged rod (fig. 65) is brought near an uncharged conducting proof sphere, the two kinds of electricity in the latter become separated, and move into new positions. The lines of force proceeding from the little sphere to the charged rod are more concentrated in one direction than those from the sphere to the other surroundings, since the rod is nearer; consequently the sphere is attracted. If the charged and uncharged bodies are brought very close together, the density of the lines between the two becomes so great that the insulating power of the air breaks down (p. 24) and a charge passes across, usually to the accompaniment of a spark. If the charged rod is then taken away, the proof body is left with an excess of the electricity of the same sign as that on the rod. We have charged the sphere by "contact". Uncharged *insulators* are also attracted, but the explanation of this must be postponed (see p. 94).

**Charging by Induction.**—We can now also understand more exactly the phenomenon of charging by induction, described on p. 56. The conductor shown in figs. 63 and 64 clearly carries equal amounts of positive and negative electricity, since just as many lines of force leave it as come to it from the inducing charge. The two kinds of electricity, however, are pulled in opposite directions by the lines of force, and thus become separated. If the strength of the external field diminishes, the opposite charges move nearer and partially neutralize each other again; when the external field disappears altogether, the neutralization becomes complete, and the conductor is left without charge as at first. The sequence of changes in the field which occur when a conductor is divided into two parts can now be explained. While the external field is still present, we separate the two portions of the conductor in the way shown in fig. 66.

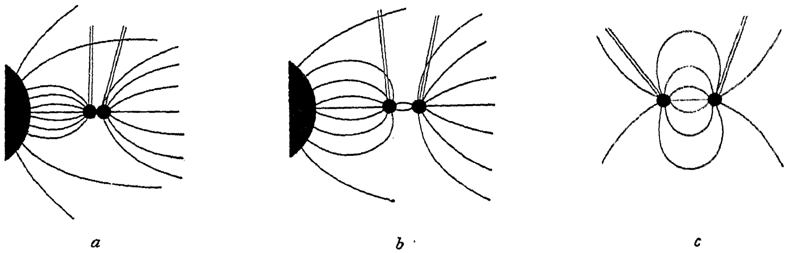


Fig. 66.—Division of a conductor into two parts in an electric field

The general disposition of the field after this has been done is shown in fig. 66*b*; by comparing with fig. 66*a*, we see that the separation has made very little difference. We now remove the external field, say by earthing the conductor carrying the inducing charge. The induced charges on the two portions of the conductor can no longer combine, since they are now separated by an insulator, viz. air. We therefore obtain fig. 66*c*, i.e. we have two bodies carrying equal and opposite charges; if in fact we bring the two bodies into contact with each other, and then separate them, we find that the electrification has disappeared completely from both.

The experiment may also be carried out in a modified form as follows (fig. 67). The conductor on which the charge is induced is connected to its surroundings, i.e. "earthed". The lines of force which end on the surroundings will then be annulled, and with them the corresponding charges; for example, the negative charges will disappear from the conductor. The positive charges, being now freed from the attraction of the negative charges, will be displaced still

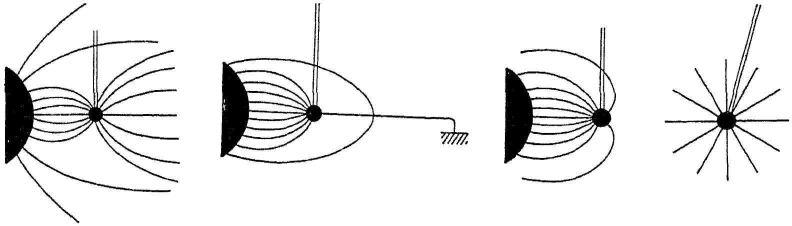


Fig. 67.—Earthing of a conductor in an electric field

nearer the inducing body (fig. 67*b*; see p. 60, where the effect is measured with the balance). We can now remove the earth connexion, without essentially altering the character of the field. If finally the inducing field is withdrawn, the conductor is left with a positive charge. This result corresponds to the experiment illustrated on p. 53.

**Electrophorus.***—The above method can be used to "generate" electricity in any quantity by means of the contrivance called the electrophorus (fig. 68). This consists of a circular disc of vulcanite H (or of glass, or of a mixture of rosin, wax, and shellac) which lies on a metal plate T, or is coated on the under side with tinfoil. On the vulcanite disc lies a flat cover D of metal, provided with an insulating handle.

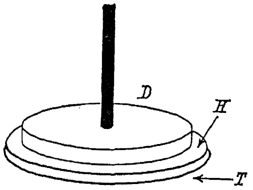


Fig. 68.—Electrophorus

The cover being lifted, the vulcanite disc lying on the plate is electrified negatively by rubbing or beating with a woollen cloth or a skin. If the cover D is now laid on the vulcanite disc, the charge on the disc acts inductively on the cover, since the cover lies in the electric field of the vulcanite disc. Consequently the lower side of the cover becomes positively charged, and the upper side negatively. If the cover is then touched by the finger, the free negative charge on its upper side flows off to earth. The cover is now lifted by the insulating handle; it carries a positive charge, with which we can make such experiments as we wish. If an earthed conductor is brought up close to the cover D, the latter is usually discharged by a spark a little before contact. The process can be repeated as often as we please, so that in this way any quantity of electricity can be "generated". During the whole process the electrical condition of the electrified vulcanite disc remains unchanged; for only the *inductive* action of this charge is used for the separation of the charges.

With regard to the peculiar phenomenon that no process of neutralization takes place between the negatively electrified vulcanite disc and the positive induced charge on the cover, even though the two bodies stand for days in contact with each other, see p. 105 (electrical double layer).

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* Gr., *phóros*, carrier.

The source of the energy of the electrical charge on the cover is mechanical work, as may be proved by the following simple experiment (fig. 69). The cover of an electrophorus is hung from one side of the beam of a balance and insulated. The electrified vulcanite disc lying on the conducting plate is then placed in contact with the under side of the cover. The other scale of the balance is now loaded until a further load of say 1 gm. wt. would suffice to lift the cover. If the cover of the electrophorus is then touched in the usual way, it is found that a distinctly greater weight is required to raise the cover. The extra work thus done is the equivalent of the potential energy of the charge which is left on the cover after it has been lifted.

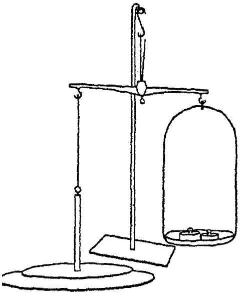

Fig. 69.—Work done in the "generation" of electricity by means of the electrophorus.

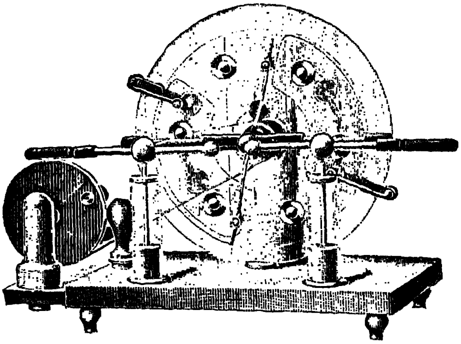

Fig. 70.—Holtz electrical machine

**Influence Machines.**—In the influence machine, which is a development of the electrophorus, the inductive action of an initial charge is used to "generate" fresh charges of electricity. Influence machines have been devised by Varley, Holtz, Töpler, and Wimshurst, and have reached a high degree of perfection.

**The Holtz Machine, with Töpler's Self-excitation.**—This consists (fig. 70) of a fixed circular glass disc, to which are attached two strips (or coatings) of paper in the form of quadrants of a circular ring, with the edges rounded off. Strips of tinfoil are held underneath the paper. Close in front of the fixed disc there is a second circular glass disc which can rotate about an axle; the axle passes through a large central opening in the fixed disc. To the movable disc several (usually six) small circular pieces of tinfoil ("sectors") are attached, carrying projecting metal buttons. To the fixed disc, at the two places where the strips of paper are first reached when the rotating disc is turned, two bent pieces of metal are fastened. These curve round the other disc, and end close up to it in fine metal brushes (lametta fibres). When the disc in front rotates, the metal brushes come into contact with the projecting metal buttons of the small tinfoil "carriers". In front of the rotating disc, exactly at the level of the middle of the paper strips, there are two collecting combs, which are connected with the inner coatings of two small Leyden jars,* and with discharging knobs the distance between which can be varied by means of insulating handles. A transverse conducting arm is also provided, carrying at its ends collecting combs and small metal brushes. This arm is so placed that it stands opposite those ends of the paper strips on the fixed disc which the rotating disc reaches last. The conducting arm is fastened to the axle of the machine, and is in electrical connexion with it, and therefore with the earth also.

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* These are glass jars, coated with conducting material outside and inside, used for storing up large quantities of electricity. They are not essential to the action of the machine, and a detailed description with an explanation of their action is therefore postponed till p. 87.

The principle on which the machine works can be seen clearly from fig. 71. The two discs are shown as glass cylinders, to prevent any of the parts from being concealed. $B_1$ and $B_2$ are the paper strips on the fixed disc (shown here as the outside cylinder). $A_1$, $A_2$, . . . , $A_6$ are the small tinfoil strips on the rotating disc (the inside cylinder). $P_1$ and $P_2$ are the brushes at the ends of the curved metal pieces; $E_1$ and $E_2$ are the brushes, connected to earth, at the ends of the transverse conductor. $K_1$ and $K_2$ represent the collecting combs with their discharging knobs.

Let $B_1$ have a small negative charge; this may either be given it from outside or be produced by the friction with the brush $P_1$. Then an inductive effect occurs in $A_1$, so that this sector becomes positively charged, while $A_4$, being connected with $A_1$ by the transverse conductor E, receives a negative charge. We now suppose the inner disc to be rotating, and follow the sector $A_1$. Carrying its positive charge, it reaches $A_2$ when, by means of the brush $P_2$, part of its charge passes to $B_2$, which accordingly becomes positively charged. The sector comes next to $A_3$. By the action of the positive charge on $B_2$, the side of the sector next $B_2$ becomes negatively charged, and the side next $K_2$ positively. The inductive action between the points and the sector develops an intense field, and the air becomes conducting (p. 24), so that the positive charge passes (at least in part) from the sector at $A_3$ to the comb $K_2$. In the position $A_4$ the portion of the positive charge which is still left on the sector becomes neutralized by the negative charge repelled from that sector which is then at $A_1$; the latter sector thus has its charge increased. At $A_5$ the sector, now strongly charged negatively, comes in contact with the metal brush $P_1$, through which it transfers a part of its negative charge to $B_1$. Finally, at $A_6$ a part of the negative charge is transferred to $K_1$, on the principle already exemplified at $A_3$. The potential difference between the coating $B_1$ and the sector becomes decidedly greater when the sector passes from $A_1$ to $A_2$, on account of the lengthening of the lines of force; it follows that $B_2$ becomes charged as the sectors continually advance to a high potential relative to $B_1$. At the same time the quantity of electricity affected by induction at $A_4$ increases, and with it the potential difference between $A_4$ and $B_2$. From the lengthening of the lines of force, the sector has at $A_5$ a still higher potential relative to $B_2$, and $B_1$ also becomes presently charged to this higher potential. The rising potential of $B_1$ relative to the sectors is further raised by the lengthening of the lines of force from $A_1$ to $A_2$. Hence the potential difference between $B_1$ and $B_2$ goes on continually increasing, and would do so indefinitely but for the breakdown of the insulation of the materials and the air. A large fraction of the potential difference between A and B exists also between $K_1$ and $K_2$, from which it can be drawn for use wherever it may be required.

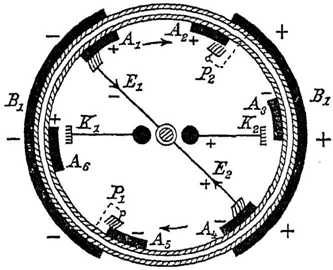
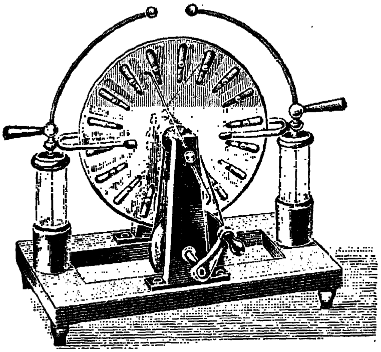


Fig. 71.—Diagram of the Holtz electrical machine

It will be seen that the potential difference is essentially due to the stretching of the lines of force between $A_1$ and $A_2$, and between $A_4$ and $A_5$. To raise the potential in this way the expenditure of work is necessary. This work is supplied by the power turning the machine. It is readily observed that it is more difficult to turn the machine by hand when it is supplying electricity than when it is running without load, as e.g. when the two coatings $B_1$ and $B_2$ are earthed.

**The Wimshurst Machine.** — This machine* is extremely reliable in its action and convenient to work, and on this account is very widely used.

It consists (fig. 72) essentially of two rotating discs of glass or vulcanite, facing one another and having a large number (18 in the figure) of small sectors of tinfoil attached as coatings to their outer sides. Two collecting combs are bent round so as to face the discs at the level of the axle; these are usually connected with small Leyden jars, and are attached to two discharging knobs. Two transverse conductors are also provided; these end in metal brushes, and are in conducting connexion with the frame of the machine and therefore with the earth. Their position is sufficiently illustrated by fig. 72. The disc in front turns in the clockwise direction, the one behind in the opposite direction.



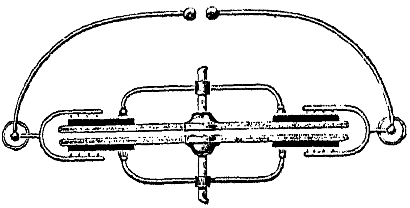


Fig. 72 and 73.—Wimshurst electrical machine

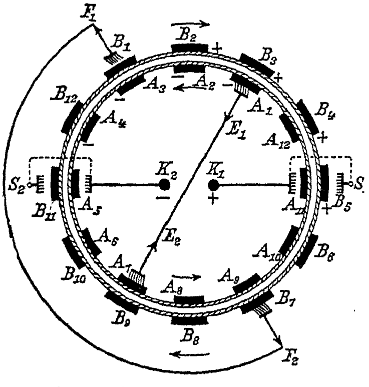


Fig. 74.—Diagram of the Wimshurst electrical machine

Fig. 73 shows the same machine diagrammatically in plan; this brings out the position of the collecting combs, the coatings and the metal brushes. For clearness of exposition of the action of the machine, fig. 74 is drawn, as in the previous case, with the coated discs represented as cylinders. The disc corresponding to the inner cylinder carries the tinfoil coatings $A_1, A_2, \ldots, A_{12}$, and turns in the anti-clockwise direction; the other disc with the coatings $B_1, B_2, \ldots, B_{12}$ turns in the clockwise direction. The metal brushes of the earthed transverse conductors are $E_1$, $E_2$, and $F_1$, $F_2$. $S_1$ and $S_2$ are the collecting combs, which are connected to the discharging knobs $K_1$ and $K_2$; the Leyden jars are not shown. Let $B_1$ acquire a small positive charge by friction with the brush $F_1$. By the rotation $B_1$ comes to the position $B_2$, then to $B_3$. Here the sector $A_1$ is opposite to it, and by induction acquires a negative charge, while the corresponding positive charge passes to $A_7$ through the transverse conductor. We follow $A_1$ on its way to $A_2$ and $A_3$. At $A_3$ the charge of the sector $A_3$ induces a positive charge in $B_1$ which is exactly opposite it, while the corresponding negative charge passes to $B_7$. The positive charge induced in the sector at $B_1$ comes to $B_3$. A negative charge is then again induced in $A_1$. But now $B_2$ and $B_4$ are also already positively charged, so that a portion of their lines of force end on $A_1$. The charge induced in $A_1$ is therefore greater than the one previously induced in the similar position. Consequently the positive charge which $A_3$ afterwards induces in $B_1$ is also greater, and this in turn leads to a further increase at $B_3A_1$.

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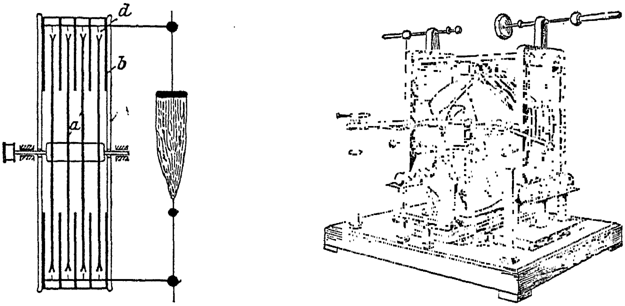
* Wimshurst first published a description of his machine in 1883.

It is easily seen that these processes are reinforced still further by the exactly analogous phenomena occurring in the lower half of the machine, so that the charge, and therefore also the potential difference, as increased by the stretching of the lines of force, mounts higher and higher, until a limit is set to it by the imperfect insulation of the disc and the air between the coatings or the knobs $K_1$, $K_2$. The supply of electricity from the machine is taken off through the collecting combs in the manner described on p. 24.



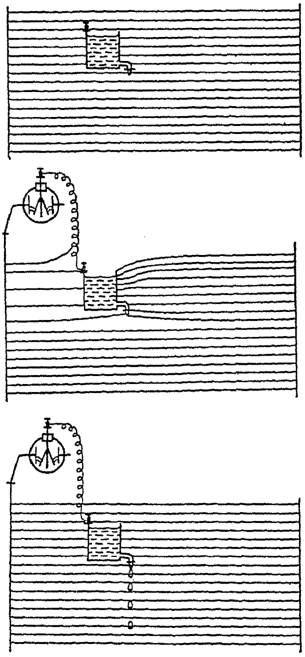
Figs. 75 and 76.—Wommelsdorf multi-disc electrical machine

**Improved Types of Wimshurst Machine.**—Within recent years a further development has taken place in the construction of influence machines. The discs are made of vulcanite and the metal sectors are incorporated in the body of the discs. The electricity generated is either (as in Wehrsen's machine) drawn off by brushes from small metal knobs which are connected with the sectors and project from the vulcanite disc; or (as in H. Wommelsdorf's machines) it is conducted away by steel wires *d* (fig. 75) which make contact with the disc at a groove in its outer edge. In the multi-disc machine (fig. 76) of the latter type, a number of fixed and rotating discs, *a* and *b* (fig. 75), fit into each other alternately at small distances, as in a condenser, thus increasing the inductive effect.

**Application of Induction to the Measurement of Field Strength.**—Field strength, being the rate of fall of potential per cm. in the direction of the electric force (p. 31), can be measured with the help of an electrometer. For this purpose conductors are placed at two different points of the field, and connected to the electrometer. The conductors assume the potentials corresponding to their positions in the field, and hence the measured potential difference, divided by the projection of the distance between the two points upon the direction of the field, gives the mean field strength in this region.

The conductors which are thus placed in the field so as to assume the potentials corresponding to their positions are known as **test-bodies or collectors.** In order to understand the principle of their action, we will consider the following arrangement.

As test-body we use a metal vessel filled with a liquid. If it is chosen sufficiently small, it assumes, when introduced into an electric field, the potential corresponding to its position. This potential may be reckoned relative to any reference body, e.g. relative to the left electrode in fig. 77, where the field is represented as homogeneous. On connecting the test-body with an electrometer (as in fig. 78), we obtain a throw which is smaller than would correspond to the potential at the position of the test-body, because the field is disturbed by the electrometer. A large fraction of the lines of force starting from the electrode connected with the electrometer will be shortened, and the potential of the test-body relative to the other electrode has increased correspondingly.



Figs. 77, 78, 79.—Drop collector

We now allow the liquid to flow out in drops through an opening in the bottom of the vessel. As each drop is formed, it is under conditions similar to those of the vessel; thus in the example shown in the figure more lines of force from the negative electrode end upon the drop than lines of force from the positive electrode. These conditions remain unchanged as the drop separates from the container; it therefore falls with an excess of lines of force from the negative electrode ending upon it, i.e. it falls positively charged. This positive charge is derived, however, from the system vessel-electrometer. Hence the number of lines of force from the negative electrode ending upon this system must have been diminished by an amount corresponding to the positive charge carried away by the drop. This process is repeated with the fall of each successive drop, until eventually all the excess induced charge has been removed and the conditions at the point of separation of the drops have become perfectly symmetrical, i.e. until the numbers of lines of force reaching this point from the positive and negative electrodes have become equal, as in the undisturbed field. When this is so, the potential shown by the electrometer is that of the point of separation of the drops, when the field is undisturbed; for this point is in conducting connexion with the electrometer (fig. 79). This arrangement is known as the drop collector.*

Other methods are also frequently used to remove the excess charge. We may mention, for example, flames (fig. 80), whose glowing soot particles and products of combustion carry away the charge, and also radioactive substances, which render the air conducting in the neighbourhood of the test-body.

The difference of potential between two points 1 cm. apart in the direction of the field gives the mean field strength in this region. Instead of measuring this directly, the difference of potential may be measured between two test-bodies, and the result divided by the projection of their distance apart in the direction of the field.

This method of measuring intensity is often employed in investigations of atmospheric electricity (Vol. V).

Concerning intensity measurements of this type see p. 333.

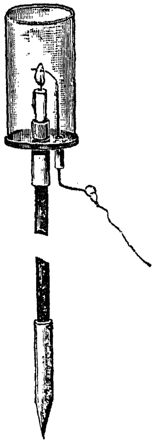


Fig. 80.—Test-body with flame for measurements of atmospheric electricity.

## 11. The Electrical Displacement as a Measure of the Field

**Measurement of Electrical Displacements.**—We have already seen (p. 19) that it is possible to describe the electric field by means of the *displacement*. We will now go on to develop the previous results in a quantitative way.

We have to determine quantitatively how many lines of force (or unit tubes of force) pass through unit area at the point under consideration. As we have seen, this can be done most easily by cutting the lines of force and observing how many ends, i.e. how many units of charge, are obtained per cm.$^2$ of the surface employed. We make use of the method given on p. 19, i.e. we introduce a double plate into the field, separate the halves there (fig. 81), and measure the charge on one half with the help of a cup electrometer. Provided, then, that the plate was at right angles to the lines of force at the moment of separation, the measured charge, divided by the

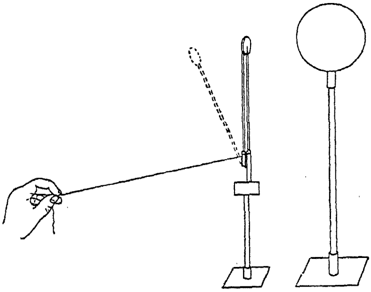


Fig. 81.—Mie's double plate

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* The original idea is due to Lord Kelvin.

area of the plate (i.e. the surface density $\sigma$ on the plate), is a direct measure of the magnitude of the electrical displacement D at the particular point of the field. If the plate was not at right angles to the field, the measurement gives the magnitude $D_n$ of the displacement in the direction of the normal to the plate (see p. 19). Thus the observed surface density is

$$\sigma = D_n.$$

**Quantitative Relation between Displacement and Intensity.**—On p. 20 we saw that the displacement and intensity are proportional to one another, i.e. that

$$\mathbf{D} = K_0\mathbf{E}.$$

Let us consider the intensity and the displacement in the homogeneous field between plane plates. The charges residing upon the plates send their lines of force parallel to one another through the homogeneous field. (Edge effects may be neglected; they can be eliminated by suitable devices.) In this case the field strength or intensity is measured by the number of field lines per 1 cm.$^2$ of a surface parallel to the plates. In the same way, the displacement is measured by the corresponding number of lines of force (p. 20). Now the magnitude of the displacement is equal to the number of unit charges residing upon unit area of the plates; for all these charges send out their lines of force normal to the surface. Hence the displacement is numerically equal to the *surface density* $\sigma$ of the charge on the condenser plates, and we have

$$\sigma = D = K_0 E.$$

The factor $K_0$ is the number of lines of force producing such a displacement in space devoid of matter that the intensity is equal to 1 volt/cm., i.e. that there is one field line per 1 cm.$^2$ of a surface normal to the field. In order to arrive at the value of $K_0$ in the customary units, we have to determine the charge density in coulombs per cm.$^2$ which produces an intensity of 1 volt per cm. in the plate condenser.

The intensity (in volts/cm.) is most easily measured from the potential difference U of the condenser and the distance $s$ between its plates.

The surface density $\sigma$ or the magnitude D of the displacement may be measured, as explained above, with a double plate.

Measurements of this kind have led to the value:

$$\mathbf{K_0 = 0{\cdot}886 \,.\, 10^{-13}\ (coulombs/cm.^2)/(volts/cm.).}$$

This value is a constant of nature; it expresses the relation between electric displacement and intensity, when these are measured in our units. It is sometimes referred to as the **absolute dielectric constant of the ether.**[*] (See further p. 74.)

If the charge be measured in elementary units (electrons), then using the data given on p. 51 we obtain the value $K_{0e} = 5{\cdot}56 \,.\, 10^5$ $(e/\text{cm.}^2)/(\text{volts/cm.})$. Thus a charge density of 556,000 elementary charges per 1 cm.$^2$ of a plate condenser produces in this a field strength of 1 volt/cm.

**Charge and Flux of Force.**—On account of their great importance we will consider the above results from a somewhat different point of view.

For this purpose we consider the flux (p. 14) of the vector **D**, i.e. the number of lines of force passing through an arbitrary surface situated in the field.

**The number of lines of force passing through a surface is called the flux of force $\Phi_e$ through the surface.**

The flux of force $\Phi_e$ can also be expressed as follows in terms of the field strength:

Since the density of the lines of force is proportional to the field strength E, the flux of force through the element of area $dA$ at right angles to the lines of force is

$$d\Phi_e = K_0 E \, dA = D \, dA.$$

If the element of area is not at right angles to the lines of force, we must take the projection of it in this direction. If $(\mathbf{E}, \mathbf{n})$ is the angle between the normal to the surface and the direction of the lines of force, then

$$d\Phi_e = K_0 E \, dA \cos(\mathbf{E}, \mathbf{n}).$$

When the different elements of a larger surface are inclined at different angles to the direction of the lines of force, the total flux of force through the surface is given by

$$\Phi_e = K_0 \int E \cos(\mathbf{E}, \mathbf{n}) \, dA = \int D \cos(\mathbf{D}, \mathbf{n}) \, dA = \int D \, dA_n,$$

where $dA_n$ stands for the projection of $dA$ upon a plane at right angles to **D**.

In this connexion we must adopt a convention as to the direction of the normals. In the case of a closed surface, for example, we will take the outer side and the outward normal as positive. Thus the flux of force through such a surface is positive in the direction from inside towards outside. It follows that the magnitude of the charge upon the test plate, as measured (p. 65) by means of the cup electrometer, is the numerical measure of the flux of force through the area of the surface occupied by the test plate at the moment of separation.

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[*] Concerning the significance of this name, see p. 84. This factor $K_0$ is a fundamental constant, like the velocity of light (p. 196) or the gravitational constant (Vol. I, p. 177). The accurate determination of its numerical value is therefore of great importance. In the so-called electrostatic system of measurement $K_0$ is put exactly equal to $1/4\pi$ (p. 82).

*Flux of Force through a closed Surface.*—As is shown by Faraday's ice-pail experiment, the total flux of force through a closed surface is equal to the magnitude of the enclosed charge (fig. 82). This result is not affected by the form of the surface or of the field. Due account must be taken of the sign, as determined by our convention concerning the direction of the normals.

To determine the total flux of force we form the integral over all the elements of surface. For a *closed* surface (to indicate which we draw a small circle through the symbol of integration) this gives the total enclosed charge Q, i.e.

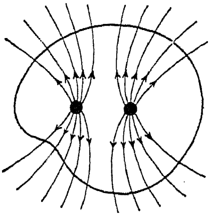

Fig. 82.—Flux through a closed surface due to enclosed charges

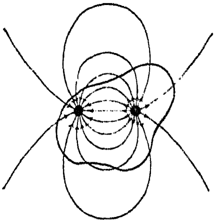

Fig. 83.—Zero total flux of force through a closed surface due to equal and opposite enclosed charges.

$$\oint \mathrm{D}\,d\mathrm{A}_n = \mathrm{Q} = \mathrm{K}_0 \oint \mathrm{E}\,d\mathrm{A}_n.$$

This equation is the mathematical expression of Faraday's ice-pail experiment.

It is important to pay due attention in the integration to the sense of the normals and the sign of the displacement. Thus in fig. 83, for example, the total flux of force through the closed surface is zero.

The last equation shows that the displacement produced by charges in a *region of space* can be obtained by integration over a *surface.* This is known as **Gauss's theorem**; it is of great importance for the treatment of specially distributed charges.

When several charges are situated within a closed surface, each of them $\mathrm{Q}_a$ sends a flux of force $\Phi_{ea} = \mathrm{Q}_a$ through the surface, and the total flux of force is therefore $\Phi_{ea} = \Sigma\mathrm{Q}$.

The distribution of the charges in the space enclosed by the surface has no influence upon the magnitude of this total flux of force; nor has the shape of the surface. The total flux of force is obtained by *algebraic* addition of the charges enclosed by the surface.

*Note.*—The fact that the total flux of force is obtained by scalar addition, although the forces add together as vectors, is due to the use, in the calculation of the flux of force, of the projections of the elements of area at right angles to the lines of force; these projections all have the same direction near any given point and therefore add together algebraically.

*Flux of Force through a Closed Surface due to Charges situated Outside.*—Each line of force entering the surface leaves it again at some other point; for, by assumption, the charges are situated *outside* the closed surface (fig. 84). It is easily seen that the flux of force entering is equal to that leaving the surface, so that the total flux through the surface is zero. Hence in calculating the flux of force through any closed surface no account need be taken of charges outside the surface.

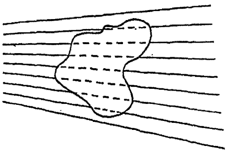

Fig. 84.—Zero flux of force through a closed surface due to charges lying outside.

*Volume Density of Electric Charge.*—The volume density $\rho$ of a charge distributed uniformly over a space (so-called *space charge*) is defined as the quantity of charge per unit of volume, i.e. $\rho = Q/V$, where Q is the total charge and V the total volume. The total flux of force due to unit volume is therefore $\rho$.

*Examples of the Application of Gauss's Theorem.*—Gauss's theorem is often used, especially in electrical engineering, for the calculation of the nature of fields.

1. *Infinitely Large Plane with Constant Surface Density* $\sigma$.—From each unit of area emanate $\sigma$ lines of force. Since the lines of force do not intersect one another, considerations of symmetry show they are in equal numbers on each side of the plane, and are at right angles to it. The density of lines of force on each side of the plane is therefore $D = \frac{1}{2}\sigma$; this value is independent of the distance from the plane. It follows from this that the field strength is also constant at all points in the neighbourhood of the plane.

2. *Two very large Parallel Planes.*—Let the planes be charged uniformly with the same surface density $\sigma$, but with *opposite sign.* The field can then be regarded as the superposition of two fields of the kind considered in example (1). Hence between the plates we have a double field for which $D = \frac{1}{2}\sigma + \frac{1}{2}\sigma = \sigma$. Outside the space between the plates the partial fields are of opposite sign and neutralize one another, i.e. $D = 0$. The field strength is thus constant at all points between the two plates, and its magnitude is independent of the distance between the plates. The lines of force run normally to the plates. Outside the space between the plates the field strength is zero. In this example the area of the plates is to be taken so large in comparison with their distance apart that edge effects (p. 12) are negligibly small (plate condenser).

## 12. The Energy Content of the Electric Field

**Force in a Plate Condenser at Constant Charge.**—Work is required in order to separate charges of opposite sign. The corresponding energy is stored up in the field. For the elongation of lines of force, for example in a plate condenser, produces an electrical displacement which previously did not exist in the space. Every volume element of the field therefore has a certain energy content, the magnitude of which must obviously depend upon the magnitude of the corresponding electrical displacement.

In order to calculate the work $W = Px$ required to fill a certain volume with a known displacement, we will separate the plates of an insulated plate condenser by the distance $x$ in the direction of the field. The new volume which is thus filled with electrical displacement is $xA$, where A is the area of each plate.

We obtain the force P of attraction between the condenser plates by applying the equation $P = QE$. We may regard the charge Q of the one plate as situated in the field of the other plate. Now according to p. 69 the partial field strength due to each plate is one half of the total field strength E in the condenser. The force on each plate is therefore $P = \frac{1}{2}QE$.

The surface density being $\sigma = K_0E$, the charge is $Q = K_0EA$ and the force

$$\mathbf{P} = \tfrac{1}{2}\mathbf{K}_0\mathbf{E}^2\mathbf{A}.$$

If Q is measured in coulombs and E in volts/cm., the value to be used for $K_0$ is $0{\cdot}886 \,.\, 10^{-13}$. In this case *the force is obtained in multiples of* $10^7$ *dynes* (p. 50), i.e. $P = 0{\cdot}443 \,.\, 10^{-13}E^2A\,[10^7 \text{ dynes}]$.

For the plate condenser we have also $E = U/s$ (p. 33), where U is the potential difference between the plates. Making use of this, we obtain for the force of attraction between the plates

$$\mathbf{P} = \tfrac{1}{2}\mathbf{K}_0\frac{\mathbf{U}^2}{s^2}\mathbf{A}.$$

**The Field Energy.**—It follows from above that the work expended in separating the plates by a distance $x$ is

$$W = Px = \tfrac{1}{2}K_0E^2Ax = \tfrac{1}{2}K_0E^2V,$$

where V is the increase in volume of the space between the plates. Since no further changes have been brought about in the field, this amount of energy is stored up in the volume V.

*Hence at constant charge the increase of field energy is equal to the work of separation.*

The energy per unit volume of the electric field, i.e. **the energy density**, is therefore

$$\mathbf{S} = \frac{\mathbf{W}}{\mathbf{V}} = \tfrac{1}{2}\mathbf{K}_0\mathbf{E}^2 = \tfrac{1}{2}\frac{\mathbf{D}^2}{\mathbf{K}_0} = \tfrac{1}{2}\mathbf{DE}.$$

**The energy density at any point of the field is equal to one half of the product of the displacement and the intensity.**

When E is measured in volts/cm. and $K_0$ is put equal to $0{\cdot}886 \,.\, 10^{-13}$, the energy is obtained in *joules* and the energy density in *joules/cm.³*.

**Total Field Energy.**—If $V_t$ is the total volume of the plate condenser, then $W = \frac{1}{2}DEV_t$ is the total energy contained in the field, i.e. the work required to charge the condenser. Since $V_t = Ad$ (where $d$ is the distance between the plates) and also $Ed = U$ and $DA = Q$, we can write

$$\mathbf{W = \tfrac{1}{2}QU.}$$

In the customary units we have

$$W \text{ (joules)} = \tfrac{1}{2}Q \text{ (coulombs)} \times U \text{ (volts)}.$$

This equation is of quite general validity.

*The work required in order to produce a potential difference* U *between two charges* Q *is equal to one half of the product of charge and potential difference.*

The field between the two originally uncharged plates can be built up gradually by transferring the charge Q in successive small elements $dQ$ from the one plate to the other. The element of work done in each such transference is given (p. 29) by $dW_m = U_m dQ$, where $U_m$ is the potential difference at the moment of the transference. At the beginning the work of transference is zero; at the end it is $UdQ$: the average is therefore $dW = \frac{1}{2}U\,dQ$. Hence the total work is $W = \frac{1}{2}UQ$.

**Magnitude of the Force at Constant Potential Difference.**—The plates of the condenser are now connected to a source of constant difference of potential. As we separate the plates by a distance $x$, the field remains homogeneous and the field strength is given at every moment by $U/s$, where $s$ is the corresponding distance between the plates. Now U remains (by assumption) constant, while $s$ increases to $s + x$; therefore E must decrease to $Es/(s + x)$.

From this it follows that the surface density, which was originally $\sigma = K_0E$, must also have diminished to $\sigma s/(s + x)$. Similarly the energy density must have decreased from its original value $\frac{1}{2}K_0E^2$ to the value $\frac{1}{2}K_0E^2s^2/(s + x)^2$.

*When the distance between the plates of a condenser is increased at constant potential difference, the charge density on the plates and the energy density of the field are decreased.*

For a calculation of the energy conditions in a similar case see p. 98.

**Attracted Disc Electrometer.***—The attraction between the plates of a plate condenser can be measured with a balance. Assuming that the field is really homogeneous, the equation $P = \frac{1}{2}K_0U^2A/s^2$ may be used to calculate the potential difference between the plates from the measured values of P, A and $s$, i.e. from purely mechanical quantities. We have

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* First described by Sir William Thomson (born 1824 at Belfast, died 1908), who was Professor of Physics at Glasgow, latterly with the title Lord Kelvin.

$$U = s\sqrt{\frac{2P}{K_0 A}}\ \text{volts.}$$

Thus the plate condenser may be employed as an electrometer. It is very useful when no standard cell is available; its chief value, however, is for the measurement of large potential differences. For this reason it is sometimes referred to as an **absolute electrometer.** For accurate measurements it is essential that the field shall be homogeneous; this is ensured by the use of a **guard-ring** (fig. 85). Fig. 86 shows a practical arrangement of the apparatus.

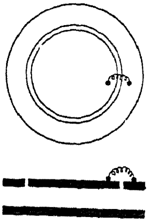

Fig. 85.—Condenser with guard-ring

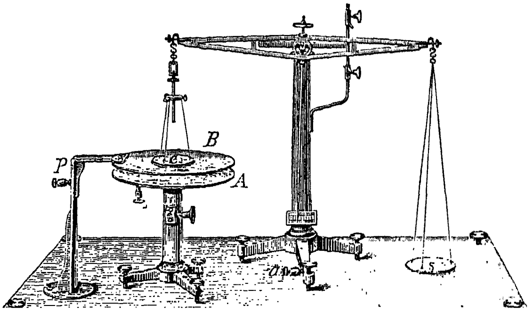

Fig. 86.—Attracted-disc electrometer

The lower condenser plate A is supported upon an insulating stand, and can be moved up towards the upper plate C at will by means of an adjusting screw. The plate C is suspended from one arm of a sensitive balance and is brought into equilibrium when the condenser is uncharged. This plate is surrounded by a ring B (the guard-ring) of the same thickness, the distance between plate and ring being small. The ring, which is fixed, is connected to earth by means of a wire attached to the binding screw P. In addition there is a conducting connexion between B and C, either by means of a wire, which must be so thin that it does not interfere with the free motion of the plate C, or through the earthed balance.

The lower plate A is connected to one pole of the system whose potential difference is to be measured. The other pole of the system is earthed, so as to eliminate disturbances due to the surroundings. The purpose of the guard-ring B is to remove the unavoidable edge-effects from the attracted disc C, so that the electrostatic field of force between A and C may be regarded as homogeneous.

## 13. Capacity

**Relation between Charge and Potential Difference.**—The total field can be well characterized by means of the potential difference and the charge (total flux of force). For any given arrangement, these two quantities are not independent. We will make this clear by considering the plate condenser.

The intensity in the condenser is determined by the surface density of the charge according to the equation $\sigma = K_0E$. Hence *at given surface density* the potential difference U depends only upon the distance $s$ between the plates:

$$U = Es = \frac{\sigma s}{K_0} = \frac{Q}{A} \cdot \frac{s}{K_0}.$$

For a given area A of the plates and a given distance $s$ between them, a definite quantity Q of electricity upon the plates therefore causes a definite potential difference between them. As the formula shows, this potential difference is proportional to the charge. Similarly the charge Q of a plate condenser of given area A and thickness $s$ is proportional to the potential difference. We may therefore write

$$Q = CU.$$

Thus for a given potential difference the system can only take up a certain quantity of electricity; this quantity is greater, the greater the value of C. For this reason the factor C is called the **capacity** of the system.

**A system has unit capacity when one coulomb of electricity produces in the system a potential difference of one volt. This unit of capacity is called 1 farad,* or abbreviated 1 f.**

This capacity far exceeds that of the arrangements of conductors ordinarily met with. Hence the unit usually employed is one million times smaller; it is called 1 microfarad (1 $\mu$f. $= 10^{-6}$ f.). Even this unit is too large for many purposes, in which case the capacity is expressed in terms of a unit one million times smaller still. This is written as 1 $\mu\mu$f. $= 10^{-12}$ f., but unfortunately has no special name.

**Condensers.**—Devices so constructed as to have a large or an accurately defined capacity are called *condensers*. We will now consider the most important types.

*Plate Condenser.*—We have already become acquainted above with the properties of this arrangement. From the previous discussion it follows that the capacity is given by

$$\mathbf{C = K_0A}/s.$$

Thus the capacity of a plate condenser is directly proportional to the area of the plates and inversely proportional to the distance between them.

Measuring A in cm.$^2$, $s$ in cm. and putting $K_0 = 0{\cdot}886 \cdot 10^{-13}$

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* In honour of FARADAY, p. 6.

(p. 66), we obtain C in *farads*. When $A = 1$ cm.$^2$ and $s = 1$ cm. we have

$$C = 0{\cdot}886 \,.\, 10^{-13} \text{ farads.}$$

Hence $K_0$ can be expressed in the units farads/cm. instead of (coulombs/cm.$^2$)/(volts/cm.). The numerical value is the same in both cases. We thus obtain the value

$$\mathbf{K_0 = 0{\cdot}886 \,.\, 10^{-13} \textbf{ farad/cm.}}$$

for the fundamental electrostatic constant $K_0$, which expresses the relation between field strength and electrical displacement.

In fig. 12 (p. 12) the change of capacity was obtained by altering the distance between the plates. In most modern instruments, however, this effect is produced by rotation of a movable set of plates relative to a fixed set (as in fig. 87).*

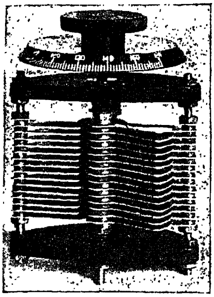


Fig. 87.—Condenser with variable capacity

In order to store up as large a quantity of electricity as possible, the area of the plates must be made large and the distance between them small. Under these conditions the field strength, for any given potential difference, becomes very great. Now air at atmospheric pressure becomes ruptured at field strengths greater than about 28,000 volts/cm. Substances of better insulating power are therefore used. But the insertion in the condenser of material substances modifies the conditions (p. 82), so that we must postpone the discussion of such types until later.

*Spherical Condenser*.—This type consists of a sphere surrounded by a concentric spherical shell (fig. 88). We will consider the case in which the space between the spheres is filled with air. The inner sphere is charged by means of a thin wire passing through the outer sphere and insulated from it.

The lines of force leaving the inner sphere, which carries the charge Q, run radially outwards and all end on the outer sphere (fig. 89). Hence the density of the lines of force, and therefore also the field strength, diminish inversely with the square of the distance from the common centre of the spheres (p. 16). Since the surface density upon a sphere of radius $r$ carrying the charge Q is $\sigma = Q/4\pi r^2$, the field strength at the surface of the inner sphere of radius $r$ is $E = \sigma/K_0 = Q/4\pi r^2 K_0$, and the field strength at any distance $x$ greater than $r$ is $Q/4\pi x^2 K_0$. The radius of the outer sphere being R, the potential difference is therefore given by the line integral

---

* The rotary variable condenser was described by Koepsel in 1901.

$$U = \int_r^R \frac{Q}{4\pi x^2 K_0}\,dx = \frac{Q}{4\pi K_0}\int_r^R \frac{dx}{x^2} = \frac{Q}{4\pi K_0}\left(\frac{1}{r} - \frac{1}{R}\right).$$

Hence for the capacity C we have

$$C = \frac{Q}{U} = 4\pi K_0 \frac{Rr}{R - r} = 4\pi K_0 \frac{r}{1 - (r/R)}.$$

When $R - r$ is very small in comparison with $r$, the variation of the field strength may be neglected and the field regarded as homogeneous. The formula for the plate condenser (see above) can then be applied.

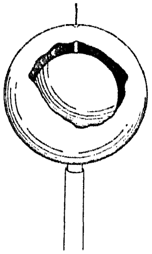


Fig. 88.—Spherical condenser

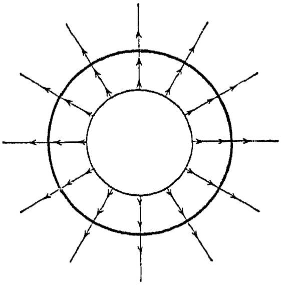


Fig. 89.—Diagram of the lines of force in a spherical condenser

Special importance attaches to the case in which R is very large in comparison with $r$. When this is so, $r/R$ becomes very small and can be neglected in comparison with unity. We then obtain

$$\mathbf{C = 4\pi K_0 r \text{ farads.}}$$

This case is important because the lines of force from an insulated charged conductor of sufficiently small dimensions end upon the surroundings, and give rise to a field which can be regarded as spherically symmetrical in the neighbourhood of the conductor. Use is frequently made of the term "capacity of a conductor", the capacity really referred to being that of the system conductor + distant surroundings. Thus we can say:

*The "capacity of a sphere" is proportional to its radius.*

An isolated sphere of 1 cm. radius has a capacity of about $10^{-12}$ farad (more exactly $1{\cdot}11 \,.\, 10^{-12}$), i.e. about 1 $\mu\mu$f.

It is important to bear in mind that the "capacity of a conductor" is dependent upon the nature of its surroundings.

In order to convince ourselves of this, we may perform the following experiment. We set up a conducting ball upon an insulating stand and connect it with one pole of an electrometer. When the ball is then charged up, the electrometer indicates a certain definite potential. But if we then bring the hand up towards the ball, the potential falls perceptibly, the reason being that a considerable fraction of the lines of force now end upon the hand, which, together with the ball, forms a condenser of perceptibly greater capacity.

*Cylindrical Condenser.*—This consists of an inner cylinder or rod of radius $r$, surrounded by an outer cylindrical shell of radius R. (See, for example, fig. 1, p. 299.) Comparing with the investigation for the spherical condenser given above, we see that in the present case we have

$$U = \int_r^R \frac{Q}{2\pi x l K_0}\, dx = \frac{Q}{2\pi l K_0} \log_e(R/r).$$

Hence the value of the capacity is, in farads,

$$C = 2\pi l K_0/\log_e(R/r),$$

where $l$ is the length of the condenser.

This formula shows that here the effect of increasing R does not become negligibly small when R is very large, as was the case for the spherical condenser. For $l$ must always be large in comparison with R; otherwise the number of lines of force proceeding from the ends increases and becomes more and more important as R is increased, until at last when R is large compared with $l$ we have practically spherical symmetry. It is very instructive to think this out in terms of the disposition of the lines of force.

If the field of force is to preserve cylindrical symmetry, $l$ must also be increased as R is increased. Hence the capacity of a cylindrical condenser does not tend to a limiting value, as does that of a spherical condenser, but goes on increasing indefinitely.

**Surface Density of Charge and Radius of Curvature.**—It follows from the formula for the capacity of a sphere that for a given potential the surface density of an *isolated* sphere is inversely proportional to its radius. In fact,

$$Q = 4\pi r^2 \sigma = CU = 4\pi K_0 r U,$$

and hence $\sigma = K_0 U/r$.

Hence if we connect together two spheres which are at a great distance from their surroundings, and then charge them to the potential U, the surface densities of charge upon them will be inversely proportional to their respective radii. The smaller sphere will therefore be more densely covered with electricity than the larger sphere. This can easily be verified by testing in a cup electroscope two balls of unequal sizes which have been previously charged up while in conducting connexion with one another.

Now the greater the surface density, the greater also the field strength in the immediate neighbourhood; for $\sigma = K_0 E$. In the case of a body with variable surface curvature the surface density of charge, and consequently also the field strength, are greatest at those regions which have the smallest radius of curvature. A sharp point is a region where the radius of curvature is very small. The surface density at a sharp point is therefore very great, as was shown by the experiments described earlier in this book (pp. 24 and 56).

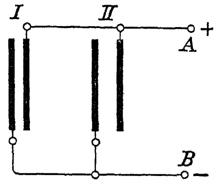


Fig. 90.—Two condensers arranged in parallel

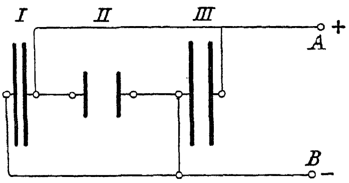


Fig. 91.—Example of several capacities arranged in parallel

The high surface density at corners and edges (e.g. of a cubical box of sheet metal, p. 56) can also be explained as due to the great curvature, i.e. small radius of curvature, at these places.

**Arrangement of Capacities.**—Two condensers may be connected together in two different ways: (1) with their similarly charged plates connected (so-called arrangement *in parallel*, figs. 90 and 91) or (2) with their oppositely charged plates connected (so-called arrangement *in series*, fig. 92).

*Arrangement in Parallel.*—On account of the conducting connexion both condensers have the same potential difference U. The quantity of electricity in the condenser I is $Q_1 = C_1U$: similarly the quantity in the condenser II is $Q_2 = C_2U$. There is no movement of electricity through the conducting connexion, because U is the same for both. Hence the total charge $Q_t$ is equal to $Q_1 + Q_2 = (C_1 + C_2)U = C_tU$, where $C_t$ stands for the total capacity of the system.

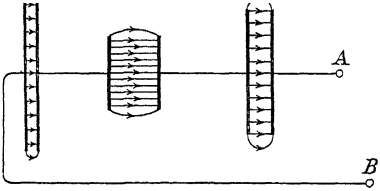


Fig. 92.—Capacities arranged in series

In general for $n$ condensers arranged in parallel we may prove in this way that

$$C_t = \sum_n C_n.$$

*The capacity of a system of condensers arranged in parallel is equal to the sum of the individual capacities.*

*Arrangement in Series.*—The potential difference U is applied to the points A and B (see fig. 92). This results in the appearance of surface charges corresponding to the lines of force drawn in the figure. Since the central part of the system is insulated, it follows that the absolute magnitude Q of the charge must be the same on each plate.

In the example shown in the figure the potential difference U is the sum of three partial potential differences $U_1$, $U_2$, and $U_3$, which are given by the equations $U_1 = Q/C_1$, $U_2 = Q/C_2$, and $U_3 = Q/C_3$. Thus we have

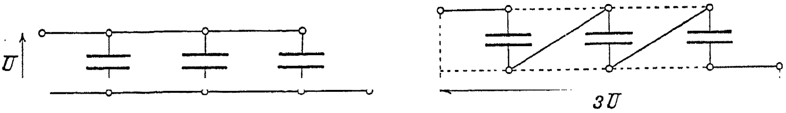
$$U = U_1 + U_2 + U_3 = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = Q/C_t,$$

and therefore
$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

In general for $n$ condensers arranged in series

$$\frac{1}{C_t} = \Sigma \frac{1}{C_n}.$$

*The reciprocal of the total capacity of a system of condensers arranged in series is equal to the sum of the reciprocals of the individual capacities.*



Figs. 93 and 94.—Increase of potential difference by rearrangement of condensers

Thus the total capacity of a system of condensers arranged in series is *smaller* than the capacity of any one of the condensers.

In this arrangement the *greater* the capacity of an added condenser, the *less* the consequent alteration of the total capacity.

**Distribution of Potential Difference in Condensers arranged in Series.**—The last result can be considered from a different and instructive point of view. The arrangement of the condensers in series causes a subdivision of the potential difference. The *smaller* the capacity of any condenser, the *greater* the share of the potential difference which it carries; for $U_n = Q/C_n$. Hence the addition of a new condenser in series with the original system has the effect of reducing the potential difference of that system, the amount of the reduction being equal to the share of the potential difference taken up by the added condenser. This share is very small when the added condenser has a very large capacity; in this case, therefore, the original system is very little affected.

**Increase of Potential Difference by means of Condensers.**—In order to produce a larger potential difference from a given difference U, the latter may be used to charge a system of condensers arranged in parallel (fig. 93). The system is then rearranged (fig. 94) in series. (The condensers must be well insulated from one another, except for the connexions mentioned.) Such an arrangement is sometimes called a *cascade battery*. When $n$ similar condensers are employed, the potential difference after the rearrangement into series is $n$U. The quantity of electricity which can now be obtained from the system is, however, only $1/n$ of the original quantity. For the system is completely discharged by connecting together the two extreme plates, upon which resides only $1/n$ of the quantity of electricity originally imparted to the condensers at the lower potential difference.

**Capacity of Electrometers.**—Every electrometer is an arrangement with a certain capacity, which indeed varies slightly with the throw of the instrument. Every measurement of potential difference necessitates the transference, to the electrometer and its leads, of a quantity of charge corresponding to their capacity. This charge is derived from the system under investigation. If therefore the measurement is to indicate the potential difference of the system *before* attachment of the electrometer, we see that (when the system is not connected to some source of constant potential difference) the quantity of electricity passing over into the electrometer must be small in comparison with the total quantity which the system contains.

If this condition is not fulfilled, it is necessary to know the capacity of the electrometer in order to determine the true potential difference.

Let the capacity of the system under investigation be C and the total quantity of electricity associated with it be Q. The original potential difference U is then given by the equation $Q = UC$. After connexion with the electrometer, whose capacity we will call $C_{el}$, the charge Q is distributed over the composite system of capacity $C + C_{el}$. The observed potential difference U′ is given by the equation $Q = U'(C + C_{el})$. Equating the two values for Q we obtain $UC = U'(C + C_{el})$, whence we have at once for the original potential difference the value

$$U = U'\left(1 + \frac{C_{el}}{C}\right).$$

When C is large compared with $C_{el}$, the term $C_{el}/C$ is very small compared with unity, and the observed potential difference is practically identical with the original. If the capacity of the system under investigation is unknown, it can be measured by connecting a known capacity to it while it is in connexion with the electrometer. It is possible to calculate the unknown capacity of the original system from the observed fall in the potential difference. Of course all these experiments necessitate very good insulation.

The order of magnitude of the capacity of the commonest electrometers is about 1/10 $\mu\mu$f.; that of quadrant electrometers is up to 10 times greater.

**Measurement of Charge.**—We have already become acquainted with a satisfactory method of measuring charge, but up to the present we have only been able to express charges in relative terms. We will now discuss how the cup electrometer, our instrument for the measurement of charge, may be calibrated in coulombs.

For this purpose we first calibrate the cup electrometer in volts with the help of a standard cell. We then connect the electrometer with a system of known capacity C and proceed to transfer to the cup the charge $Q_x$ whose magnitude we wish to measure. This causes a throw of the electrometer corresponding to a certain pressure U. From this we obtain at once the desired value of the charge in coulombs, viz. $Q_x = U(C + C_{el})$, where $C_{el}$ is the capacity of the electrometer. In order to avoid the determination of $C_{el}$, we may choose C so large that $C_{el}$ can be neglected in comparison.

It is possible to obtain condensers of known capacity, so-called

**standard condensers** (concerning their standardization see p. 476). We may use a standard condenser of 0·01 μf., for example, in connexion with the electrometer. In this case the capacity of the condenser is about 1000 times greater than that of the electrometer, so that the charge can be measured with an accuracy of 0·1 per cent when the electrometer capacity is neglected. This arrangement can only be used, however, for measuring large charges, because otherwise the potential difference is too small. For small charges the electrometer must be employed alone (without condenser), and its capacity must be known.

To determine the capacity of the electrometer we make use of the equation $U = U'[1 + (C_{el}/C)]$ (see above). The electrometer, which has been calibrated in volts, is charged up to a certain pressure U. The charging source is then removed and a known capacity C connected to the electrometer. The diminished pressure U′ is observed and the electrometer capacity $C_{el}$ calculated from the above equation.

*Example.*—We charge a ball of 10 cm. radius (see fig. 81, p. 65) to a potential of 10,000 volts by means of an electrical machine and place a double plate of 3 cm.$^2$ area at a distance of 20 cm. from it. The test plate gives a throw corresponding to 1 volt in a cup electrometer of known capacity equal to $3 \,.\, 10^{-11}$ farad. The charge on the plate is therefore $3 \,.\, 10^{-11}$ coulomb, and the electric displacement at a distance of 20 cm. from the charged ball is $10^{-11}$ coulomb/cm.$^2$.

According to p. 75 the capacity "of the ball" (i.e. of the system ball + distant surroundings) is equal to $1{\cdot}11 \,.\, 10^{-12} \,.\, 10 = 1{\cdot}11 \,.\, 10^{-11}$ f. Thus at a potential of 10,000 volts the ball carries the charge $Q = UC = 1{\cdot}11 \,.\, 10^{-7}$ coulombs. Hence the surface density of charge is $1{\cdot}11 \,.\, 10^{-7}/4\pi r^2 = 0{\cdot}886 \,.\, 10^{-10}$ coulomb/cm.$^2$. At a distance of 20 cm. from the surface (i.e. $3r$ from the centre) the corresponding surface density is $0{\cdot}886 \,.\, 10^{-10}/9 = 10^{-11}$ coulomb/cm.$^2$ approximately, as found in the experiment described above.

## 14. The so-called Absolute Electrostatic System of Measurement

Nowadays all instruments are calibrated in volts, coulombs, and farads and all measurements carried out on the basis of these units. The volt, coulomb, and farad are, however, arbitrary units; they were chosen so as to coincide as nearly as possible with the earlier historical units, so as to approximate to the orders of magnitude commonly met with, and so as to be reproducible with exactitude and ease. The standard cell was devised as a means of reproducing the volt. We shall become acquainted later with the practical method of reproducing the coulomb. The farad is determined by means of the volt and coulomb. Thus standard condensers, calibrated in farads, can be used in conjunction with standard cells as standards of charge.

Before these units had been fixed, and in the times when the present methods of specification were impossible (on account, for example, of insufficient chemical purity of the substances employed), attempts were made to express electrostatic quantities in terms of units derived from the most accurate methods then known, namely the measurement of forces. For this purpose the natural course for us to-day would be to take the forces in a homogeneous field as basis, and to define charge and potential difference in terms of the forces in an attracted disc electrometer, using the C.G.S. units. In the earlier times we are considering, however, Coulomb's law had been chosen as the basis for the formulation of the properties of the electric field, in analogy to Newton's law for the gravitational field. Hence GAUSS and WEBER pursued the following line of thought in setting up the first electrostatic system of measurement.

Coulomb's law for space devoid of matter, viz. $P = fQ_1Q_2/r^2$ (p. 16), contains only one electrical quantity (except for the factor $f$); this is the charge. The units of the other quantities are defined in the C.G.S. system. It is possible therefore to express the charge in C.G.S. units, provided that the factor $f$ be fixed arbitrarily. The simplest course is to put $f = 1$. Then we have:

**The C.G.S. unit of charge is that quantity of charge which exerts a force of 1 dyne upon a similar quantity of charge situated at a distance of 1 cm. from it.**

This is called the electrostatic C.G.S. unit of charge or of quantity of electricity.

Its dimensions (when $f$ is put equal to a pure number, namely unity) are [$\text{cm.}^{\frac{3}{2}}$, $\text{gm.}^{\frac{1}{2}}$, $\text{sec.}^{-1}$].

The *electrostatic C.G.S. unit of potential difference* is defined as that potential difference which must exist between two points in order that the quantity of work required to transfer the *unit of charge* from one point to the other may be 1 *erg*.

The dimensions of this unit of potential difference are therefore [$\text{cm.}^{\frac{1}{2}}$, $\text{gm.}^{\frac{1}{2}}$, $\text{sec.}^{-1}$].

The *electrostatic C.G.S. unit of capacity* follows at once from the units already defined. Thus the C.G.S. capacity of a system is unity when the electrostatic unit of charge produces in the system the electrostatic unit of potential difference.

It follows that the dimensions of the electrostatic C.G.S. unit of capacity must be [cm.].

Hence the "capacity of a sphere" (p. 75), expressed in electrostatic units, is $C = r$, i.e. numerically equal to the radius in centimetres. For this reason the electrostatic unit of capacity is often called "1 centimetre". We have, therefore,

$$1 \text{ cm.} = 1{\cdot}11 \, . \, 10^{-12} \text{ farad} \quad \text{or} \quad 10^{-12} \text{ farad} = 0{\cdot}9 \text{ cm.}$$

Comparison of the formulations of Coulomb's law in the two systems of measurement gives the following relations:

1 electrostatic unit of charge $= \frac{1}{3} \, . \, 10^{-9}$ coulomb.
1 electrostatic unit of potential difference = 300 volts.
1 electrostatic unit of capacity $= 1{\cdot}11 \, . \, 10^{-12}$ farad.

Expressed in the electrostatic C.G.S. units, the fundamental constant $K_0$ has the value $1/4\pi$.

This absolute electrostatic system of measurement has never been in practical use; but it is still employed almost exclusively in theoretical treatises. In order therefore to arrive at the formulæ corresponding to the readings of practical measuring instruments (namely, coulombs, volts and farads), it is necessary to replace $1/4\pi$ by $K_0$ in theoretical formulæ which are expressed in terms of the so-called absolute electrostatic system of measurement.

For the sake of convenience the most important formulæ giving the relationship between the various systems of electrical units are collected in a Table in the Appendix.

## 15. The Electrostatic Field in Space filled with Matter

**Charges in Systems with Equal Potential Differences.**—We begin with the following experiment, the essential idea of which is due to Faraday. An electrically charged ball (fig. 95) is supported upon an insulating stand. At suitable distances on either side of it are two exactly similar metal vessels, both connected to earth. One of the vessels is filled with paraffin oil, the other with air. Two rigid conducting wires project from the ball into the interiors of the respective vessels. We now take two similar proof balls, mounted upon insulating handles, and dip one into each of the vessels so as to touch the wires simultaneously. Both proof balls are thus brought to the same potential. We now investigate their respective charges with the help of a cup electrometer. We find in this way that the proof-ball which has been dipped into the vessel containing the paraffin oil carries a charge $2\frac{1}{2}$ times greater than that of the other ball which has obtained its charge from contact with the wire in the vessel filled with air.

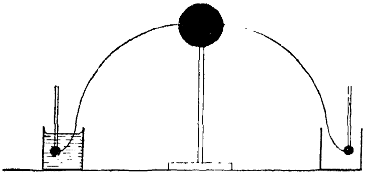


Fig. 95.—Influence of the dielectric

Other liquids may be used; in every case there is an *increase* of the charge in the vessel containing the liquid. The magnitude of the increase is dependent to a large extent upon the chemical nature of the liquid.

Since the two test-balls were in conducting connexion with one another, they must have had the same potential relative to the vessels, which were also in conducting connexion. From the similarity of the vessels and from the fact that the potential difference is defined as the line-integral of field strength, it follows that the relation between charge density and field strength (p. 66) must be different in the new material medium from what it is in air. As the above experiment shows, the charge density of the condenser, for a given field strength, is always *greater* in the new material medium than in air (or vacuum). We can express this result as follows:

*For a given potential difference, the charge density of a condenser is greater when the condenser is filled with a material medium than when it is evacuated.*

**Potential Difference for Equal Charges.**—We can also test this relationship by measuring the potential differences in the two cases when the charge remains constant.

We measure the potential difference between the insulated plates of a charged plate condenser by having them connected to an electrometer throughout the experiment (see fig. 96). We note the reading when the space between the plates is filled with air; and then (without altering the charge) we lower a glass plate upon two strings, as shown in the figure, so that it passes between the two plates without touching either of them. The deflection of the electrometer is observed to decrease. From this it follows, in analogy to the result stated above, that:

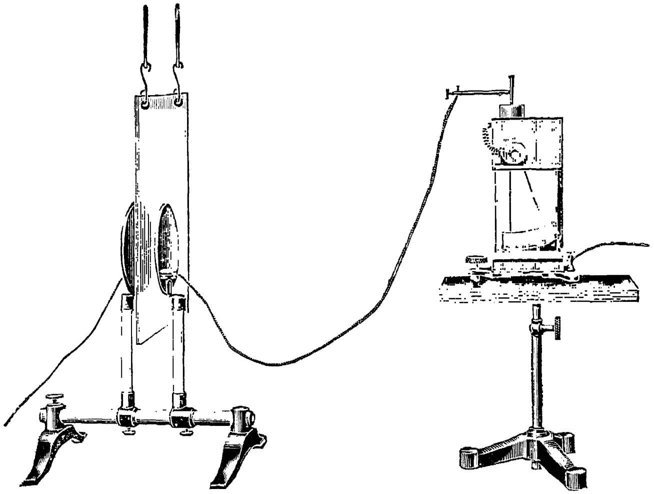


Fig. 96.—Influence of the dielectric upon the potential difference of a charged condenser

*For a given charge the field strength, or intensity, is smaller in the material medium than in vacuo.*

**Relation between Displacement and Field Strength.**—When the space between the plates of a plate condenser is *devoid of matter*, the field strength and the surface density of charge are related to each other by the equation $\sigma = K_0E = D$. The above experiments have shown that the relationship between $\sigma$ and E is not the same in a material medium as it is in a vacuum.

Following the course taken by MAXWELL, we can describe the experimental facts by adopting the following formal rule: The electric charges are characterized by the total electrical displacement of the ether, indeed they may in a certain sense be equated to this total displacement. Since now in the last experiment described above the charge remains constant, the total displacement in the condenser must be considered as the same after the introduction of the material medium as before. The field strength, however, is smaller after the material medium has been inserted. We can express this result by writing

$$\boldsymbol{\sigma} = \mathbf{D} = \mathbf{K}_0\mathbf{KE}$$

(so-called *material equation*). The factor K is equal to unity in space devoid of matter; but in material media K is always greater than unity, because in this case the value of E for a given value of $\sigma$ is always smaller than in a vacuum. The factor K is called the **dielectric constant** or the **specific inductive capacity** of the material medium.

**The dielectric constant expresses how many times smaller the field strength is in a space filled with the substance than in a vacuum, the displacement being the same in both cases.**

The justification for describing this influence of the medium by a simple factor of proportionality lies in the fact that the proportionate increase of charge for a given potential difference is independent of the magnitude of that potential difference. Thus in the experiment shown in fig. 95 (p. 82), for example, the charge is always increased $2\frac{1}{2}$ times relative to air or vacuum, no matter whether the constant potential difference is chosen very large or very small. (See also § 17, p. 99.)

**Condenser with Dielectric.***—Making use of the equation $\sigma = K_0 K E$, we obtain for the capacity of a plate condenser filled with matter (compare p. 73) the value $C = K_0KA/s$. Thus the capacity is proportional to the dielectric constant of the dielectric.

*The capacity of a condenser is increased by the factor K when the condenser is filled with a substance whose dielectric constant is K.*

The capacity of the condenser in a vacuum being known, the dielectric constant of a substance can be determined by measuring the capacity of the condenser when filled with the substance as dielectric. We have already become acquainted with an elementary example of this method (fig. 95, p. 82); for other methods see p. 476 and Chapter XVII, p. 617 *et seq.*

Table I contains the dielectric constants of some important substances.

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* A substance acting as medium for an electric field is known as a *dielectric*. (Gr., *diá*, through.)

TABLE I.—DIELECTRIC CONSTANTS

| Substance | K | Substance | K |
|---|---|---|---|
| Rock salt | 6·12 | Water at 18° C. | 81·1 |
| Sylvite | 5·03 | Ice | 3·2 |
| Calcite | ‖ 8·0 | Dimethyl sulphate | 54·8 |
| Calcite | ⊥ 8·5 | Nitrobenzene | 36·5 |
| Mica | 4 to 8 | Methyl alchohol | 33·7 |
| Gypsum | 5·66 | Ethyl alchohol | 26·0 |
| Sulphur | abt. 4 | Hexane | 20·4 |
| Flint glass | up to 11 | Bromine | 4·6 |
| Ordinary glass | 5 to 7 | Ethyl ether | 4·4 |
| Porcelain | 4 to 5·5 | Benzene | 2·3 |
| Sealing-wax | 4·5 to 5·2 | Acetone | 1·85 |
| Beeswax | 4·75 | Castor oil | abt. 2·3 |
| Gutta-percha | 2 to 4 | Oil of turpentine | 4·9 |
| Vulcanite | | Petroleum | abt. 2 |
| Caoutchouc | | | |
| Shellac | 2·5 to 3·7 | | |
| Silica, fused | 3·6 | | |
| Amber | 2·8 | $CO_2$ at 0° C. | 1·000975 |
| Rosin | abt. 2·6 | Air ,, | 1·000585 |
| Paper | 1·8 to 2·5 | $H_2$ ,, | 1·000273 |
| Paraffin wax | 1·7 to 2·3 | He ,, | 1·000074 |

By choosing a substance of high dielectric constant as dielectric it is possible to " condense " a very great quantity of electricity upon a surface. The construction of practical condensers varies according to the potential difference for which they are to be used. In order to obtain high capacities, it is usual to connect a large number of condensers in parallel (see, for example, fig. 100, p. 86).

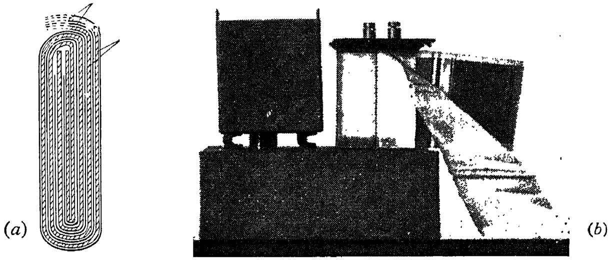


Fig. 97.—Paper condenser
(a) Diagram showing the winding (b) Practical form of construction

For relatively small potential differences the dielectric used is generally paraffined paper, covered on both sides with tin foil or aluminium foil. In this way it is possible easily to construct condensers of fairly high capacity, say 2 μf. when charged to a potential difference of 200 volts, which only weigh 100 gm. wt. (fig. 97).

*Mica* is another excellent material on account of its high dielectric constant and very great resistance to rupture by electric discharge (see Appendix). In order to avoid the effect of atmospheric humidity, such condensers are often enclosed in air-tight casings (fig. 98). Standard condensers are also frequently constructed with mica as dielectric, because air condensers of high capacity would be too large (on account of the low dielectric constant of air and its small resistance to rupture by electric discharge).

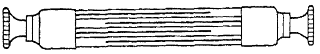
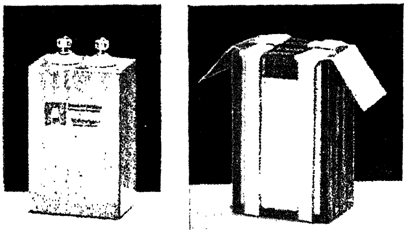


Fig. 98.—Enclosed mica condenser (Minko)

For higher E.M.F.s use is made of glass or oil. Special kinds of glass have been prepared, having particularly high dielectric constants and resistance to rupture (minos glass of Schott). These are employed in the form of plate condensers for potential differences up to several thousand volts (figs. 99 and 100). For such high E.M.F.s use is often made of jar-shaped condensers, in which the jar, which is usually cylindrical, is partially coated with metal (e.g. tin foil) on both sides so as to make it conducting, a wide margin being left uncoated so as to insulate the inner and outer sides from one another. This type of condenser (see figs. 101 and 102) is called a Leyden jar.[*]



Figs. 99 and 100.—Glass plate condenser

Plate condensers are also constructed for potential differences as high as 100,000 volts; the plates are then embedded in vulcanite so as to prevent discharge between them.

Transatlantic cables and land cables may be regarded as condensers of very great capacity, the inner plate being the conductor, the outer plate the conducting sea water (or the earth), and the dielectric the insulation of the cable. The capacity of such a cable may amount to hundreds of microfarads—a circumstance attended by considerable disadvantages (p. 210). Concerning a method of suppressing these effects, see p. 580.

A condenser of very great capacity can be obtained by placing two pieces of sheet aluminium opposite one another in an aqueous solution of borax and boric acid. If a potential difference of about 100 volts is applied to the pieces of aluminium, an extremely thin layer of aluminium hydroxide is rapidly formed upon the surface of the positive plate. This layer insulates very effectively and forms the dielectric of a condenser whose one plate is the aluminium and the other the conducting liquid. The thickness[†] of the layer is about 100 mμ; when suitably treated it will bear potential differences up to 1000 volts. The capacity obtainable in this way is from $\frac{1}{2}$ to 6 μf. per dm.$^2$.

**Polarization of the Dielectric.**[‡]—A plate condenser is connected with an electrometer as in fig. 96 (p. 83) and an insulated sheet of glass is brought between its plates, which are then pressed close up to the glass. The condenser is now charged to a certain potential difference, after which its plates are removed from the glass and connected together so that their charges neutralize each other. The neutral plates are now brought back again close to the glass plate, whereupon the electrometer indicates a potential difference between them. This difference is smaller than the original one, but nevertheless of comparable magnitude. It follows, therefore, that:

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[*] The name "Leyden jar" came into use as a result of the lectures of Nollet (1700–1770).

[†] $1\ \mathrm{m}\mu = 10^{-3}\mu = 10^{-6}\ \mathrm{mm.} = 10^{-9}\ \mathrm{m.} = 10$ Ångströms. (The notation μμ is also frequently used with the same meaning as mμ.)

[‡] The condenser was invented almost simultaneously (1746) by the cathedral dean Ewald Georg von Kleist at Cammin in Pomerania and by Cunaeus in Leyden (Holland). Both had attempted to electrify the water in a jar by dipping into it a needle connected to an electrical machine, the jar being held in the hand. On touching the needle with the other hand, they felt a strong electric shock. Benjamin Franklin

*The dielectric is the seat of an electric field.*

The charges associated with this field have obviously been produced by induction. The same effect can indeed be obtained, though to less marked extent, by bringing the condenser plates near to but not quite in contact with the glass plate after the original charging of the condenser. The recharging of the neutral condenser plates shows that the induction effect produced by the electric field in the dielectric persists for a certain time; this behaviour is unlike that found in the case of metals. Another related phenomenon is the reappearance of a potential difference some time after a condenser with dielectric has been discharged. A Leyden jar which has been strongly charged up and then discharged by momentary connexion of its plates gives after a while a quite noticeable shock if both plates are touched simultaneously (electric after-effect).

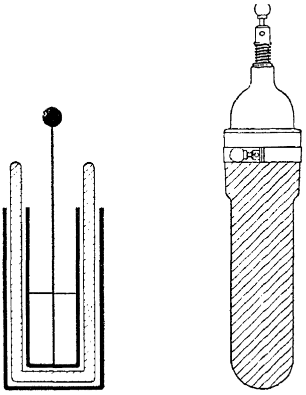
Fig. 101.—Principle
Fig. 102—Practical form of construction

Figs. 101 and 102.—Leyden jar

This leads us to the view that charges are induced in the dielectric by the influence of the field between the plates (see the remarks at fig. 64, p. 57). Since the dielectric does not conduct, i.e. since the

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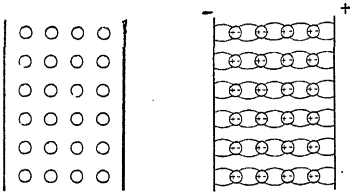
(1706–90) was the first to connect together a number of jars, the inner covering of the one with the outer covering of the next. Franklin also constructed the first condenser with separable parts (1749), with the help of which he proved that the glass also carried a charge. Wilcke observed (1762) that the one plate of a condenser can be removed and discharged, and that it becomes charged again to the same extent as before when the condenser is reassembled. He was thus the inventor of the electrophorus; this was modified by Alessandro Volta (1745–1827), who used a cake of rosin cast in the form of a plate. From the electrophorus Volta developed the condenser with separable parts, connecting it directly with the electroscope in 1787. In the same year the English clergyman Bennet invented his duplicator, which is essentially the same as Volta's condenser.

particles of electricity (electrons, p. 49) are not capable of free motion in the dielectric, we must assume that *it is the molecules of the dielectric which are affected by the field.* A transference of electricity from one molecule to another is not in general possible in insulators. The molecules therefore assume polar properties under the influence of the field; they become positively charged at one end and negatively charged at the other (*dielectric polarization*). Such a system, which carries two equal and opposite charges at a distance apart which is small in comparison with the extent of the field, is called a **dipole**. We can thus get a rough picture of the dielectric molecules as conducting spheres, which become dipoles under the influence of the field (figs. 103 and 104).



Figs. 103 and 104.—Diagram of dielectric polarization by induction

There is another possible kind of dielectric polarization in a field, namely that which occurs when the positive and negative charges in the neutral molecules are asymmetrically distributed even before the field is applied. Molecules of this kind are thus permanent dipoles. In absence of the field, however, their random disposition resulting from thermal agitation renders the external effect zero. Under the influence of the applied field the dipoles orientate themselves in such a manner that their positive ends point towards the negative side of the field and their negative ends towards the positive side (figs. 105 and 106). For the time being we need not differentiate between these two possibilities: the result is the same in both. Concerning a possible differentiation, see p. 102.

Figs. 105 and 106.—Diagram of the polarization of a dielectric with permanent molecular dipoles

Hence the structure of a polarized dielectric may be represented diagrammatically as in figs. 104 and 106. Between the condenser plates are chains of dipoles arranged head to tail.

**Definition of Field Strength and Displacement in a Dielectric.**— In space devoid of matter the field strength or intensity was measured by means of the force upon a test charge, and the electric displacement by the charge induced per unit of area. We will now inquire further into the nature of electric fields in dielectrics, with a view to extending the above concept to the case of space filled with matter.

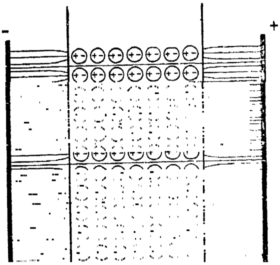


Fig. 107.—Charge in space containing matter

The displacement associated with the charges produces the polarization in the dielectric. The result of this is a surface charge upon the dielectric, where the polarized dipole chains terminate. A certain fraction of the lines of force are, as it were, cut short by these surface charges upon the dielectric, which (as is seen from fig. 107) neutralize the effect of a corresponding fraction of the charge upon the condenser plates.

The total charge upon the condenser plates is called the **true** charge. The ends of the dipole chains upon the surface of the dielectric give rise to the so-called **apparent** charges (see below). That part of the true charge upon the condenser plates which is not neutralized by the induced charge on the dielectric is called the **free** charge.

Let $\sigma$ be the surface density of the true charge and $\sigma_D$ the surface density of the induced (apparent) charge of opposite sign very near to the condenser plate. Then the surface density $\sigma_0$ of the free charge is given by $\sigma_0 = \sigma - \sigma_D$.

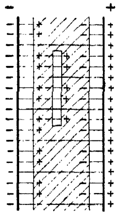


Fig. 108.—Diagram of the measurement of displacement in a dielectric.

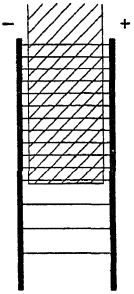


Fig. 109.—Field lines in a condenser with dielectric

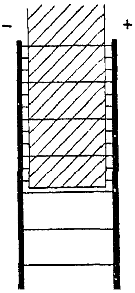


Fig. 110.—Field lines in a condenser with dielectric

From this it follows that the electric displacement in the field is made up of two parts: firstly, that due to the true charge, which acts as in a vacuum; and secondly, that due to the polarized dipole chains.

We measure *the displacement* D, as in space devoid of matter, by marking off a plate-shaped space at right angles to the direction of the field and observing, by the introduction of a double plate, the magnitude of the density of the lines of force. As can be seen from fig. 108, the space under consideration is permeated both by the lines of force corresponding to the surface charges of the dielectric at its boundaries (i.e. the lines of force from the apparent charge), and also by the lines of force which pass uninterrupted through the dielectric from one condenser plate to the other. The density of the lines of force in the plate-shaped space corresponds, therefore, to the sum of the apparent and free charges, i.e. to the *true* charge.

*The true charge is the source of the displacement* D.

From this it follows that Gauss's theorem is also true for the lines of force, i.e. for the vector **D**, *in space filled with matter.*

*The field strength* or *intensity* E is found experimentally (p. 97) to be reduced by the factor K (dielectric constant) in space filled with matter as compared with vacuum, the true charge of the condenser plates being the same in both cases; for the length of the field lines remains unaltered, but the introduction of the dielectric causes the potential difference, which is the line-integral of the field strength, to diminish to 1/K of its value in space devoid of matter. This is understandable in the light of the considerations put forward above. That fraction of the lines of force which is cut short by the dipole chains is of no effect as far as the electric force inside the dielectric is concerned.

*Only the free charge is the source of the vector* **E**.

We will consider these results more closely, taking the experiment represented in fig. 96, p. 83, as example. Fig. 109 shows the lines of force (vector **D**), fig. 110 the field lines (vector **E**). The glass plate is imagined to have been lowered half-way down between the condenser plates.

1. Consider first the upper part of the figures. According to what has been said above, the vector **D** is of the same magnitude both in space devoid of matter and in the dielectric. The vector **D** is unaffected by the transition from vacuum to dielectric. As is seen from fig. 109, the normal component of **D** (i.e. the component at right angles to the boundary surface) remains unaltered as the lines of force intersect the surface of the dielectric. We may express this by the equation

$$\mathrm{D}_{0n} = \mathrm{D}_{\mathrm{D}n},$$

the subscripts 0 and D referring to vacuum and dielectric respectively, and the subscript $n$ denoting the normal component.

It is otherwise in the case of the field strength. The vector **E** is greater in space devoid of matter than it is in the dielectric. The magnitude of its normal component changes suddenly at the boundary of the dielectric, its value in the dielectric being only 1/K of that in vacuum. We have therefore

$$\mathrm{E}_{\mathrm{D}n} : \mathrm{E}_{0n} = 1 : \mathrm{K}.$$

A surface density $\sigma_D$ appears, however, upon the boundary surface. To the left of the first boundary the field strength is $E_{0n}$, to the right $E_{Dn} = E_{0n}/K$. Hence

$$E_{0n} - E_{Dn} = (\sigma/K_0) - (\sigma_0/K_0) = \sigma_D/K_0.$$

*The surface density at the boundary is proportional to the difference between the magnitudes of the normal components of the field strength on the two sides.*

2. Consider now the boundary surface parallel to the field. Since the condenser plates are made of metal, the potential difference between them is the same whatever points be chosen. But this potential difference is the line-integral of the field strength: this line-integral must therefore have the same value in the vacuum as in the dielectric. Hence the average density of field lines must be the same in the bottom half of fig. 110 as in the top half, since the distance between the condenser plates is everywhere the same.

In the case when the space between the condenser plates and the dielectric is very small, the field strength in the lower half (vacuum) is exactly the same as in the upper half (dielectric); for then the large field strength in the very narrow space between condenser plates and the dielectric, being independent of the width of this space, contributes only a negligible amount to the line-integral.*

Travelling down the figure midway between the condenser plates, the magnitude of the field strength parallel to the boundary surface (i.e. the tangential component $E_t$) does not alter as we pass across the boundary. The tangential component is therefore continuous:

$$E_{0t} = E_{Dt}.$$

It is otherwise in the case of the *displacement*. In the lower part, where there is no apparent charge, the sources of **E** and **D** are identical. Hence in this part the diagrams of lines of force (fig. 109) and field lines (fig. 110) are equivalent. In the upper part, however, the density of the lines of force is K times greater than in the lower part (considering the field to be homogeneous as for E above, see footnote). Hence the magnitude of the tangential component of **D** changes as we pass through the boundary surface of the dielectric from the top half of the figure to the bottom, the change being represented by the relation

$$D_{Dt} : D_{0t} = K : 1.$$

We may also express the above results as follows: The external electric field may be characterized either by the vector **D** or by the vector **E**. The former is that characteristic of the external field whose *normal* component extends continuously into the dielectric: the latter is that characteristic whose *tangential* component extends continuously into the dielectric.

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* In regarding the field as everywhere homogeneous, we are neglecting the fact that the introduction of the dielectric has a deforming effect (p. 93).

**Boundary between two different Dielectrics.**—It is at once clear, and may easily be verified by drawing the field lines and lines of force, that the above treatment of the boundary surface vacuum-dielectric can be extended to the boundary between two dielectrics of different dielectric constants. We obtain

$$E_{1n} : E_{2n} = K_2 : K_1; \quad D_{1t} : D_{2t} = K_1 : K_2;$$

and also

$$E_{1t} = E_{2t} \text{ and } D_{1n} = D_{2n},$$

where the subscripts 1 and 2 refer to the two dielectrics.

*Note.*—In the interior of metals we have $E_{\text{met}} = 0$. Considering the boundary between a dielectric and a metal, we have therefore $E_{1n} : 0 = K_{\text{met}} : K_1$, from which it follows that metals behave in a static field as though their dielectric constant were infinite. From the relation $E_{\text{met}} = 0$ it follows further that $E_{\text{met},\,t} = E_{1t} = 0$. But this is the result, frequently referred to above, that the lines of force stand at right angles to the surface of a conductor.

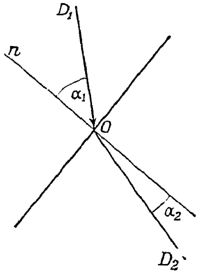

Fig. 111.—Refraction of field lines

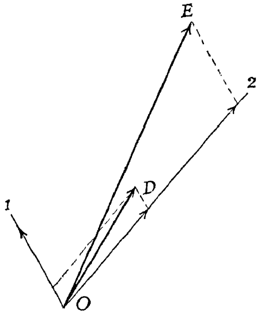

Fig. 112.—Difference in the directions of field strength and displacement in crystals

**Law of the Refraction of Lines of Force.**—The lines of force may be inclined at any angle to the boundary surface between two insulating media. It follows from the above conditions of continuity that the lines of force (and therefore also the field lines, which have the same direction as the lines of force) must be refracted. Fig. 111 shows a line of force meeting the boundary (to which $On$ is normal) between two dielectrics with the dielectric constants $K_1$ and $K_2$. The lengths $D_1O$ and $D_2O$ represent the magnitudes of the electric displacement in the respective media. Calling the angle of incidence $\alpha_1$ and the angle of refraction $\alpha_2$, we have

$$\tan \alpha_1 = D_{1t}/D_{1n} \text{ and } \tan \alpha_2 = D_{2t}/D_{2n}.$$

Now $K_1D_{2t} = K_2D_{1t}$ and $D_{1n} = D_{2n}$, and therefore

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{K_1}{K_2}.$$

*The tangents of the angles of incidence and refraction are in the ratio of the dielectric constants.*

**Field Strength and Displacement in Crystals.**—In certain crystals (e.g. calcite, p. 85) the dielectric constant has different values in different directions. In such crystals **E** and **D** no longer have the same direction. This can be understood easily with the help of fig. 112. Let OD be the direction of the lines of force. The component of the field strength or intensity in any direction is proportional to the component of the displacement in this direction, the factor of proportionality being $1/(KK_0)$ (p. 84). We resolve into components in two directions in which the dielectric constants are different ($K_1$ and $K_2$). The component of the field strength parallel to the direction 1 is $D_1/(K_1K_0)$, that parallel to the direction 2 is $D_2/(K_2K_0)$. The factors of proportionality are different in the different directions. In fig. 112, for example, $1/(K_1K_0)$ has been made equal to 2 and $1/(K_2K_0)$ equal to 4. The resultant field strength is obtained by vector addition of the components. As is seen from the figure, its direction does not coincide with that of **D**, as would be the case in an isotropic medium in which $K_1 = K_2$.

**Deformation of the Field due to Dielectric.**—In consequence of the refraction the magnitude of **D** is altered. This can be seen at once from fig. 113, in which the displacement is greater to the right than to the left, although the number of lines of force is the same in both cases; for the magnitude of the displacement is determined by the number of lines of force passing through unit area of a plane at right angles to them.

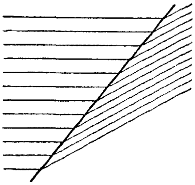

Fig. 113.—Different displacements in contiguous dielectrics.

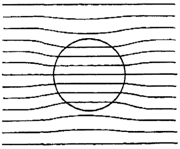

Fig. 114.—Sphere of insulating material in a medium of smaller dielectric constant. The figure shows field lines.

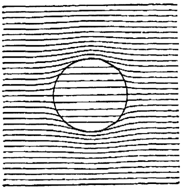

Fig. 115.—Sphere of insulating material in a medium of greater dielectric constant.

On account of the refraction of the lines of force, their density is greater, for example, inside a solid sphere than outside, when the dielectric constant of the substance of the sphere is greater than that of the surrounding medium (fig. 114). The reverse is true (fig. 115) when the dielectric constant of the sphere is smaller than that of the surrounding medium.

Simple considerations show that these phenomena, and indeed all refraction of the lines of force, are due to the apparent surface charges of the bodies concerned.

**Force at the Interface between two Dielectrics.**—In a given electric field there is a *free* charge at the interface between two media of different dielectric constant. The surface density $\sigma_0$ of this free charge is given by the difference of the surface densities $\sigma_1$ and $\sigma_2$ of the apparent charges upon the two media at the interface, i.e. $\sigma_0 = \sigma_1 - \sigma_2$. When $K_1 > K_2$, $\sigma_0$ is positive in sign. The field exerts a force upon this free charge, the direction of which is from 1 to 2 in the case shown in fig. 116. If the direction of the field be reversed, the force is still directed from 1 to 2, as can best be seen by considering fig. 116 (see also below). Since the tangential component of the field strength is the same in both media, the force has no component along the boundary surface, and is therefore normal to this surface. The magnitude of the free charge at the interface is proportional to the field strength and the force upon unit charge is again proportional to the field strength, so that the force upon the free charge is proportional to the *square of the field strength.* This accounts at once for the fact that the direction of the force is independent of the direction of the field. Summing up, we may say:

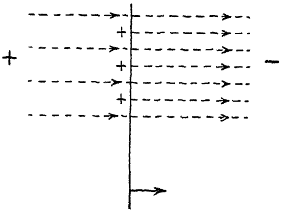

Fig. 116.—Force at the interface between two dielectrics. The figure shows field lines. $K_1$ (left) > $K_2$ (right).

*At the boundary of two media of different dielectric constants in an electric field there is a force acting normal to the interface and directed towards the medium of smaller dielectric constant.*

Hence every body embedded in a medium of different dielectric constant experiences in a homogeneous field a force of extension or compression. This is known as **electrostriction.*** When the field is not homogeneous, a body of greater dielectric constant than its surroundings will be driven into the region of greater field strength, and a body of smaller dielectric constant than its surroundings will be driven into the region of lesser field strength.

These forces of electrostriction can be demonstrated very elegantly by the following experiment (after QUINCKE):—Between the plates of a condenser (fig. 117), which is immersed in an insulating liquid, is an air bubble. It is connected by a glass tube to a small manometer in the manner shown in the figure. Under ordinary conditions the manometer indicates a pressure corresponding to the liquid column above the air bubble. But when the condenser is connected to a source of potential difference, the pressure immediately increases considerably.

**Pyro-electricity.**† — Many crystals (e.g. tourmaline, pentaerythritol, lithium sulphate monohydrate, &c.) have the peculiar property of acquiring charges of opposite signs at opposite points of their surface when warmed or cooled (*pyro-electricity*). This phenomenon has been investigated with particular thoroughness in the case of tourmaline. Fig. 118 shows a tourmaline crystal. The appearance of the charges can be made visible in the following way. The crystal is suspended by a loop of thread in a drying oven and heated to about 120° C. After temperature equilibrium has been attained, the crystal is removed from the oven and suspended outside. Then, while it is cooling, a mixture of red lead and sulphur powder is allowed to fall upon it out of a bag made of cotton stuff. By friction with the cotton the sulphur becomes negatively charged and the red lead positively. Thus the sulphur is attracted and held by the positively charged parts of the crystal surface and the red lead by the negatively charged parts. The cooling tourmaline crystal becomes covered with red powder at the one end (A), indicating negative surface charge, and with yellow powder at the other end (B), indicating positive surface charge. By warming the cold crystal the sign of the electrification is reversed.

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* Lat., *stringere*, to draw tight.

† Gr., *pyr*, fire.

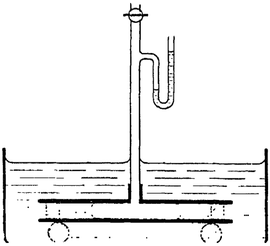

Fig. 117.—Electrostriction in non-homogeneous medium

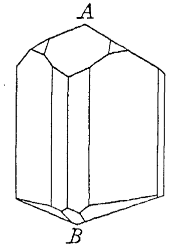

Fig. 118.—Tourmaline crystal

These phenomena suggest the following hypothesis, which is supported in the main by further experiments. The individual volume elements of the tourmaline are electrically polarized in absence of an electric field. On account of the regular arrangement of these elements in the crystal this polarization is uniformly directed throughout large pieces of the substance. Tourmaline crystals ought, therefore, to exhibit permanent equal and opposite electrification at their ends, just as a steel magnet has permanent opposite magnetic poles. The ends of the chains of polarized elements form surface layers of apparent charge. But on account of the conductivity of the air and the film of moisture on the surface of the crystal true charges are superimposed upon these surface layers so as to compensate them. Hence under normal conditions the surface charges due to the chains of polar elements are not observable at constant temperature. When the temperature is altered, however, the polarization also alters and with it the surface density of the apparent charge. If the alteration be carried out fairly rapidly, the free charges cannot compensate it quickly enough, because the air and the film of moisture are poor conductors. The charge upon the surface can then be observed. This is the explanation of the experiment described above (but see also below).

**Piezo-electricity.***—If pressure be applied to the surface of a crystal belonging to a crystal system without centre of symmetry, the surface becomes electrically charged (*piezo-electricity*). This can be demonstrated directly in the case of a quartz plate, cut out of a crystal in the manner shown in fig. 119. The plate is loaded with several kilograms weight (see fig. 120) and its two sides connected to a sensitive electrometer. At the moment when the load is applied the electrometer indicates a potential difference (about 5 volts for a pressure of 0·1 kg. wt./mm.$^2$); this disappears again after a while owing to the incomplete insulation.

* Gr., *piézein*, to press.

Conversely if a potential difference be applied to the plate, the latter contracts or expands according to the direction of the applied voltage.

In addition to quartz, these properties are also possessed in marked degree by Rochelle salt.

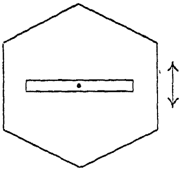

Fig. 119.—Position of a piezoelectric quartz plate in a quartz crystal. (The small arrowheads show the piezoelectric direction.)

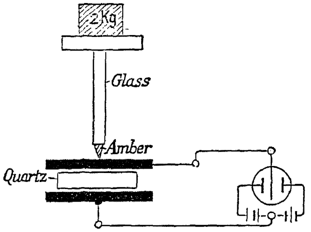

Fig. 120.—Demonstration of piezoelectric charge

The above results suggest an explanation of the appearance of pyro-electricity as due to the elastic forces caused by the thermal expansion of the crystals (false pyro-electricity). Recent investigations appear to indicate that about $\frac{9}{10}$ of the effect in the case of tourmaline is due to elastic strains accompanying the thermal change of volume.

Piezo-electric effects have recently found an important technical application on account of the oscillations which can be produced in pieces of crystal by means of rapidly varying fields (quartz oscillator).

## 16. The Energy of the Field in Space filled with Matter

Consider, as on p. 70, a plate condenser filled with a substance having the dielectric constant K. According to p. 84 the field strength or intensity is $E = Q/(AK_0K)$, the potential difference $U = Qs/(AK_0K)$, the capacity $C = AK_0K/s$ and the displacement $D = \sigma = Q/A$.

The work required to charge the condenser with a quantity of electricity Q is $W = \frac{1}{2}QU = \frac{1}{2}U^2C$ (p. 71). In the present case we have, therefore,

$$W = \tfrac{1}{2} \cdot Q \cdot \frac{Qs}{AK_0K} = \tfrac{1}{2} \cdot \frac{Q}{A} \cdot \frac{Q}{AK_0K} \cdot As = \tfrac{1}{2}DEV,$$

where V is the volume between the condenser plates.

Since the field is confined to the space between the plates, the energy density in the field must be

$$\mathbf{S = \tfrac{1}{2} DE = \tfrac{1}{2} KK_0 E^2.}$$

Thus we obtain the same relation as that already stated on p. 70 for space devoid of matter.

**The energy density of an electric field is equal to one half of the product of the displacement D and the field strength E at the point under consideration.**

This relation therefore holds in all cases.

As in the treatment of space devoid of matter (p. 70), we will consider two cases in the light of the above result.

1. Let the charge Q on the insulated condenser plates be constant. In the absence of dielectric let the magnitude of the field strength be $E_0$, the displacement $D_0 = K_0E_0$, and the energy density $S_0 = \frac{1}{2}K_0E_0^2$. Now imagine the dielectric to be introduced so as completely to fill the condenser. The displacement D remains unaltered, since the charges on the plates are constant. The field strength becomes $E = E_0/K$. Similarly the potential difference falls to $U = U_0/K$, because the distance between the plates is constant, but the field strength has been reduced everywhere by the factor K. It follows that the new energy density after introduction of the dielectric is

$$S = \tfrac{1}{2}KK_0E^2 = \tfrac{1}{2}K_0E_0^2/K = S_0/K.$$

*When the space occupied by the electric field is filled with a dielectric of dielectric constant* K, *the charges being kept constant, the field strength, potential difference and energy density are all reduced to* 1/K *of their former value.*

Hence, in this case, energy is liberated when the dielectric is introduced. This corresponds to the fact that the process of introduction of the dielectric is capable of yielding work. According to the discussion given on p. 94 the dielectric is attracted into the field, since the field strength outside is smaller (namely, zero) than inside. From the relationships given on p. 94 it is possible to prove that the diminution of the energy density of the field is quantitatively equivalent to the work done in pulling in the dielectric. The details of the calculation cannot, however, be given here.

2. Let the condenser plates be connected with a source of constant potential difference, i.e. let $U = U_0$.

In this case $E = E_0$. But $D = KK_0E_0$, and the charge also changes from $Q_0$ to $Q = KQ_0$ on introduction of the dielectric. For the energy density we have

$$S = \tfrac{1}{2}KK_0E_0^2 = KS_0.$$

*When the space occupied by the field is filled with a dielectric of dielectric constant* K, *the potential difference being kept constant, the displacement, charge and energy density are all increased to* K *times their former value.*

Although work must be performed in order to increase the energy density of the field, the introduction of the dielectric is accompanied by a gain of mechanical work even greater than in case 1; for during the process of introduction the density of the lines of force continually increases. The work required to bring the field into its final state of increased energy density must be provided by the source of potential difference to which the plates are connected all the time. The final state differs from the initial state in that the quantity of charge $(K - 1)Q_0$ has been brought on to the condenser plates at the potential difference $U_0$. Since $U_0$ is constant throughout, the corresponding work of transference is $W = (K - 1)Q_0U_0$. The total increase in field energy is

$(K-1)S_0V = (K-1)S_0As$. But $S_0As = \frac{1}{2}Q_0U_0$. Therefore $(K-1)S_0V = \frac{1}{2}W$. Thus the increase of the field energy is equal to one half of the work provided by the source of potential difference. The other half is obtained as mechanical work during the process of introducing the dielectric.

*Example.*—Consider a Leyden jar of diameter $2r = 10$ cm., made of glass ($K = 6$) of thickness $s = 0{\cdot}3$ cm. and covered with tinfoil to a height $h = 20$ cm. We may regard the field as homogeneous and put the capacity $C = AK_0K/s = \pi r(r + 2h)K_0K/s$. Substituting the numerical values, we obtain $C = 1{\cdot}25 \,.\, 10^{-9}$ farad.

In order to charge up the jar to 30,000 volts the work

$$W = \tfrac{1}{2}U^2C = \tfrac{1}{2}9 \,.\, 10^8 \,.\, 1{\cdot}25 \,.\, 10^{-9} \text{ joule} = 0{\cdot}56 \text{ joule}$$

must be performed. Hence the energy density in the glass is $S = 2{\cdot}7 \,.\, 10^{-3}$ joule/cm.$^3$ = $2{\cdot}7 \,.\, 10^4$ ergs/cm.$^3$.

**Force per Unit Area exerted by the Lines of Force.**—The equation derived on p. 70 can be written in the form

$$P = \tfrac{1}{2}K_0E^2A = SA.$$

Hence the force of compression or tension per unit area is $p = P/A = S$. This relation also holds for space filled with matter, as can be proved at once by substituting the proper values of E and D. Quite generally, therefore:

*The force exerted by the lines of force upon unit area normal to the direction of the field is numerically equal to the corresponding energy density.*

The advantage of describing the field in a dielectric formally by means of the vector **D** is clearly apparent in this formulation. We are enabled still to represent the field by means of lines of force having their ends only upon true charges, and to regard the medium as perfectly continuous, without taking account of its polarization in detail. Gauss's theorem holds, as in space devoid of matter, for the vector **D**; it takes account only of the true charges within the closed surface (p. 68). It must be borne in mind, however (see above), that:

*For a given distribution of charge the tension along the lines of force and the lateral repulsion between them are reduced, in a medium of dielectric constant* K, *to* 1/K *of their values in a vacuum.*

In the dielectric, therefore, Coulomb's law becomes (cf. p. 16)

$$P = \frac{1}{4\pi KK_0} \cdot \frac{Q_1Q_2}{r^2} \,.\, 10^7 \text{ dynes.}$$

## 17. Molecular Processes in the Dielectric.* The Electrical Structure of Matter

The description of the behaviour of dielectrics by means of the dielectric constant is purely formal in character. It takes no account of the details of the elementary processes in the medium which cause the alteration of the field. Based as it is upon considerations of force effects and charge variations in regions which are very large in comparison with molecular dimensions, it conveys very little direct information as to what properties of matter are concerned.

We made the first step towards discovering the molecular mechanism of dielectric effects when we introduced the hypothesis of dielectric polarization. This involved the quite new recognition of the presence of electrical charges *within* the molecules. We will now take up in more detail the question of the effects of the molecular structure of matter upon the electric field, i.e. the quantitative treatment of the molecular processes underlying the dielectric constant.

**Polarization.**—The magnitude of the displacement in the disc-shaped space considered on p. 89 was made up of two terms, the displacement due to the free charge and that due to the apparent charge. The part due to the free charge of surface density $\sigma_0$ is $K_0E$. The surface density $\sigma_D$ of the layer of apparent charge produced by the polarization of the molecules gives rise to a displacement which we will call the *polarization* and designate by **I**. The polarization is a vector, being associated with a direction coinciding (in non-crystalline material) with the field direction. Its magnitude I corresponds to the surface density $\sigma_D$ upon a surface at right angles to **I**. Hence we have

$$\mathbf{D} = K_0\mathbf{E} + \mathbf{I}.$$

It is possible, however, to obtain a measure of I in another way. The effect of a dipole at a distance depends upon the product of the magnitude Q and distance apart $l$ of its two charges. For the farther the charges are apart, the greater the preponderance at any point (excepting in the plane of symmetry, where the actions of the charges are equal) of the influence of the nearer pole upon a test charge, the opposite influence of the other pole decreasing as the distance between the poles is increased. The product $Ql = \mathbf{m}_e$ which determines the field of the dipole is called its **electric moment**. It is a vector, since the factor $l$ is associated with a direction (direction of the line from the negative pole to the positive pole) as well as a magnitude.

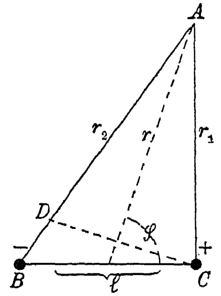

Fig. 121.—Field of a dipole

**Field of a Dipole.**—Consider a point A (fig. 121) at distances $r_1$ and $r_2$ respectively from the poles C and B of the dipole. The potential at A is

$$V = \frac{Q}{r_1} - \frac{Q}{r_2} = \frac{Q(r_2 - r_1)}{r_1 r_2}.$$

Let distance of A from the middle point of BC be $r$. Further, let AD = AC = $r_1$.

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* The first attempts at a quantitative treatment of the individual molecular processes in a dielectric were made by CLAUSIUS. The modern theory, the fundamental principles of which are given in this paragraph, has been built up chiefly by DEBYE.

Then when $r$ is very great compared with $BC = l$, the triangle BDC has practically a right angle at D. In this case

$$\cos DBC = BD/BC = (r_2 - r_1)/l.$$

Also the angle DBC becomes equal to $\varphi$ (see figure). We have therefore

$$V = \frac{Ql \cos\varphi}{r_1 r_2} = \frac{m_e \cos\varphi}{r^2},$$

putting $r_1 = r_2 = r$ and substituting $m_e$ for the product $Ql$. The magnitude of the field strength $E_r$ in the direction of $\mathbf{r}$ ($\varphi =$ constant) is

$$E_r = -\frac{d}{dr}\left(\frac{m_e \cos\varphi}{r^2}\right) = \frac{2m_e \cos\varphi}{r^3}.$$

Keeping $r$ constant, the point A moves by an amount $r\,d\varphi$ when $\varphi$ changes by $d\varphi$. Hence the magnitude $E_\phi$ of the field strength in the direction perpendicular to $\mathbf{r}$ ($r =$ const.) in which $\varphi$ increases is

$$E_\phi = -\frac{dV}{r\,d\varphi} = \frac{m_e \sin\varphi}{r^3}.$$

The magnitude of the resultant field strength of the dipole is therefore

$$E = \sqrt{E_r^2 + E_\phi^2} = \frac{m_e}{r^3}\sqrt{4\cos^2\varphi + \sin^2\varphi} = \frac{m_e}{r^3}\sqrt{3\cos^2\varphi + 1}.$$

In a homogeneous field a dipole experiences a turning moment which rotates it until the line joining its positive pole to its negative pole lies along a line of force but in the opposite sense (see fig. 122). The force acting upon each pole is $\pm\mathbf{P} = \pm Q\mathbf{E}$. If $\varphi$ is the angle between the line joining the charges and the lines of force of the field, the magnitude of the turning moment of the couple is

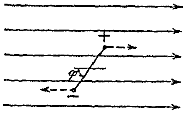

Fig. 122. — Turning moment acting upon a dipole in a homogeneous field.

$$R = m_e E \sin\varphi.$$

In the case of a chain of several dipoles placed parallel to one another and head to tail, the total turning moment is the sum of the individual moments (Vol. I, p. 114).

Imagine a cube of 1 cm. edge to be cut out of the homogeneous field in such a way that two of its faces are at right angles to the lines of force. These faces are covered with charges. A charge $+dQ$ on the one face corresponds to a charge of $-dQ$ on the other. The moment of the pair is $dM_e = dQ \cdot 1 = dQ$.

The magnitude of the total moment of the cube is therefore $\Sigma dQ = Q$. But Q is the surface density $\sigma_D$ due to the ends of the polarized chains of dipoles, and is therefore equal to I, the polarization.

*The magnitude of* **I** *is thus equal to the magnitude of the electric moment per unit volume.*

Hence the electric moment of a volume element $dv$ of the dielectric is $dM_e = I\,dv$.

*Dependence of Polarization upon Field Strength.*—The polarization I of the dielectric is proportional within wide limits to the field strength E at the point considered. We may therefore put $\mathbf{I} = \varepsilon\mathbf{E}$.

Hence we have (as to $K_0$ see pp. 66, 82)

$$\mathbf{D} = \mathbf{E}(K_0 + \varepsilon) = \left(1 + \frac{\varepsilon}{K_0}\right) K_0 \mathbf{E}.$$

But $\mathbf{D} = \mathrm{KK_0}\mathbf{E}$. Therefore the dielectric constant K and the polarization I are connected by the equation

$$\mathrm{K} = 1 + \frac{\varepsilon}{\mathrm{K_0}} = 1 + \frac{1}{\mathrm{K_0}}\frac{\mathrm{I}}{\mathrm{E}}.$$

From this we can calculate the density of the surface charge of the dielectric. The free charge density is $\sigma_0 = \mathrm{K_0E}$; the apparent charge density is $\mathrm{I} = \varepsilon\mathrm{E}$; the total charge density is $\sigma = \mathrm{D} = \left(1 + \frac{\varepsilon}{\mathrm{K_0}}\right)\mathrm{K_0E}$. We can therefore write

$$\mathrm{I} = (\mathrm{K} - 1)\mathrm{K_0E} = \frac{\mathrm{K} - 1}{\mathrm{K}}\,\mathrm{D}.$$

**Internal Field.**—The field strength **E** at a point of the dielectric may be defined as the force exerted upon a unit point charge situated in a narrow tube-shaped space free of matter in the interior of the medium, the direction of the tube coinciding with that of the field. The field strength acting upon a molecule within the dielectric will be different from this, because of the additional influence of the surrounding charges of the polarized molecules. This latter is called the "internal" field strength **F**.

In order to calculate F we proceed as follows: Around the molecules under consideration we draw a sphere (fig. 123), whose radius is great in comparison with the distances between the molecules, but still small in comparison with the extent of the external field. The other molecules within the sphere can be regarded as without effect when the external field is present, provided that they have no action when the field is absent. This is true, for example, for all gases, for a number of crystals and for many liquids. It is not true, on the contrary, for substances whose molecules normally group themselves together (i.e. associate). We shall leave all such substances out of account. With this reservation, the particular molecule we are considering is acted upon by the field strength **E**, and in addition by the forces arising from the surface charges of the sphere.* The field strength arising from these charges is $\frac{1}{3}\sigma_D/\mathrm{K_0}$, i.e. $\frac{1}{3}\mathrm{I}/\mathrm{K_0}$ in the same direction as **E**. (The derivation of this cannot be given here.) Thus for the internal field strength we have

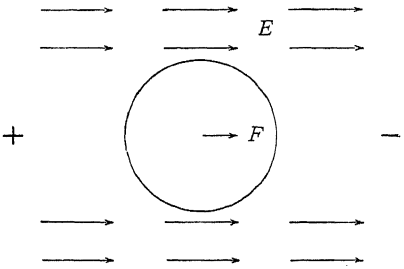

Fig. 123

$$\mathbf{F} = \mathbf{E} + \tfrac{1}{3}\mathbf{I}/\mathrm{K_0}.$$

*Polarization by Induction.*—We will first consider the molecule to be a conducting sphere. Under normal conditions, therefore, the substance has no molecular dipoles. Under the influence of a field, however, there is a displacement of charge (figs. 103 and 104, p. 88). The dipole moment of the resulting dipole can be put proportional to the field strength producing it: $m_e = \alpha\mathrm{F}$. The factor $\alpha$ is called the *polarizability* of the molecule. If $n$ is the number of molecules in 1 cm.$^3$ (i.e. a measure of the density), then the total moment per 1 cm.$^3$ is $nm_e$. But this is the polarization I. Hence we have

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* In defining **E** above we eliminated the effect of these surface charges by means of the narrow tubular shape of the vacuous space enclosing the unit point charge.

$$I = nm_e = n\alpha F = n\alpha\left(E + \frac{1}{3K_0}I\right).$$

But also
$$D = K_0E + I.$$

Substituting in the first equation the value of I obtained from the second, and also putting $D = KK_0E$, we obtain

$$\frac{K-1}{K+2} = \frac{1}{3K_0}n\alpha.$$

(Clausius-Mossotti equation.)

In this form the equation expresses the dependence of the dielectric constant upon the density for a substance with the polarizability $\alpha$. The polarizability $\alpha$ is a constant, because it is an experimental result that the dielectric constant is independent of the field strength within very wide limits.

We will now transform the equation. According to Vol. I, p. 53, we have $nM/\rho = N$, where M is the molecular weight, $\rho$ the density, $M/\rho$ the molar volume and N the Loschmidt number. ($N = 6{\cdot}06 \,.\, 10^{23}$.) Hence we can write

$$\frac{K-1}{K+2}\cdot\frac{M}{\rho} = \frac{1}{3K_0}N\alpha = P.$$

The quantity P is called the *molecular polarization.* According to the above derivation it signifies the polarization of a volume containing one gramme-molecule of the substance when the internal field strength is unity. It is an important molecular constant of fundamental significance for the treatment of the electrical properties of molecules. Considering the molecules to be conducting spheres (see above), it is possible to calculate $\alpha$. We cannot go into the details of the calculation, but will simply state the result that for a conducting sphere of radius $r$ the electric dipole moment in a field of strength E is $m_e = 4\pi r^3K_0E$. Thus in the field of strength F the dipole moment of a molecule would be $\alpha F = 4\pi r^3K_0F$, where $r$ is the radius of the molecule. Hence $\alpha = 4\pi r^3K_0$. Substituting this in the expression for P above, we have $P = \frac{4}{3}N\pi r^3 = v_M$, i.e. P is numerically equal to the total volume $v_M$ of the molecules contained in a gramme-molecule.

Since P can be calculated from the dielectric constant, a possibility is provided of testing the assumption that the molecules behave as if they were conducting spheres. It is found that P and $v_m$ * are in good agreement for a large number of non-associating substances (especially gases and vapours). Examples of such substances are hydrogen, argon, ethylene, pentane (Table II, p. 104). Thus the electric behaviour of these substances can be accounted for very well by considering them to be made up of conducting spheres of the volume of the molecules.

Certain other gases, however, have quite different (actually, considerably greater) values for P. Examples of this class are ammonia, water, hydrogen sulphide. Now it is found that it is just these substances which show a marked variation of molecular polarization with change of temperature. In the above treatment, however, no mention was made of the temperature; and indeed those substances whose molecular volume $v_M$ is equal to P show *no variation* of polarization with temperature (see also fig. 124). There must, therefore, be a quite different phenomenon associated with the polarization of the molecules with abnormal behaviour. We proceed now to consider this.

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* $v_M$ can be determined, for example, from the constant $b$ of the van der Waals equation ($v_M = \frac{1}{4}b$) (Vol. II, p. 91).

*Polarization by Orientation.*—We will turn our attention to the second case mentioned above, namely the orientation, by the field, of the molecular dipoles * already existing before the field was present. In this case each molecule of the substance possesses an inherent electric dipole moment $\mu_e$ in absence of the field. Such molecules are said to be **polar.** The orientation of the dipoles caused by the field is continually being disturbed by thermal agitation. If at a certain instant the electric moment is inclined at an angle $\varphi$ to the direction of the field, its component in the direction of the field is $m_e = \mu_e \cos\varphi$.

The number of molecules in 1 cm.$^3$ being $n$, the electric moment per unit volume is therefore $I = \mu_e \sum_n \cos\varphi$. In absence of the field this sum vanishes for gases and liquids (which we alone consider here). In presence of the field we have to take account on the one hand of the directing force of the field, on the other hand of the disturbing effect of thermal agitation. We will not go into the details of this calculation, which was first carried out by LANGEVIN. For field strengths which are not too great and for temperatures which are not too low, it gives the mean value

$$\overline{\Sigma \cos\varphi} = \frac{\mu_e F}{3kT},$$

where T is the absolute temperature and $k$ the Boltzmann constant (Vol. II, p. 60). Hence the polarization is

$$I = \frac{n\mu_e{}^2 F}{3kT} = n\bar{m}_e,$$

where $\bar{m}_e$ represents the mean component electric dipole moment of a molecule in the direction of the field. Hence

$$\bar{m}_e = \frac{\mu_e{}^2 F}{3kT}.$$

The result of the discussion is, therefore, that the polarization is still proportional to the field strength (compare p. 100) when the molecules have permanent dipole moments.

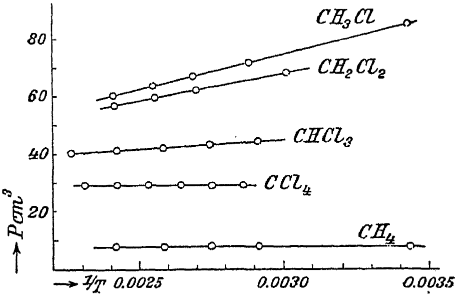

Fig. 124.—Dependence of molecular polarization of some substances upon the temperature

In addition to being orientated, the polar molecules are also deformed by the electric field. Hence the general expression for the mean component electric moment of a molecule in the direction of the field is

$$\bar{m}_e = \left(\alpha + \frac{\mu_e{}^2}{3kT}\right) F.$$

Similarly, the polarization is given in general by

$$P = \frac{N}{3K_0}\left(\alpha + \frac{\mu_e{}^2}{3kT}\right).$$

Fig. 124 shows graphically the behaviour of some simple carbon compounds.

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* The distribution of the charges may also be more complicated than that corresponding to a dipole. We cannot, however, go into this further here.

In the case of substances with symmetrical molecules, as for example methane and carbon tetrachloride, the molecular polarization is independent of the temperature. In absence of an external field they therefore possess no electric dipole moment. The centres of the positive and negative charges are coincident, as is to be expected from the chemical formulæ of these compounds and the tetrahedral symmetry of the carbon atom. The substances of unsymmetrical structure, e.g. $CHCl_3$, $CH_2Cl_2$ and $CH_3Cl$, show, on the contrary, a permanent polarity. The agreement of the theory with the experimental results is seen from the figure to be very good. The change of polarization is proportional to 1/T, as demanded by the above theoretical investigation.

Thus it is possible to obtain far-reaching information about the electrical structure of matter from measurements of dielectric constants under different conditions.

**Conclusions as to the Electrical Structure of Matter.**—The permanent electric dipole moments of a large number of substances have actually been determined.

TABLE II*

PERMANENT ELECTRIC DIPOLE MOMENTS OF FREE MOLECULES

| Substance | Formula | $\mu_e$ | | |
|---|---|---|---|---|
| | | Coulomb . cm. . $10^{-27}$ | Elementary Charge . cm. . $10^{-8}$ | C.G.S. . $10^{-18}$ |
| 1–8-dinitronaphthalene | $C_{10}H_6(NO_2)_2$ | 2·4 | 1·5 | 7·1 |
| *o*-dinitrobenzene .. | $C_6H_4(NO_2)_2$ | 2·0 | 1·3 | 6·0 |
| *m*-dinitrobenzene .. | $C_6H_4(NO_2)_2$ | 1·3 | 0·8 | 3·8 |
| *p*-dinitrobenzene .. | $C_6H_4(NO_2)_2$ | 0 | 0 | 0 |
| Benzaldehyde .. | $C_6H_5CHO$ | 0·93 | 0·59 | 2·8 |
| Acetone .. .. | $CH_3 . CO . CH_3$ | 0·90 | 0·56 | 2·7 |
| Water .. .. .. | $H_2O$ | 0·60 | 0·38 | 1·8 |
| Sulphur dioxide .. | $SO_2$ | 0·57 | 0·36 | 1·7 |
| Ethyl alcohol .. .. | $C_2H_5OH$ | 0·57 | 0·36 | 1·7 |
| Ammonia .. .. | $NH_3$ | 0·50 | 0·31 | 1·5 |
| Ethyl ether .. .. | $C_2H_5 . O . C_2H_5$ | 0·40 | 0·25 | 1·2 |
| Iodine .. .. .. | $I_2$ | 0·40 | 0·25 | 1·2 |
| Hydrogen iodide .. | HI | 0·13 | 0·08 | 0·4 |
| Carbon monoxide | CO | 0·03 | 0·02 | 0·1 |
| Carbon dioxide .. | $CO_2$ | 0 | 0 | 0 |
| Stannic iodide .. | $SnI_4$ | 0 | 0 | 0 |
| Benzene .. .. | $C_6H_6$ | 0 | 0 | 0 |
| Nitrogen .. .. | $N_2$ | 0 | 0 | 0 |
| Hydrogen .. .. | $H_2$ | 0 | 0 | 0 |

Table II gives a small selection. A knowledge of them permits us to draw certain conclusions as to the special arrangement of the atoms in the molecules, e.g. as to the shape of the molecule of water $H_2O$ or of ammonia $NH_3$. The table shows that the value of $\mu_e$ is about $1 . 10^{-27}$ coulomb . cm. for substances of simple structure. Since the diameter of these molecules is about $10^{-8}$ cm., we shall not go far wrong in assuming that the distance $l$ between the positive and negative charges is of this order of magnitude. Making use of the equation

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* Since $\mu_e$ is usually expressed in the literature in C.G.S. (electrostatic) units, these values are also included in the table.

$\mu_e = Ql$ we obtain a value of Q about $10^{-19}$ coulomb, i.e. exactly the order of magnitude of the electronic elementary charge, which we found (p. 51) to be $1{\cdot}59 \,.\, 10^{-19}$ coulomb.

From this we deduce the following important result:

*The electric charges of material molecules are elementary charges. The number of elementary charges per molecule affected by external fields is not large.*

## 18. Methods of Electrification

**Double Layers on Boundary Surfaces.**—The behaviour of matter in an electric field indicates very clearly that electricity plays a fundamental part in the structure of matter and that the electrical properties of matter must be intimately related to the other properties. We have already become acquainted above with some of the properties of molecules (or atoms) underlying the electrical phenomena. In a later volume (Vol. V) additional experimental evidence will be brought forward, and the theory will then be developed further.

We are already in a position, however, to consider in greater detail the processes underlying " electrification ".

The fundamental phenomenon is as follows:

**When a body is brought into contact with another body composed of a different material, the two bodies become oppositely electrified.**

In the case of insulators the following relation, discovered by Coehn, appears to be very generally true.

*The substance with the greater dielectric constant becomes positively electrified.*

This can be demonstrated by dipping into water a number of balls made of substances which are not " wetted " by water.* The balls must be carried upon good insulating handles. After being withdrawn from the water they are found to be negatively electrified. (Water has a greater dielectric constant than any other substance.) The charge can easily be detected in a sensitive cup electrometer. The positive charge of the water can also be demonstrated by placing the vessel containing the water within the cup of the electrometer. (This experiment is due to Pohl.) In this case the electrometer shows no deflection at first, and still remains undeflected when the ball is dipped into the water. As soon, however, as the ball is withdrawn from the water and removed out of the cup, the electrometer shows a deflection corresponding to a positive charge. If the ball is then brought back into the cup but without touching the water, the deflection disappears. The two charges (on water and ball respectively) are therefore equal and opposite. If finally the ball be dipped into the water again, there is no change; the electrometer remains undeflected.

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* It is convenient to employ glass balls which have been previously coated by being dipped into a melted sample of the substance to be tested (e.g. paraffin wax, shellac).

According to a hypothesis first put forward by Helmholtz, it is to be assumed that the opposite surface charges are still separated when the ball is in the water. Such a field of molecular dimensions, produced and maintained by the inherent electrical forces of the matter, is called an **electrical double layer.** The experiments to be discussed below show that the existence of such double layers can be demonstrated; at the same time they give us information as to the site of the double layers.

**Waterfall and Thunderstorm Electrification.**—No double layer is formed at the interface between water and air. If jets of water are allowed to pass through the air, no electrification occurs (Lenard).

On the contrary, small droplets of diameter less than about $10^{-6}$ cm. prove to be negatively charged when detached from the water surface. Such a breaking up of the surface occurs during the bubbling of gases through water or when water drops are blown to pieces by a very variable air current.

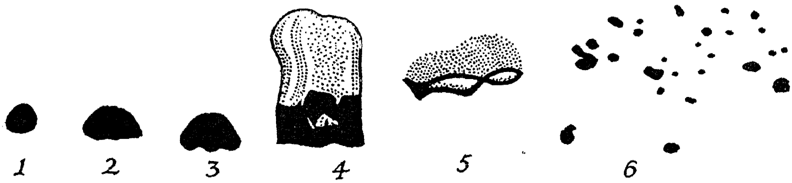

Fig. 125.—Electrification of water drops on being blown to pieces. (Lenard and Hochschwender). [From *Annalen der Physik,* Vol. 65, 1921, published by J. A. Barth, Leipzig.]

As Lenard and Hochschwender were able to show, raindrops are blown to pieces by an ascending air current of variable strength in the manner illustrated in fig. 125. After being blown out into the shape of a hat, the drops break up, forming a fine mist consisting partly of very small droplets derived from the surface and partly of drops of somewhat larger dimensions. The very small droplets, which are negatively electrified, are carried upwards by the ascending air current, while the larger drops sink. In this way the charges are separated and the lines of force elongated enormously, thus giving rise to correspondingly great potential differences.

In all probability it is processes of this kind that produce the potential differences of millions of volts which occur in thunderstorms. It is indeed often possible, by illuminating the rain with a pocket lamp, to recognize quite clearly the simultaneous presence of very fine mist and larger drops of water during a thunderstorm at night.

The water drops of waterfalls are also broken up in a similar manner. Consequently the neighbouring air is found to be electrically charged.

It is probable that *solids* also have double layers at their surfaces;

for electrification may also be found in the case of very fine particles removed from the surface. VOLTA observed that an electroscope becomes charged when the particles scraped off from sugar are allowed to fall upon its plate. When the charges are separated to large distances, very high potential differences may be produced. This is the reason why organic dust (e.g. sugar, flour, coal dust) often explodes: the surface charges are separated by the swirling of the dust, and in this way such great differences of potential are produced that a spark is formed. On dark cold nights high up in the mountains the discharges of the ice dust from certain kinds of avalanches can actually be observed.

According to LENARD, this is to be explained as follows. At the surface of a solid or liquid the molecules are subject to special forces. We have already become acquainted in Vol. I, p. 294, with the effect of these forces in producing surface tension. Another of their effects is that the molecules in the surface layers have in general quite definite orientations. As we have seen on p. 103, the (neutral) molecules of many liquids are polar in character, i.e. the "centres of gravity" of their equal positive and negative charges do not coincide. Water is a particularly good example of this kind of liquid. The orientation of the surface molecules is associated, therefore, with an orientation of the dipoles, which produces an electric field at the surface. The electrical double layer, whose thickness is approximately equal to the radius of the molecular sphere of action (Vol. I, p. 262), is so constituted that the negative charges are situated on the outside, while the positive parts (which are principally associated with mass, see p. 345) are drawn rather into the interior of the liquid.

*Thus on the surface of every liquid there is an electrical double layer with its negative side towards the exterior of the liquid.*

As is shown by the above experiments, electrical double layers of this kind are also present in the case of solids.

In materials of higher dielectric constant the attraction between the opposite charges of the double layer is smaller than in materials of smaller dielectric constant; for the higher the dielectric constant, the smaller the field strength. Hence if two substances of different dielectric constant are brought into contact and then separated again, the substance of smaller dielectric constant is able to remove with it the negative elementary charges (i.e. the electrons), since these can exist free from matter. For this to happen, it is obvious that the contact must be very intimate; the molecules of the one body must approach about as close to the negative charges of the other body's surface layer as the molecules of the other body themselves, i.e. to approximately $10^{-8}$ cm.

The "production of electricity by friction" probably depends only upon the fact that the rubbing together of the substances makes their contact so intimate that the transference of electricity from the one to the other becomes possible. The rubbing itself has nothing to do with the process of electrification; the electricity is not "produced", but a separation of the charges occurs by virtue of the forces inherent in the matter. As the bodies are separated after rubbing or contact, the lines of force, whose initial length in the double layer is about $10^{-7}$ to $10^{-6}$ cm., are drawn out to an enormous extent. Consequently the potential difference is correspondingly increased (p. 39) and may (see, for example, the formula for the capacity of a spherical condenser on p. 75) reach very large values.

*The "production of electricity by contact" (i.e. friction, induction, galvanic cells) involves nothing but an elongation of molecular lines of force.*

**Electrokinetic Phenomena.**—When a solid particle moves through a liquid, it generally carries the surrounding "skin" of liquid along with it (Vol. I, p. 264). Hence, in consequence of the double layer with its positive side as a rule towards the interior of the liquid, the particle is always accompanied by a charged envelope, usually of negative sign.

The electrification of particles in liquids plays an extremely important part in nature. The normal course of life processes is probably dependent to a large extent upon the nature of the charges upon the particles present in the body fluids. The stability of colloidal solutions (Vol. I, p. 334) is due to this cause; some have positively charged and some negatively charged particles.

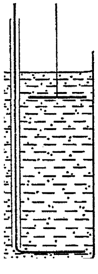

Fig. 126
Cataphoresis

*Cataphoresis.**—The electrification of small particles can be very easily demonstrated by suspending some fine, well washed clay or washed peat fibres in water and bringing the suspension into an electric field (fig. 126). After the liquid has come to rest, it can be seen with a strong magnifying glass that the particles move upwards against gravity to the anode. Reversal of the field causes reversal of the direction of motion of the particles.

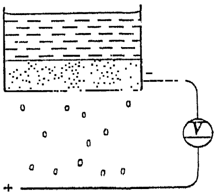

Fig. 127.—Potential difference due to flow

*Electro-osmosis.*—If the particles be held at rest, the oppositely charged liquid is set in motion. Thus, for example, water is driven out of damp peat when the latter is placed between two wire gauzes and an E.M.F. is applied. This method is employed on a large scale for the removal of water from peat (electro-osmotic process of Count SCHWERIN).

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* Gr., *kata*, along; *phoros*, carrying.

*Potential Difference due to Flow* (QUINCKE, 1859).—A liquid becomes appreciably charged when pressed through a finely porous, somewhat conducting substance, especially if the charges accumulating upon this substance can be removed (fig. 127).

**Double Layer Metal-Insulator.**—Electrical double layers are also formed between a metal and an insulator. For example, when a ball of sulphur is dipped into mercury, the surface of which is very clean, a mutual electrification occurs. Since metals give up their electrons with relative ease, they always become positively charged in such processes (see also p. 40).

In the case of steam flowing out of a metal container, a potential difference can also be detected between the container and the water droplets which are carried along in the vapour. They come into contact with the wall of the outlet tube, their surfaces with the negative charge remain partly adhering there, and the remainder is carried along in the steam current. Faraday actually constructed a steam electrical machine on this principle.* Its mode of action can be seen from fig. 128.

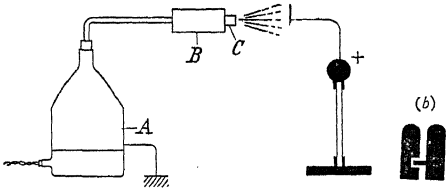

Fig. 128.—Principle of Faraday's steam electrical machine

(A) Steam generator. (B) Box with cold water running through it to condense a part of the vapour. (C) Exit nozzle. (*b*) Enlarged diagram of exit nozzle.

Electrification effects of this kind can be observed very well during the emptying of a carbon dioxide cylinder into a bag, as in the production of solid carbon dioxide. As a result of the good insulation at the low temperature (absence of films of moisture) sparks several centimetres in length may be seen to pass between the bag and the surroundings, which are in conducting connexion with the cylinder.

The electrification of benzene flowing through a metal tube may often become so great as to be dangerous. The same is true of dust particles which are carried along by the hydrogen used to fill airships. Several large airships have been destroyed by fires started in this way.

**Contact Difference of Potential between Metals.**—Contact potential differences also occur between metals. It is not so easy, however, to demonstrate the charges in this case as it is in the case of insulators, because metals cannot be separated from one another so uniformly. Although a portion of the surfaces has already been moved relatively far apart (compared with the contact distance of $10^{-8}$ cm.), a small elevation may still cause a conducting connexion between the metals, so that in spite of the relatively great separation of the surfaces only the potential difference (of the order of 1 volt) corresponding to the small region of contact is obtained. Hence whereas in the case of insulators potential differences of 10,000 volts can readily be obtained by extension of the lines of force, it is hardly possible to exceed 100 volts with metals, and then only by employing suitable artifices (p. 39).

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* The electrification of steam during efflux was discovered by chance in 1840 by a workman at Newcastle. He noticed that he could draw sparks from a boiler with his hand by holding his other hand in the escaping steam. Faraday recognized the phenomenon underlying this.

The potential difference of direct contact can be measured with the help of a calibrated electrometer. It is found to be about 1 volt. Since contact means an approach to a distance approximately equal to molecular dimensions, i.e. $10^{-8}$ cm. (Vol. I, p. 260), the corresponding field strength is of the order of magnitude of $10^8$ volts/cm. The production of a contact potential difference must be imagined as due to a passage of negative elementary charges (electrons, p. 360) from the one metal to the other, until a certain potential difference (dependent upon the particular metal and hence a material constant) is reached. Concerning the cause of this transition see p. 366.

We cannot discuss how the process depends upon material constitution until we have learned more about the structure of metals (Vol. V).

The values given above are for metals in ordinary (i.e. moist) air, in which case a film of moisture must be assumed to be present upon the metal surfaces. When the metals are very thoroughly dried the potential difference becomes much smaller. The values of the contact differences of potential between perfectly pure and dry metals are not yet known accurately, because they are very much affected by adsorbed gases and other surface layers. They appear, however, to be of the order of magnitude of a few tenths of a volt.

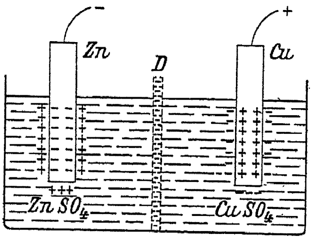

Fig. 129.—Diagram of the electrical double layers in a Daniell cell

**Double Layer Metal-Liquid.**—The influence of moisture indicates that electrical double layers must also exist between metals and liquids. These double layers underlie the action of the galvanic cells described on p. 40 *et seq.* As will be discussed in detail later (p. 280), the double layer is dependent to a large extent upon the composition of the liquid. The simplest case is that of a metal dipping into a solution of one of its salts. It becomes positively or negatively charged relative to the solution, according to its position in the electromotive series. Fig. 129 is a diagram of a Daniell cell. Concerning the magnitude of the potential difference of the double layer and for further details see p. 279 *et seq.*

CHAPTER II

# The Magnetic State of the Ether

## 1. Fundamental Phenomena of Magnetism

In addition to the electrical state, the ether may assume another state, namely the **magnetic state.**

Certain naturally occurring substances (e.g. magnetite $Fe_3O_4$, magnetic pyrites $6FeS \cdot Fe_2S_3$) possess the property of attracting neighbouring particles of iron over considerable distances. Such bodies are called **magnets.** If a steel rod be stroked with such a natural magnet, it also assumes the property of attracting particles of iron. In what follows we shall perform our experiments exclusively with such magnetized steel rods (so-called *bar magnets.*)

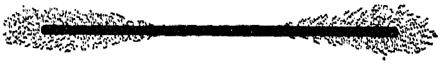

Fig. 1.—The attraction exerted by a bar magnet is greatest near its ends

"Magnetic" action is independent of the presence of air and occurs also in a vacuum. Hence magnetism is a state of space devoid of matter, i.e. a state of the ether.

A magnet is thus surrounded by a **magnetic field of force.** As in the case of an electric field, we will characterize this magnetic field by the magnitude and direction of the force acting at each point.

We can at once say something about the *form* of the magnetic field of a magnetized steel bar. Such a bar attracts iron filings chiefly at its two ends, showing only weak magnetic action in the middle (see fig. 1). The field is therefore stronger near the ends than near the middle of the bar.

In order to define the *sense* of a magnetic field of known direction, we proceed as follows. We hang up a bar magnet (which for reasons which will become clear later should be as long as possible) so that it can turn freely, and bring up another close to it (as in fig. 2). We observe that the two ends of the magnets have different properties. Confining our attention to one end only of the suspended magnet, we find that there is attraction or repulsion at that end according as we bring the one or other end of the other magnet towards it. Similarly one and the same end of the magnet which we hold in our hand exerts opposite forces upon the two ends of the suspended magnet. Each bar thus exhibits a polar character, its two ends having opposite magnetic properties. The two most intense regions of the field are oppositely directed, the force being towards the bar in one case and away from it in the other.

We can also observe this in the case of a single bar magnet. We suspend it freely from its middle point and observe that it takes up a north-south direction, to which it returns after a few swings when disturbed. Moreover by colouring one end of the bar red or by marking it in some other way, we can see that it is always one and the same end that points towards the north. Since an unmagnetized bar does not exhibit this behaviour, we must assume that *there is a magnetic field of force at the earth's surface, running approximately north-south.*

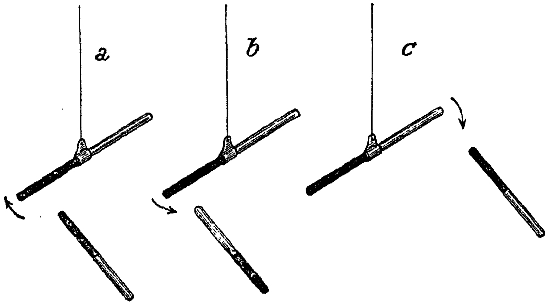

Fig. 2.—Attraction and repulsion by bar magnets

The magnetic properties associated with the end of a bar magnet which points to the north are said to be due to the **north polarity** of this end. The other end is said to exhibit **south polarity**. As remarked above, the magnetic attractions and repulsions are confined more or less to certain regions of the bar; these regions are therefore called **poles.** Thus we speak of the **north pole** and the **south pole** of a magnet.

By bringing up towards a freely suspended bar magnet another bar magnet whose north and south poles have been previously determined by hanging it up freely and noting which end points to the north, we obtain the **fundamental law of magnetic action:**

**Like poles repel one another; unlike poles attract one another.**

Two equal bars, magnetized to the same degree, show no magnetic action when placed one upon the other with opposite poles together. Hence north polarity and south polarity behave as quantities of opposite sign, just like positive and negative electric charges.

When brought near to the pole of a magnet, a piece of soft iron or steel itself becomes a magnet and able to attract and hold fast other pieces of iron. The end nearer to the pole of the original magnet becomes magnetized oppositely from that pole. The reverse is true of the end farther from the pole of the original magnet (*magnetic induction*). After removal from the influence of the inducing pole the soft iron loses its magnetism almost completely, but the steel remains magnetized (**permanent** magnetism).

## 2. Magnetic Lines of Force

We are now in a position to trace out the directions of the magnetic forces, just as we did in the case of an electric field.

The direction and sense of a magnetic field at any point can be most easily determined with the help of a small bar magnet (magnetic needle).

**The direction of a magnetic field at any point is the direction taken up by a magnetic needle subject to no other forces.**

**Representation of the Field.**—The investigation of the whole space around a magnet by the above method shows that the needle takes up a perfectly definite direction at every point of the field. The needle must be mounted (as in fig. 3) so as to be able to turn in all directions. The positions of the needle in the field are tangential to a system of curved lines running from the region of the north pole to that of the south pole.

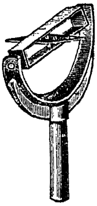

Fig. 3.—Magnetic needle mounted so as to be capable of assuming any direction.

A more convenient method is to use iron filings. As stated above, each particle becomes a small magnet when placed in the field, and two particles lying end to end have opposite poles together. Hence, owing to the attraction between these poles, the particles become joined together into rows, the whole system possessing a certain rigidity.

In order to do away as far as possible with the influence of gravity, which hinders the free arrangement of the filings, we lay the magnet horizontally upon the table, cover it with a sheet of stiff (not too smooth) paper, and sprinkle the iron filings upon the paper. We then disturb the paper a little by tapping it with a non-magnetic object, whereupon the particles of iron rotate and move slightly, arranging themselves into lines. These lines join the one end N of the bar magnet to the other end S in curves as shown in fig. 4 (see also fig. 11, p. 116).

Figs. 5 and 6 show the nature of the magnetic lines of force between two unlike and two like poles respectively. We see that they are essentially the same as the diagrams representing the lines of force between unlike and like electric charges (figs. 9 and 10, pp. 10 and 11). It remains now to fix the sense of the magnetic lines of force. It has been decided to define the positive sense as that from the south pole towards the north pole of a small freely suspended magnetic needle situated at the point considered. From this definition it follows that:

**The lines of force begin at the north pole of a bar magnet and end upon the south pole.**

Thus the magnetic state of the ether can be represented by lines of force, just as in the case of the electrical state. It is found, indeed, that the same "fictitious" elastic properties can be ascribed to the statical magnetic lines of force as to the electrical lines of force (p. 21), namely, a tension in the direction of the lines and a lateral repulsion at right angles to them. Thus the attraction in fig. 5 is due chiefly to the tension of the lines of force; the repulsion in fig. 6 results in the main from the lateral pressure or repulsion. These similarities between the two kinds of field justify the treatment of the magnetic field in terms of concepts corresponding to those used for the electric field. In this connexion, however, we meet with a peculiar difficulty. This must first be explained.

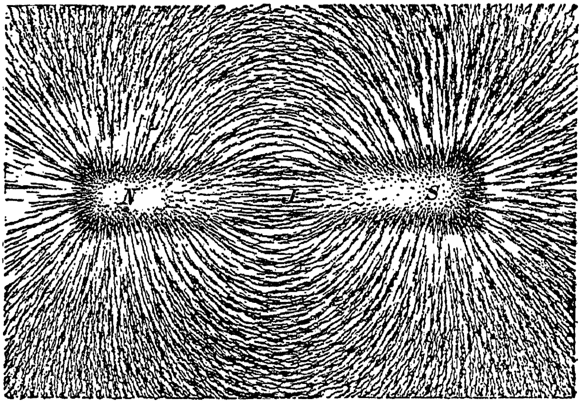

Fig. 4.—Diagram of the lines of force in the case of a bar magnet

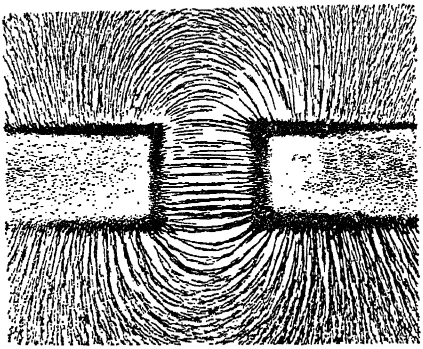

Fig. 5.—Field between two unlike magnetic poles

**Difference between Electric and Magnetic Fields.**—The lines of force of a static electric field begin and end upon electric charges. To obtain an electric field analogous to the magnetic field of a bar magnet, we will consider an electrified bar whose lines of force in the surrounding space have the form shown in figs. 4 and 7. The corresponding charge distribution is that indicated in fig. 7. In order that this distribution may persist, the bar must be made of insulating material.* If we break the bar in two (observing the necessary precautions) and remove one of the halves, we obtain for the other half a distribution of lines of force like that shown in fig. 8. Starting from the electric charge, the lines of force proceed outwards to infinity (or to the surroundings).

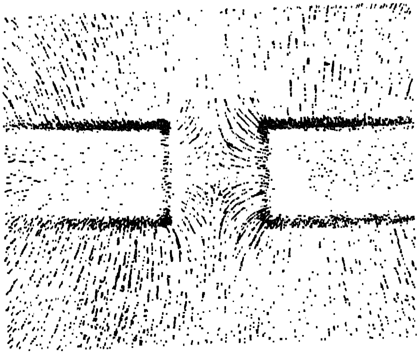

Fig. 6.—Magnetic field between like poles

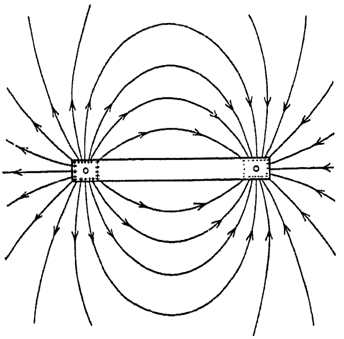

Fig. 7.—Field of force in the space surrounding a rod charged electrically as a dipole.

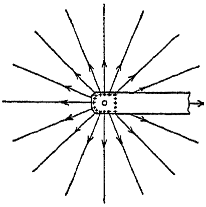

Fig. 8.—Field of force of one half of the rod shown in fig. 7.

We now make the same experiment with a bar magnet. We break it into two pieces through the middle (fig. 9), and find that each of the separate halves is a complete new magnet with opposite poles (fig. 10). The same is true if the halves be further broken; each portion proves to be a complete magnet.

Figs. 11 and 12 show the form of the lines of force before and after the breaking into two and removal of one half. We observe a fundamental difference between the fields of fig. 8 and fig. 12. In the electrical case the half bar with its charge is merely the starting-point (*or* end-point) of the lines of force; but in the magnetic case there are equal numbers of lines of forces starting *and* ending upon it. Since the latter behaviour persists no matter how far the mechanical division is carried, we must conclude that the lines of force continue in the interior of the magnet so that each line is closed. It is quite impossible by any means to find the ends of magnetic lines of force. Hence whereas we can isolate positive or negative charge upon a body, we are unable to bring a body into such a state that it is magnetized with north polarity alone or with south polarity alone. A body can never be only a starting-point or only an end-point of magnetic lines of force: a certain degree of one kind of magnetic polarity is always associated with an equal degree of the other kind of polarity at some other part of the body, so that the number of lines of force starting from the body is equal to the number ending upon it. Now according to Gauss's theorem (p. 68) this is the condition for the absence of sources or sinks of lines of force in the closed region, i.e. in our case in the region of the body. Since it is also impossible to find any ends to the lines of force in the external space, it follows that:

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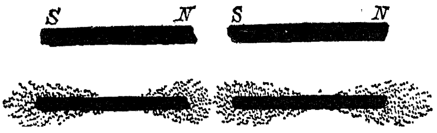
* For the sake of simplicity its dielectric constant is assumed to be unity.



Figs. 9 and 10.—Magnetization of the portions of a magnet broken into halves

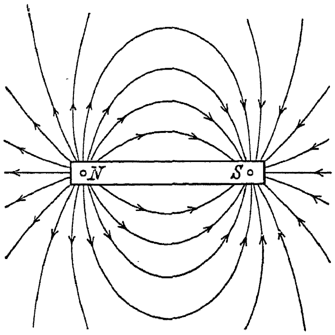


Fig. 11.—Field of a bar magnet

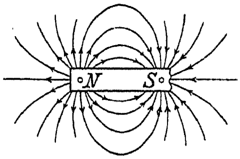


Fig. 12.—Field of one half of a broken bar magnet

*Free magnetism, i.e. a quantity of magnetism or magnetic "charge", does not exist. That is to say, the idea of substance cannot be applied to magnetism,*

A bar magnet does not lose anything of its own magnetization in magnetizing another piece of steel by contact. This peculiar fact also indicates that magnetization does not consist of the transference of a substance, as is the case in electrification by contact.

From all the above considerations we arrive at the following important result:

*Magnetic lines of force are always closed.*

This will become particularly clear later (p. 403).

## 3. Iron in a Magnetic Field

As was shown by Faraday, the course of the magnetic lines of force is somewhat affected by the presence of material bodies. In general this effect is very slight (for further details, see p. 402). The so-called **ferromagnetic** materials (p. 413) are exceptions to this rule. The most important among them is iron.

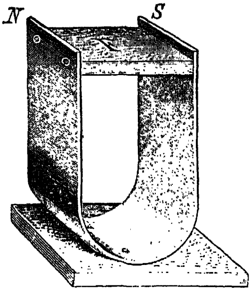


Fig. 13.—Horse-shoe magnet for the production of a homogeneous field.

**Influence upon a Magnetic Field.**—For simplicity in investigating the disturbance of the course of the lines of force we will make use of a homogeneous magnetic field, i.e. a field in which the lines of force are all parallel and of uniform density of distribution.

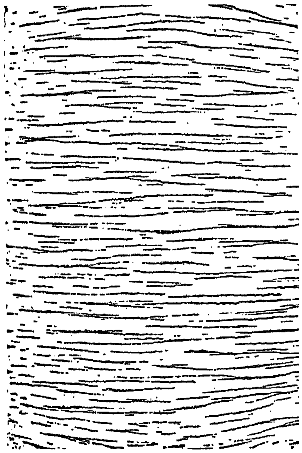


Fig. 14.—Homogeneous field in the plane F of the horse-shoe magnet shown in fig. 13.

Such a homogeneous field can only be produced approximately with magnets, because the lines of force always converge towards the regions of the poles.

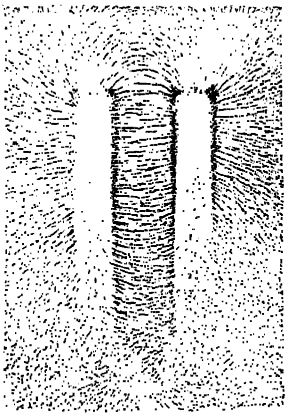


Fig. 15.—Magnetic field of a horse-shoe magnet

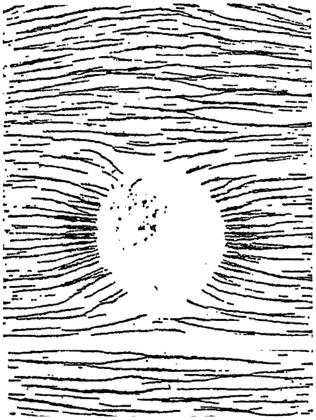


Fig. 16.—Deformation of a homogeneous field by a circular iron disc

For our practical experiments we make use of the field between the extended poles of a magnet consisting of a large rectangular sheet of steel, bent into the shape of a horse-shoe as shown in fig. 13. Between the poles N and S is fixed a board F to support the body which is introduced into the field. Making the magnetic field visible in the plane of the board F by means of iron filings, we obtain the result shown in fig. 14. Fig. 15 represents a vertical section through the field of the horse-shoe magnet. From these figures it can be seen that in the plane F in the middle of the field the lines of force are approximately straight, parallel and of uniform density.

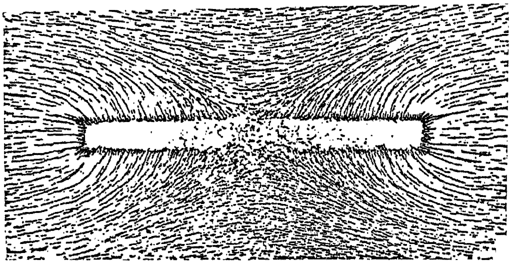


Fig. 17.—Deformation of a homogeneous field by an iron bar

An extremely marked alteration is produced when a small plate of iron is introduced into the field. Brass or wood, on the contrary, leave the field practically unaffected. Fig. 16 shows how the lines of force converge towards the piece of iron.

Fig. 17 shows the effect upon the originally homogeneous field when an iron bar is introduced. Comparing this with fig. 18, which represents the field of a steel magnet of the same size (but without superimposed field, which accounts for the differences between the two figures), we see that the bar of iron has become a magnet in the homogeneous field (p. 404). Thus we have the field representation of the phenomenon already mentioned on p. 113, namely, magnetization *by induction*.

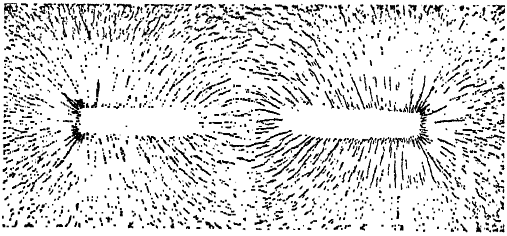


Fig. 18.—Field of a bar magnet

These figures show that iron has the property, so to speak, of sucking the lines of force through it. This can be interpreted by supposing the iron to be more permeable than air to magnetic lines of force. We say that the iron has the greater **permeability**. This property obviously corresponds to the dielectric constant in the case of an electric field. Iron offers, as it were, a smaller resistance than air to the passage of magnetic lines of force through it. Hence in the iron the lines of force are more densely packed. The mutual lateral pressure between the lines of force must therefore be smaller within the iron than in air (p. 430).

This can be demonstrated very beautifully by an analogous experiment in which the magnetic (or electric) lines of force are replaced by the lines of flow of a liquid (method of HELE-SHAW, see p. 17). The liquid flows between two parallel sheets of glass. At a certain place, however, the bottom sheet is provided with a rectangular depression and the upper sheet with a corresponding rectangular elevation, so that over this region the distance between the sheets is greater and the resistance to the flow of the liquid is less than elsewhere. Fig. 19 shows the resulting flow-line diagram, which is seen to be completely analogous to the force-line diagram of fig. 17 (compare also fig. 114, p. 93).

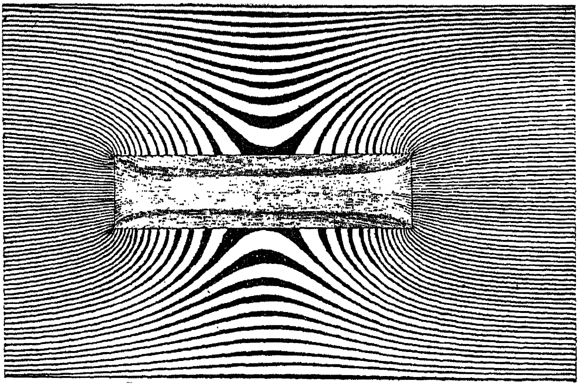


Fig. 19.—Lines of flow of a liquid in the space between two plates, the distance between which is greater in one region than elsewhere. [From Ebert, *Magnetische Kraftfelder*, published by J. A. Barth, Leipzig.]

**Magnetic Screening Effect.** — Upon the high permeability of iron depends a very important practical property, namely, its *screening action*.

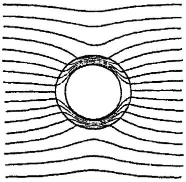


Fig. 20.—Deformation of a homogeneous field by an iron ring.

When, namely, we introduce a hollow ball or hollow cylinder of soft iron into a magnetic field, the lines of force pass into the iron and run almost exclusively in the interior of the walls of the hollow body (fig. 20), leaving the inner space practically free from lines of force, i.e. free from magnetic forces (see fig. 21). Use is frequently made of this property in order to protect bodies and apparatus from the influence of magnetic fields.

The following is another example of the high permeability of iron:

Two steel magnets in the shape of semicircles are placed with their unlike poles together. Almost all the lines of force from the north pole of the one magnet pass into the south pole of the other, and the surrounding space remains practically free of lines of force. Now imagine the two semicircular magnets to be completely united so as to form a closed ring. The lines of force now run entirely

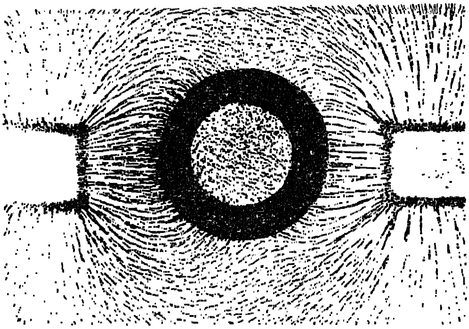


Fig. 21.—Magnetic screening action of iron

in the interior of the steel ring and we obtain a *magnet without poles,* a so-called *toroidal magnet* or *toroid.** If such a ring be cut through at some point, an extremely intense but spatially very limited magnetic field is obtained between the two poles on the opposite sides of the narrow gap (see also p. 423).

## 4. Quantitative Treatment of the Behaviour of Magnets

**The Concepts underlying the Quantitative Description of Magnetic Fields.**—The above mentioned peculiar nature of magnetic fields (p. 115) makes it impossible without further consideration to apply the same methods, based upon the concept of point charges, as were used for the description of electric fields.

The portion of the field of a bar magnet which lies outside the magnet can (as in fig. 7, p. 115) be represented in the same way as the electric field of a suitably charged insulating rod. Now it is found that the electric field of such a rod at a great distance away is not seriously affected by supposing the charges to be concentrated in very small regions (points). This can easily be shown practically to be the case. Thus in fig. 7 (p. 115) the charges can be regarded as concentrated within the small circles. In the limit we obtain a dipole (p. 88), i.e. two opposite point charges at a certain definite distance $l$ apart.

As was shown on p. 100, the behaviour of such a dipole is determined by its electrical dipole moment. Thus two dipoles, the first of charge Q and length $l$ and the other of charge 10Q and length $l/10$, have equal fields (in direction as well as in field strength or displacement) at a distance away which is large compared with $l$. They also experience equal turning moments in a given homogeneous field. Hence the field

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*Lat., *torus,* swelling.

of a dipole at a great distance and the forces to which it is subjected in a homogeneous field are completely determined by its dipole moment.

In the same way we can introduce an analogous quantity for a bar magnet, the *magnetic moment* **M**. It determines both the magnitude and the direction of the magnetic field at a great distance from the bar magnet, as well as the turning moment acting upon the magnet when placed in a homogeneous field.

**Forces Acting upon a Bar Magnet in a Homogeneous Field.**—We can consider the behaviour of a bar magnet in a homogeneous magnetic field in the light of the diagram of lines of force shown in fig. 22. The consideration is analogous to the case of electrical lines of force. According to Faraday's view there is a tension in the direction of the lines and a pressure or repulsion between them (p. 21). We see from the figure how each pole of the bar magnet is subjected on the one side to the tension along the lines and on the other side to the lateral repulsion between them. These forces form a couple tending in the case we are considering to turn the bar magnet in the anti-clockwise sense.

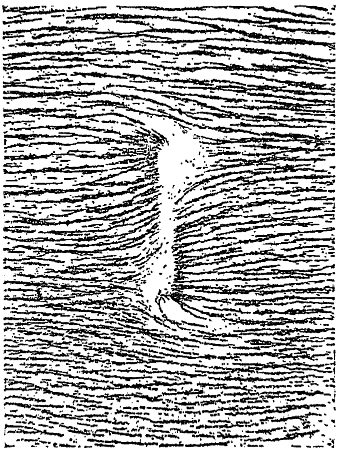

Fig. 22.—Bar magnet at right angles to a homogeneous field

The magnitude of the turning moment acting upon an electric dipole is $R_e = EM_e \sin \phi$ (p. 100). Thus in the case of dipoles of given dipole moment $M_e$ the magnitude of this turning moment is a measure of the strength of the field.

Experiment has shown that a corresponding relation exists for bar magnets. Just as in the case of a given electric dipole the turning moment $R_e$ can be chosen as a measure for the strength of the electric field in which the dipole is situated, so also can we take the turning moment acting upon a bar magnet as a measure of the strength of the magnetic field producing it. Hence in analogy to the quantity E or $|\mathbf{E}|$ appearing in the above formula we can characterize the magnetic field by a vector of magnitude $|\mathbf{B}|$ or B. This vector **B** is called the **magnetic induction** or **density of magnetic flux** (p. 377). Just as in the case of the electric field the direction of the field strength **E** or the displacement **D** is characterized by the direction taken up by the line joining the charges of a freely turning dipole (compare the exploration of the electric field by means of an electrical pointer, p. 7), so also does the final direction taken up by a freely suspended magnetic needle determine the direction of the magnetic induction **B**.

This direction is therefore the same as that of the magnetic lines of force at the point considered (p. 113).

**The magnetic lines of force are the lines of magnetic induction.**

In analogy to the electrical case we write

$$R_m = MB \sin \phi,$$

where $R_m$ is the magnitude of the turning moment upon the bar magnet, M the magnitude of the magnetic moment of the latter, B the magnitude of the magnetic induction of the field and $\phi$ the angle between magnet and field.

**Experimental Investigation.**—In order to investigate the forces acting upon a small magnetic needle, we suspend the needle from a stand by a fine wire in such a way that it can rotate over a divided circular scale (fig. 23). The whole suspending device can also be rotated and its angular displacement read off upon another graduated circular scale. In this way we can compensate the turning moment acting upon the needle by means of the torsion of the suspending wire; this is done by turning the point of suspension (torsion head) until the needle is brought back to its initial position or some desired position. The torsional rigidity $M_1$ (Vol. I, p. 208) of the suspending wire being known the total
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is
n
n

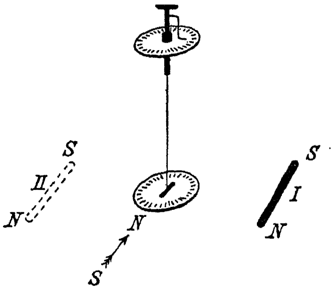

Fig. 23.—Measurement of turning moment

*Investigation of Variation of Turning Moment with Angle.*—We use here the field of a large bar magnet laid in the direction of the meridian. In order to minimize the influence of the earth's field, the magnet must be magnetized as strongly as possible. We choose a point far away from the magnet, at which the field direction coincides with the direction of the bar magnet itself and with the north-south direction. At this point we place our small needle with its torsion device. To determine the zero position of the suspension, we allow it first to come to rest without the magnetic needle. We then turn the torsion head until this zero direction of the suspension coincides with the field direction. After this preliminary procedure has been concluded, we place the needle upon the suspension. Since the field direction is the stable position of rest for the needle, the latter remains pointing in the direction into which the suspension had been previously adjusted. We now turn the torsion head through a certain known angle, simultaneously reading off the angle through which the magnetic needle has turned out of its original position of rest. We find in this way that the angle read off above is proportional to the sine of the angle read off below.

But the torsional moment is proportional to the angle (Vol. I, p. 207). It follows therefore that the turning moment acting upon the magnetic needle is proportional to the sine of the angle between the needle and the direction of the field, i.e. $R_m = C \sin \varphi$.

*Investigation of the Variation with the Field.*—We adjust the magnetic needle to make a certain angle with the field direction, and read off the angle of torsion. Then we place a second magnet, of the same shape as the first and as nearly as possible equally magnetized, at the same distance and in the same direction but on the opposite side of the magnetic needle from the first magnet (see fig. 23). In order now to adjust the needle to the same direction as before in the new field, whose direction is the same as the old field, we have to turn the torsion head farther through the same angle as before.

If the two magnets are not similar, their difference can be determined by means of the turning moment exerted upon the needle. Also by using one magnet, first in one direction and then reversed, the earth's effect (when not negligible) can be determined.

It follows from these experiments that the turning moment acting upon the needle is proportional to the induction B of the magnetic field, i.e. $R_m = MB \sin \varphi$, where M is a quantity characteristic of the particular needle used. If another magnetic needle be employed, the magnitude of the turning moment experienced at any position in the field is different from before. But the ratio of the turning moments acting upon the two needles in the same position is constant for all positions in the field. Thus M depends only upon the nature of the needle. It is called the magnetic moment and is a convenient quantity for the characterization of the needle with regard to its behaviour in a magnetic field.

**Field of a Bar Magnet.**—On account of the quantitative analogy to the behaviour of an electric dipole, we can express the magnetic induction B at a great distance from a bar magnet in the following way (see p. 100):

*The magnitude of the component of the magnetic induction of a bar magnet of magnetic moment* M *at a distance* r *and in a direction making an angle* $\phi$ *with the longitudinal axis of the bar is*

$$B = \frac{M\sqrt{1 + 3\cos^2\phi}}{r^3}.$$

**Vibrations of a Bar Magnet in a Magnet Field.**—Consider a bar magnet mounted so as to be able to rotate about its centre of gravity in a magnetic field. When displaced from its equilibrium position through an angle $\phi$ which is so small that it can be set equal to its sine, the magnet is acted upon by a turning moment $R = MB\phi$. In this case the restoring moment or couple is proportional to the angular displacement $\phi$. When released the magnet executes harmonic vibrations about its equilibrium position (Vol. I, p. 146). The period of swing can be calculated from the corresponding formula $T = 2\pi\sqrt{I/R_1}$, where I is the moment of inertia about the axis of rotation and $R_1$ the value of R when $\phi = 1$, i.e. $R_1 = MB$. We have therefore

$$T = 2\pi\sqrt{\frac{I}{MB}}.$$

The period T can be determined by direct observation, the moment of inertia I either by special vibration experiments or from the dimensions of the magnet. Hence the formula allows of the calculation of the accurate value of MB.

The formula, as given above, only holds when the magnetic field is homogeneous. In the case of an inhomogeneous field it is only valid provided we confine our attention to such a small region that the inhomogeneity of the field becomes negligibly small.

**The Unit of Magnetic Induction and of Magnetic Moment.**—Before we can evaluate the above relation numerically we require measuring units for B and M. We can arrive at these in the following way (after GAUSS).

For the sake of simplicity consider a field that is homogeneous and horizontally directed. For a bar magnet swinging in this field we have found the relation $T = 2\pi\sqrt{I/MB}$ between the period of swing T, the magnetic moment M of the magnet and the magnetic induction B of the field. From this we have $MB = 4\pi^2 I/T^2$. Since T and I can be determined experimentally, the product MB can be measured from vibration experiments.

Each of the quantities M and B can then be found, provided we have one other relation between them, for instance, if we know the value of the quotient M/B. This quotient can actually be measured by observing the deflection of a short magnetic needle by the same bar magnet of magnetic moment M, the needle being situated in the field of induction B. The way in which this is done is shown in the following calculations.

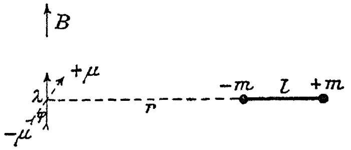

Fig. 24.—Gauss' method of measuring magnetic field strength. (First position.)

1. In fig. 24 the arrow with both head and tail represents a small magnetic needle of length λ and magnetic moment *m*. The other arrow shows the direction of the magnetic induction **B** of the field. The thick horizontal line represents a bar magnet whose length *l* is small compared with its distance *r* from the small needle. When the needle is alone in the field, its direction coincides with that of **B**. It is deflected through an angle φ out of this position by the bar magnet of magnetic moment M, placed as in the figure. The needle is in equilibrium when the turning moment ($R_1$) of the homogeneous field is equal and opposite to that ($R_2$) of the couple due to the bar magnet.

The component $B_m$ of the magnetic induction of a short bar magnet in the direction of its own longitudinal axis is equal to $2M/r^3$ (see p. 100). Since by assumption the needle is small in comparison with *r*, we can regard the values of $B_m$ at the two ends of the needle to be the same. The corresponding turning moment is then (p. 100)

$$R_2 = B_m m \sin(90^\circ - \varphi) = 2Mm\cos\varphi/r^3.$$

The opposite turning moment due to the field is

$$R_1 = Bm\sin\varphi.$$

The needle is in equilibrium when $R_1 = R_2$, i.e. when

$$Bm\sin\varphi = 2Mm\cos\varphi/r^3.$$

From this we have the quotient M/B, namely,

$$M/B = \tfrac{1}{2}r^3\tan\varphi.$$

This equation does not involve the magnetic moment of the deflected needle. The quotient M/B can be calculated from the values of the angle of deflection φ and the distance $r$ between magnet and needle, both of which quantities can be measured by direct experiment.

2. The relative position of bar magnet and magnetic needle shown in fig. 24 was called by Gauss the *first position*; that shown in fig. 25 was designated as the *second position*. The quotient M/B can also be calculated from the deflection $\psi$ produced in the second position. The result, which can be derived in the same way as that for the first position, is given by the equation

$$M/B = r^3 \tan\psi.$$

Combining now the value of the product MB obtained from vibration experiments with the value of the quotient M/B obtained from deflection experiments we obtain by multiplication and division

$$M = \sqrt{MB \times M/B}, \qquad B = \sqrt{MB \div M/B}.$$

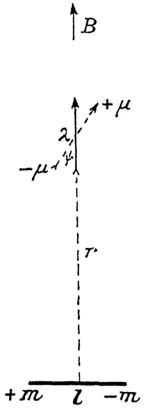

Fig. 25.—Second position of Gauss

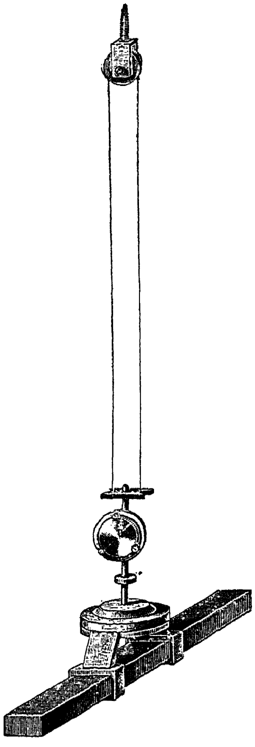

Fig. 26.—Vibration magnetometer

The right-hand sides of these equations involve only quantities whose units have already been fixed elsewhere. Hence by means of a suitable numerical factor these equations can be used to define the units of magnetic induction and of magnetic moment. Thus for example, using the results of the two preceding sections, we may write,

$$B = f\sqrt{\frac{4\pi^2 I}{T^2 r^3 \tan\psi}}.$$

In the so-called *absolute magnetic system of measurement* the numerical factor $f$ is made equal to unity and all the other quantities on the right-hand side are measured in C.G.S. units.

**The unit of magnetic induction defined in this way is called a gauss** ($\Gamma$).

From the above equation it follows that the dimensions of magnetic induction are [cm.$^{-\frac{1}{2}}$ gm.$^{\frac{1}{2}}$ sec.$^{-1}$].

Similarly we obtain a *unit of magnetic moment* in the absolute measuring system. This unit, whose dimensions are [cm.$^{\frac{5}{2}}$ gm.$^{\frac{1}{2}}$ sec.$^{-1}$], bears no special name.

For reasons to be explained later the numerical factor is put equal to $10^8$ in the *practical measuring system.*

**The practical unit of magnetic induction is 1 volt sec./cm.$^2$** (see p. 376).

Hence 1 gauss $= 10^{-8}$ volt sec./cm.$^2$.

The special instruments used for the measurement of MB and M/B are called **magnetometers** (figs. 26 and 27).

**Intensity of Magnetization.**—By dividing the magnetic moment M of a magnet by its volume V we obtain the magnetic moment per unit volume or *intensity of magnetization*, as it is called. The more slender the magnet, the greater the intensity of magnetization attainable (maximum about 750 C.G.S. units/cm.$^3$).

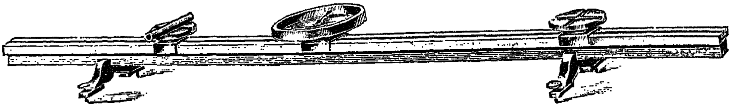

Fig. 27.—Deflection magnetometer

**Coulomb's Law for Magnetic Poles.**—The use of the magnetic moment for describing the behaviour of magnets in a magnetic field as well as the fields of bar magnets themselves is completely satisfactory and yields results of any desired degree of approximation, provided that the distance considered be correspondingly great in comparison with the dimensions of the magnet. Attempts have also been made to describe the field nearer to a magnet in the same way as we have employed above for the field at a great distance, namely, as due to a dipole. In analogy to the case of the electric dipole moment $M_e = Ql$ (p. 99), the magnetic moment M can be regarded as the product *ml* of a quantity of magnetism *m* and the distance *l* between the poles. In the case of a bar magnet of the usual shape the distance *l* between the poles is about $\frac{5}{6}$ of the total length. It can be shown that the sources and sinks of the magnetic field which are imagined to be concentrated at the point poles obey *Coulomb's law. This means (p. 16) that* fields can be produced with spherical symmetry at least in a certain region, i.e. fields which may be regarded as produced by a single point pole. It also indicates that the magnetic induction is proportional to the density of the lines of force.

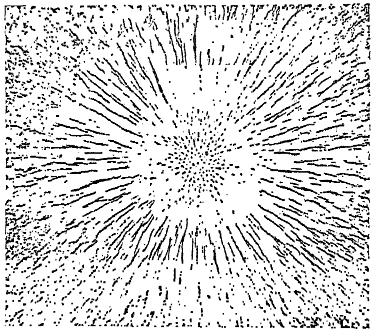

Fig. 28.—Diagram of the lines of force of a bar magnet as viewed in the direction of its length

The field of a bar magnet is spherically symmetrical within distances from the poles which are small in comparison with the length of the magnet (compare fig. 11, p. 116). This can be shown by taking a cylindrical bar magnet about 60 cm. in length, setting it up vertically with its north pole at the top, placing a sheet of smooth stiff paper horizontally upon the upper end, and scattering iron filings upon it. The filings arrange themselves in straight lines running out radially from the magnetic pole (fig. 28). These lines represent here, as in other similar filing diagrams, only the projections of the lines of force upon the plane of the paper.

As an example of actual measurement of the force between two magnetic poles, take the case of the repulsion between two like poles.

Set up two similar, similarly magnetized steel rods (knitting needles) as shown in fig. 29, with like poles one above the other. The one rod forms one half of a balance beam turning easily about a knife edge supported by the stand A (magnetic balance). Thus one pole $n_1$ is at the end of the beam and the other at the axis of rotation. The other arm of the beam is graduated as a rider scale upon which light riders, weighing whole multiples of a dyne, can be placed. The second magnetized rod is held in a horizontal position in the stand C, and can be moved up and down in a vertical direction. In this way its pole $n_2$ can be adjusted at known distances above the like pole $n_1$ of the other rod. These distances can be read off upon the vertical mirror scale B.

There is a repulsion between the like poles $n_1$ and $n_2$. By placing a suitable rider on the rider scale the balance beam can be brought back into a horizontal position. The turning moments of the rider weight and the magnetic repulsion are then equal. Hence the force of repulsion can be calculated. By altering the distance between the poles it is found that the force becomes greater as this distance is decreased, being in fact *inversely proportional to the square of the distance.*

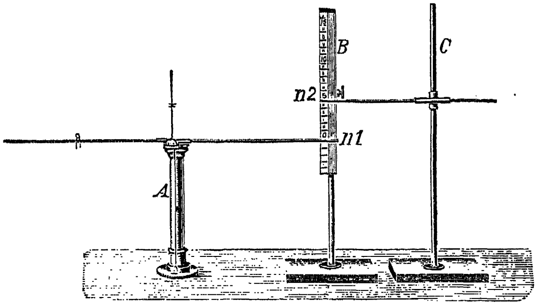

Fig. 29.—Grimsehl's magnetic balance

If the upper knitting needle be replaced by two, three or more similar magnetized needles, the force of repulsion is respectively doubled, trebled or increased in the same ratio as the number of needles. The same result is obtained if the lower knitting needle be replaced by a larger number.

These experiments provide proof of **Coulomb's Law** (Coulomb, 1784), viz.

$$P = f \cdot \frac{m_1 m_2}{r^2},$$

where $m_1$ and $m_2$ are measures of the pole strengths and $r$ is the distance between the poles. The factor $f$ depends upon the choice of the unit of pole strength (see also the note on p. 16).

The unit of pole strength can be defined in such a way as to make the factor $f$ in Coulomb's law equal to unity. We then have the following definition:

**A magnetic pole of unit pole strength is such as to repel with a force of 1 dyne an exactly similar pole placed at a distance of 1 cm. from it in a vacuum.**

Since a bar magnet of unit pole strength and with unit distance (1 cm.) between its poles has unit magnetic moment, the equation $R_1 = MB$ (p. 123) reduces in this case to $R_1 = B$. Hence the unit of magnetic induction (1 gauss) can also be defined as follows:

**The magnetic induction at a point of a field is of unit magnitude when the force exerted at this point upon a unit pole is 1 dyne.** For then $R_1 = 1 = B$.

Historical reasons account for the nature of the attempt to treat magnetic phenomena quantitatively by means of relationships analogous to Newton's law of gravitation (Vol. I, p. 176). The inclination towards this form of treatment is understandable when we remember the extraordinary impression created by Newton's discovery. It was FARADAY's power of unbiased, independent thought that first produced the theory of field action—a theory far more in harmony with the observed phenomena. Magnetic poles are not realizable in the same way as the charges of an electric dipole, but are imaginary entities introduced in analogy to the electrical case. This was indeed recognized by GAUSS, who originated a satisfactory basis of treatment in terms of magnetic moments. The derivation of the unit of magnetic field strength making use of magnetic moment (see p. 125) is completely independent of any assumption as to magnetic poles as fictitious sources of the field.

## 5. The Magnetic Field of the Earth

**The Form of the Earth's Field.**—We have already come to the conclusion (p. 112) that there is a magnetic field at the surface of the earth. Its form can be determined by observation of the direction taken up by a freely suspended magnetic needle at as many different places as possible. Such experiments have shown the earth's field to be of the type shown in fig. 30, i.e. approximately that of a magnetized sphere (fig. 31).

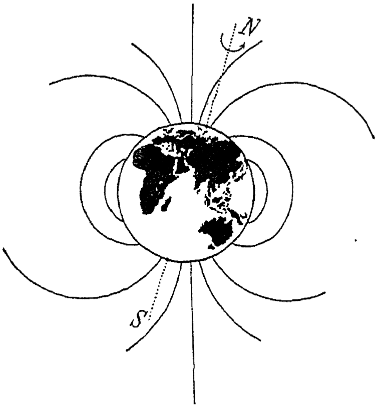

Fig. 30.—Diagram of the earth showing lines of force

The earth behaves as a magnet not only towards magnetic needles but also towards all pieces of iron. This can be seen from the fact that in the northern hemisphere every iron rod or bar standing vertical (window bars, fence posts, &c.) has a south pole at its upper end (sink of magnetic lines of force) and a north pole at its lower end (source of lines of force). This *induced* magnetization is still more intense when the piece of iron has the same direction as a freely suspended magnetic needle, i.e. when its direction coincides completely with that of the lines of force of the earth's magnetic field.

When a piece of soft iron rod (e.g. a length of gas pipe) is held in the direction of the earth's magnetic field and given a shock by being struck with a hammer, it becomes magnetized to an extent easily demonstrable with the help of a compass needle. If now the rod be turned through 180° and struck again with the hammer, its poles are reversed.

Investigation of the earth's magnetic field gives the following results.

A freely suspended magnetic needle assumes parallel positions at different places not too far away from one another. (These experiments must be performed in the open, because of the disturbing effects of the structural iron in the walls, ceilings, &c., of buildings.) Determinations of the period of vibration of a freely suspended bar magnet at different places give everywhere the same value (with the restriction given above). Within regions of not too great extent, therefore, the magnetic field of the earth can be regarded as *homogeneous.*

From this it follows that a magnetic needle can only experience a *turning moment*; there can be no force tending to produce translation, because the magnetic force upon the one pole of the needle is equal and opposite to that upon the other pole. Conversely we can deduce that the earth's field is homogeneous from the fact that even the most sensitive methods fail to detect any force tending to produce translation.

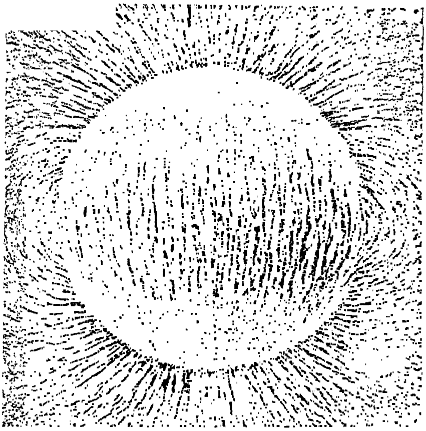
Fig. 31.—Field of a magnetized sphere

It is found further that the lines of force of the earth's field are inclined to the earth's surface (see fig. 30). In our latitude they slope downwards towards the north.

A magnetic needle capable only of rotation in a horizontal plane takes up a direction corresponding to the projection of the lines of force upon this plane. It is acted upon only by the **horizontal component of the earth's field.**

The instrument used for the investigation of the horizontal projection of the lines of force of the earth's magnetic field is called a declinometer (fig. 32), or in its simpler form a compass.* This consists of a magnetic needle mounted upon a point in a case so as to be capable of rotation in a horizontal plane over a circular scale. The case is equipped with a telescope or simple sighting tube, which can be turned in a vertical plane. The whole is supported in a horizontal position upon a tripod stand. The case is turned until the needle is parallel to the axis of the horizontal telescope. A distant point is then sighted through the latter, and the apparatus set up again there. Then a further point is sighted, the apparatus set up again at it, and so on. Proceeding in this way, the line joining all the successive points is the horizontal projection of a line of force of the earth's magnetic field.

Starting from any point on the earth's surface, the horizontal projection of the line of force through that point can be mapped out by the above method. The line obtained is called a **magnetic meridian.** Fig. 33 shows 18 magnetic meridians which are equidistant at the equator (i.e. 20° apart). All the meridians converge towards two points. The northerly point * lies to the west of the peninsular Boothia Felix at about 70° 40′ north and 96° 5′ west: the southerly point † lies in Victoria Land at about 72° south and 155° east.

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* From the Latin *cum* = with and *passus* = a step; *compassus* (late Latin), measuring instrument, pair of compasses. In the 15th and 16th centuries the name compass was used for a small pocket sundial, which was adjusted with the help of a magnetic needle. The name then passed over to the needle and its circular scale.

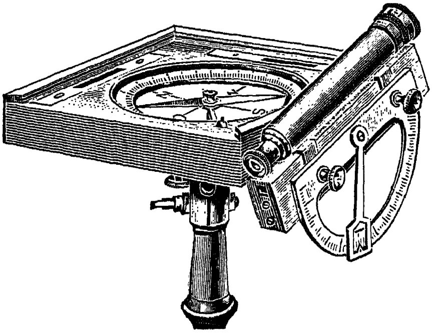

Fig. 32.—Declinometer

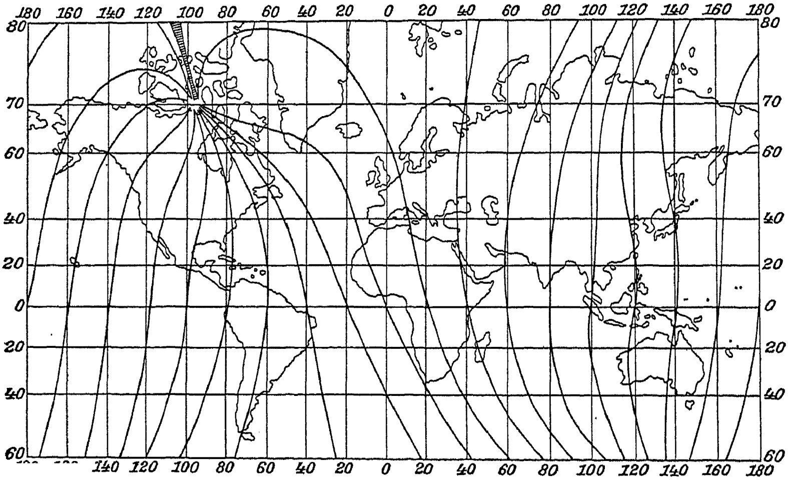

Fig. 33.—Magnetic meridians

These points of convergence are known as the **magnetic poles of the earth.** The lines of force run towards the north magnetic pole;

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*Discovered by James Clark Ross on 1st June, 1831. The above position is that found by the Gjöa expedition (Amundsen, 1903–5).

†Discovered in 1909 by Prof. David, a companion of Shackelton, during the latter's Antarctic expedition.

this therefore has southern polarity. Similarly, since the lines of force are directed away from the south magnetic pole, this has northern polarity. As is seen from fig. 30, the actual lines of force do not converge towards the magnetic poles, but only their horizontal projections. The magnetic poles are therefore the points at which the lines of force are at right angles to the earth's surface.

On account of the use of magnetic needles for the determination of directions, the knowledge of the angle between the magnetic and geographical meridians (the **declination**) is of the utmost practical importance. At London the declination amounted in 1932 to about 12° towards the west; it is not constant, however, the rate of change in 1932 being about 0·2° per annum towards the east.

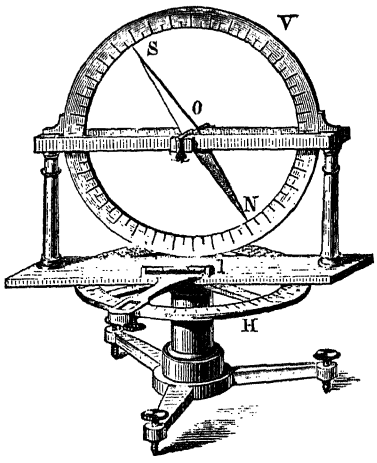


Fig. 34.—Inclinometer or dip needle

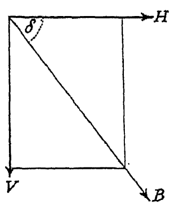


Fig. 35.—Components of the earth's magnetic field.

In investigating the lines of force of the earth's field by means of a magnetic needle suspended so as to be able to take up any direction (fig. 3, p. 113), we found that these lines of force are inclined to the horizontal. The magnitude of their slope, i.e. the angle between the needle and the horizontal plane, is called the magnetic **inclination** or **dip.**

For the more exact determination of inclination use is made of the instrument depicted in fig. 34 and known as an **inclinometer** or **dip needle**. The vertical circular scale V is brought into the plane of the magnetic meridian by means of the horizontal circular scale H. The inclination or dip can then be read off directly at the points N and S of the magnetic needle, which is suspended so as to be able to turn about the horizontal axis O.

Since it is quite possible that the centre of gravity of the needle does not lie exactly upon its axis of rotation, another reading must be taken with the needle remagnetized so that its poles are reversed. The vertical circle V should also be turned through 180° and the readings repeated. The true value of the inclination is then found by taking the average of all the observations. The value of the dip at London was about 66° 40′ in 1932.

The properties of the terrestrial magnetic field are treated at length in Vol. V, where more detailed maps and tables are also given.

**The Intensity of the Earth's Magnetic Field.**—From the map (fig. 33) showing the magnetic meridians (i.e. the horizontal projections of the lines of force), it may be deduced that the magnetic force upon a test magnet is different at different places, being in fact smallest where the lines of force are farthest apart (i.e. near the equator) and greatest where the lines of force are nearest together (i.e. near the magnetic poles). It is not possible, however, to draw any conclusions as to the exact magnitude of the intensity from the behaviour of the horizontal projections of the lines of force; the densities of the lines of force themselves must be known.

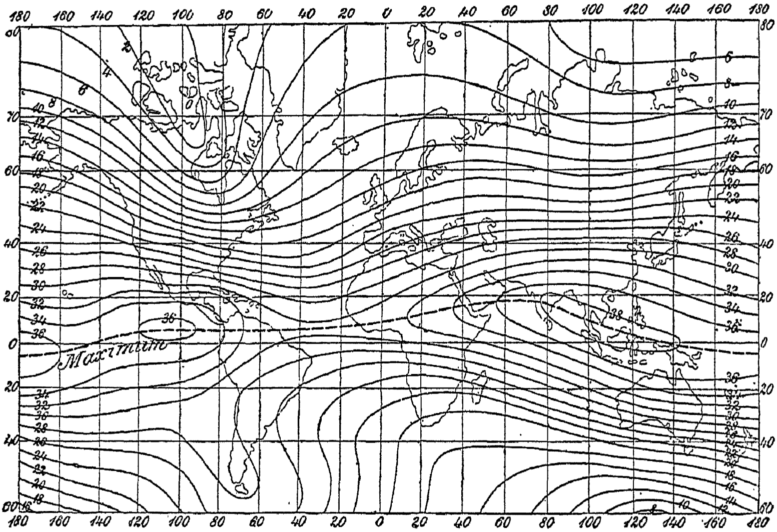


Fig. 36.—Isodynamic lines for H. (Numerical data in hundredths of a gauss)

Now from fig. 35 we see that the magnitude B of the total intensity is related to the magnitude H of the horizontal component by the simple equation $H = B \cos \delta$, where $\delta$ is the angle of inclination or dip. Hence it is easy to calculate the total intensity when the values of the horizontal intensity and the inclination are known. Very little practical importance attaches to the vertical intensity $V = B \sin \delta$.

Measurements of the horizontal intensity H were carried out systematically by GAUSS. Modern scientific determinations are still made by his methods, as described on p. 124.

Fig. 36 shows the curves obtained by joining points of equal horizontal intensity (so-called **isodynamic lines**). The value of H at London is about 0·185 gauss. (See also p. 392.)

# PART II

# STEADY ELECTRIC CURRENTS

## CHAPTER III

## Introductory Observations. Characteristic Properties of Electric Currents

### 1. Disappearance of Electric Fields and the Motion of Electricity

In the first chapter we confined our attention to electric fields, which were practically invariable with time. These fields existed between charged bodies in air or in other substances with the property of maintaining the field unchanged for an indefinite time within them. To such substances we gave the name *insulators* (p. 21). At the same time we recognized the existence of other substances in which electric fields very quickly die away; these substances are the so-called *conductors*. We saw that the characteristic of conductors is that in them the electric charges are more or less free to follow the pull of the lines of force. We now proceed to consider this process of the dying away or disappearance of electric fields and the simultaneous motion of the electric charges.

To begin with, we can easily convince ourselves that the velocity of disappearance of the field can be different in different cases. For this purpose we take a charged condenser connected to a volt-meter V (fig. 1) and connect its two terminals A and K (i) with a dry linen thread F and (ii) with a metal wire.

In the first case the dying away of the field, as indicated by the reading of the volt-meter, takes several seconds. In the second case the field disappears instantaneously.

In order that the above experiments may not be affected by involuntary "earthing" of the condenser plates, the thread or wire must be manipulated by means of well insulated handles G (say two sticks of sealing-wax) attached to the ends as shown in the figure.

Let us consider this process of discharge a little more closely. Fig. 2 shows the field between the condenser plates K and A. We establish connexion at the upper edges by means of the conductor L. As the conductor is brought up, the field is disturbed and the charges upon the conductor, being free to move, are displaced. We get a distribution of lines of force like that shown in fig. 3, in which the average distance between the lines has become greater owing to the fact that a certain number of them have been shortened and no longer exert so great a lateral pressure as before. When now the conductor is made to touch both condenser plates, the previously shortened lines of force disappear altogether, since the opposite charges have combined. Their lateral pressure is thus removed and the remaining lines of force are able to separate to still greater distances from one another, namely, by spreading out towards the side where the conductor is situated (fig. 4). Finally, the lines of force pass through the conductor, between whose two points of contact (which we will call its "ends") a potential difference therefore exists. This potential difference causes the charges to move along the conductor. Fig. 4 is very instructive. From it we see firstly that the density of the lines of force decreases towards the conductor, since on that side the lateral pressure has been removed. This distribution of lines of force does not correspond to a state of equilibrium. In order to maintain it, energy is in fact necessary (see below). Further, it should be noticed that the motion of the lines of force is *at right angles to the direction in which the charge neutralization takes place.* It is easy to show that this is always the case in all circuits in which electric currents are flowing.

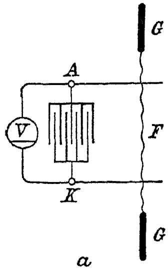
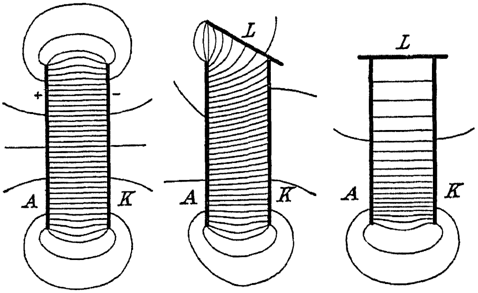


Fig. 1.—Discharge of a condenser



Figs. 2, 3, and 4.—Charges of the field during discharge of a condenser

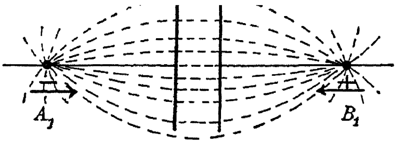


Fig. 5.—Charging of a condenser

We will now consider the process of charging a condenser by means of leads attached centrally to the plates. The flow of electricity from the source can be regarded as the rapid motion towards one another of positive and negative charges, in response to the field between them. These charges are prevented from uniting by the non-conducting dielectric within the condenser. Hence as the charges on the plates increase, an electrostatic field of ever increasing intensity is formed between them. The fields coming from the poles of the source of electricity are as it were collected together in this field within the condenser, which thus becomes the store of all the energy.

Now it is seen from fig. 5 that, as the opposite charges $A_1$ and $B_1$ approach each other from either side, the lines of force between them enter the interior of the condenser from the edges of the plates. Thus again the alteration of the field strength in the dielectric between the plates is associated with a motion of the lines of force in a direction at right angles to the field. More particularly, as the field strength increases, the lines of force move inwards from the edges of the plates; conversely, during discharge of the condenser and decrease of the field strength, the lines of force move outwards towards the edges and the distance between them increases.

**The motion of electric charges is called an electric current.***

Whenever there is a potential difference between the ends of a conductor, this gives rise to a motion of the charges situated at the ends of the lines of force: an electric current flows. This current continues until the difference of potential between the ends of the conductor has fallen to zero. In order that a current may flow continually in a conductor, a potential difference must be maintained between its ends.† This requires the expenditure of work, as is always the case in charging a body, i.e. in producing a potential difference (p. 28).

*Experiment.*—Connect the two terminals of a condenser to a voltmeter and also to the terminals of a well insulated electrical machine. Charge up the condenser by turning the machine and then connect the condenser terminals by means of a hemp string. The pressure falls off, since the charges neutralize each other through the string. In order to keep up the initial potential difference the machine must be turned, i.e. mechanical work performed.

As can easily be shown (see below), the disappearance of the electric field is accompanied by energy transformations in the conductor itself. This is only to be expected according to the principle of energy (Vol. II, p. 32); for an expenditure of energy was necessary in charging the condenser and this energy must reappear in some form or other during discharge. The actual energy transformations which occur will be discussed in more detail later (p. 137 *et seq.*).

A discharge often takes place through the electrical machine, while this is not being worked, since the materials of which it is constructed have a certain conducting power.

The above experiments show that electric fields are indeed possible within conductors, but that they always give rise to a motion of electricity. The maintenance of an electric field in a conductor necessitates expenditure of work. Only very small fields can be attained in metallic conductors (see Chap. XI, p. 360). The relation connecting the field between different points of a conductor and the corresponding current will be treated in detail in Chap. VI, p. 219.

**Switches.**—When there is a conducting connexion between two points at different potentials, so that these potentials can be equalized by the passage of a current, the system or circuit is said to be *closed*; when no such conducting connexion exists, the circuit is said to be *open*. A switch is a device for closing ("making") or opening ("breaking") a circuit. When the pressure is high, the breaking must be very rapid, in order to avoid an arc (pp. 352 and 436). Fig. 6 shows how this is ensured in ordinary lighting switches by means of a spring. Under certain circumstances the making or breaking of a circuit may be a very difficult problem of electrical engineering.

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* For a generalization of this definition see p. 167.

† In certain special cases a current can flow continually without a potential difference (super-conductivity, p. 245).

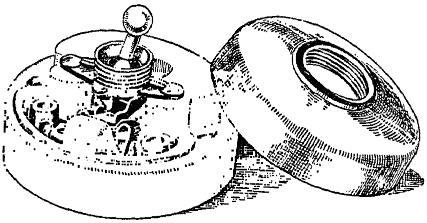
Fig. 6.—Lighting switch

The equalization of a potential difference in a conductor takes place in a similar manner to the equalization of water-level by means of a water current.

**Current Strength.**—The arrangement shown in fig. 7 consists of two glass vessels connected near their bases by means of a horizontal tube, which can be closed with a tap. We fill the one vessel with water and then open the tap. The water flows across into the other vessel until the levels in the two vessels are equal.

In the arrangement shown in fig. 8, the inner coatings of two Leyden jars can be connected by means of a thin wooden rod about two metres long. Before placing the rod so as to establish connexion, we charge up the one jar until the electroscope attached to it shows a large deflection. In order that this deflection may be definite (p. 34), the table and the outer coating of the jars must be earthed. The deflection of the electroscope is then a measure for the *potential* of the charge upon the inner coating. We now put the wooden rod in place (fig. 8, *b*). The electric charge flows across from the one jar into the other until their potentials (as indicated by the electroscopes) are equal, i.e. until both inner coatings have the same potential.

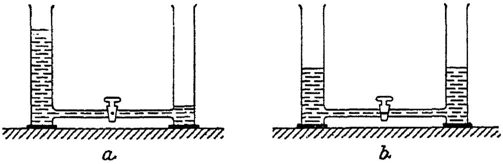

Fig. 7 (*a*) and (*b*).—Equalization of pressure in vessels of water

If we maintain a difference of level in the two vessels of fig. 7 by pumping the water out of the one on the right back into the one on the left, a current of water will flow continually through the connecting tube from the vessel with the higher water-level into the vessel with the lower.

By connecting the inner coatings of the Leyden jars (fig. 8) with the two terminals of an electrical machine, we can maintain a potential difference between them. A current of electricity then flows continually from the jar with the higher potential to the jar with the lower.

Thus just as the water current only persists so long as there is a difference of level in the two vessels and a connecting tube between them, so also does the electric charge only go on flowing from the one Leyden jar into the other so long as there is a **potential difference** and a **conducting connexion** between them. The potential difference producing the motion of electricity is frequently referred to as the **electromotive force** (E.M.F.).

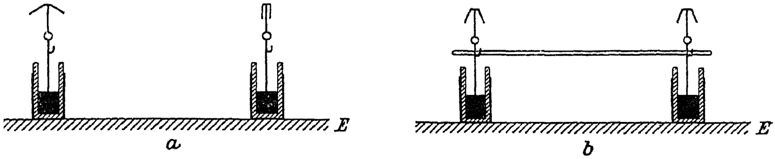

Fig. 8 (*a*) and (*b*).—Equalization of potential in condensers

We measure the strength of a current of water in a pipe by means of the quantity of water passing any section of the pipe in a second. Preserving the analogy between currents of water and of electricity, we make the following definition:

**The strength of an electric current flowing in a conductor is measured by the quantity of electricity passing any section of the conductor per second.**

## 2. Detection of Electric Currents

**Characteristic Properties of Currents.**—Up to the present we have deduced the existence of an electric current merely from the disappearance of a potential difference; for so far we have not become acquainted with any other method of determining whether a current is flowing in a given conductor or not. Making use again of the analogy of the two vessels of water connected by a tube, it is as though we had deduced the existence of a water current through the tube merely from the equalization of the water-levels in the two vessels. Extending this type of deduction, we can conclude that a current is flowing whenever we have to perform work in order to maintain the difference of potential (or of water-level). But there are other, more easily observable characteristic properties of currents, which will be found later to provide a means of treating current phenomena in a quantitative way. We proceed to consider these properties.

1. *Magnetic Field of a Current.**—*A conductor through which an electric current is flowing is surrounded by a magnetic field.*

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* This effect of a current outside the conductor carrying it was first discovered in 1820 by the Danish physicist OERSTED when experimenting with a magnetic needle placed in the neighbourhood of a thin platinum wire, through which was flowing a current strong enough to make it glow. He thought at first that the cause of the observed deflection of the magnetic needle was to be sought in the heating of the wire. But

If we take a wire through which an electric current is flowing, dip it into iron filings and remove it again, the filings remain hanging on to the wire in peculiar clusters. The particles of iron appear to arrange themselves in circular chains around the wire. If the wire be made horizontal and a piece of stiff paper laid upon it, iron filings scattered upon the paper will become arranged in short straight lines when the paper is tapped lightly. These lines are at right angles to the length of the wire, as shown in fig. 9.

The arrangement of filings shown in fig. 10 is that obtained when the plane upon which the filings are placed (paper or glass plate) is at right angles to the wire, which in this case is made to pass vertically through a hole in the plane of the figure. It is seen that the arrangement consists of concentric circles whose centre lies at the point of intersection of the wire with the plane of the figure. Since now the filings indicate the course of the lines of force of the magnetic field, it follows that a wire carrying an electric current is surrounded by magnetic lines of force in the form of concentric circles with their centre in the axis of the wire (fig. 11).

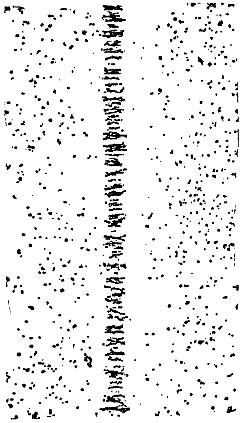

Fig. 9.—Magnetic field in the plane of a conducting wire carrying a current.

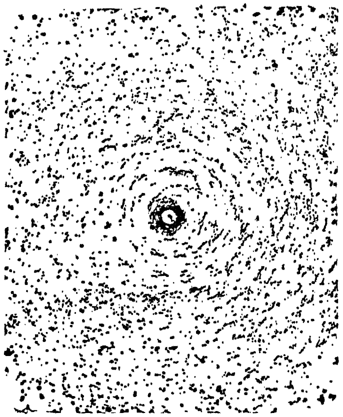

Fig. 10.—Magnetic field in a plane at right angles to a wire carrying a current

An electric current flowing near to a freely suspended magnetic needle acts upon the needle so as to tend to make it take up a position at right angles to the current. The action of the current is most marked when the wire carrying it is situated above or below the needle and in

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he soon found that the needle was also affected by weak currents flowing through thick wires in which no detectable heating effect was produced. In the same year the French physicist AMPÈRE (1775–1836) established the simple relationship connecting the sense of the deflection of the magnetic needle with the sense of the current (see p. 161).

the direction of the magnetic meridian, i.e. in such a position that needle and wire are parallel when no current is flowing.

These results can be deduced at once from the course of the magnetic lines of force, since the magnetic needle always tends to set itself in the direction of the tangent to the lines of force.

A wire wound into a coil (so-called *solenoid*, fig. 12) gives rise to a magnetic field of especially simple form (for further details, see p. 170). When a current flows through the wire, the magnetic lines of force run in exactly the same way as in the case of a bar magnet. Consequently the forces exerted upon other coils or magnets are the same as would be exerted by a bar magnet. A magnetic needle placed within the coil tends, when a current flows, to set itself in the direction of the axis of the coil, i.e. along the lines of force, which can thus be proved in this case to pass without break through the interior of the coil. A piece of soft iron is attracted into the coil.

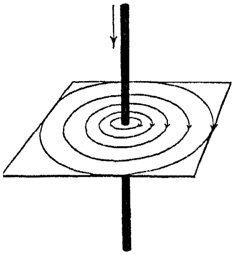

Fig. 11.—Magnetic field of a conducting wire through which a current is flowing.

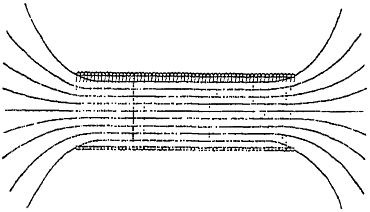

Fig. 12.—Magnetic field of a closely wound solenoid

2. *Thermal Effects.—When a current flows through a conductor, the conductor is heated.*

This effect is very widely applied in incandescent lamps.

It may be mentioned at once that an electric current is *always* surrounded by a magnetic field. Moving electric charges or moving electric lines of force are impossible without a magnetic field. Thus the magnetic field is in a certain sense only another aspect of electric lines of force in motion. An electric current without *thermal* effect is, on the contrary, quite possible, though only under special circumstances.

3. *Chemical Effects.—Chemical changes occur when an electric current passes through liquid conductors.*

*Experiment.*—Connect the two binding screws of the apparatus shown in fig. 13 with the poles of the direct current mains (say 220 volts), including a switch in the circuit. The apparatus consists of an incandescent lamp, which lights up brightly when connected directly to the mains, and two plates of platinum dipping into a vessel containing distilled water. The plates are held in stands, so that their distance apart can be varied.

When the switch is closed, the lamp does not light. There is therefore no current in the circuit. But if the platinum plates are made to touch, the lamp glows at once with normal brightness. From this it follows that the reason why no current flows when the platinum plates are not in contact is that the layer of distilled water between them does not conduct electricity.

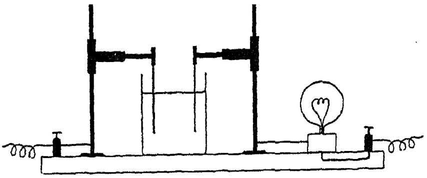

Fig. 13.—Chemical effect of an electric current

Now adjust the platinum plates at a distance of several centimetres apart and introduce a few drops of sulphuric acid upon a glass rod, stirring so as to mix with the water. The lamp immediately lights up brightly. The addition of the acid has rendered the water conducting. At the same time bubbles of gas are observed to rise from the platinum plates. These gases turn out, when tested, to be hydrogen and oxygen respectively.

Water can also be made to conduct by the addition of other substances. Not all substances produce this effect, however, and those which do are not all effective in the same degree. Thus, for example, water remains non-conducting no matter how much cane-sugar solution is added to it.

It is clear from the above experiments that the substances added to the water take an active part in the process of conducting the current. Conductivity in a liquid is thus dependent upon the presence of certain substances, and is accompanied by chemical change.

The plate which is connected to the positive pole of the source of electricity, i.e. the plate at which the positive current enters the liquid, is called the **anode.*** The other plate, at which the positive current leaves the liquid, is called the **cathode.***

The liquid conducting the current is called the **electrolyte,**† and the whole process is known as **electrolysis.**

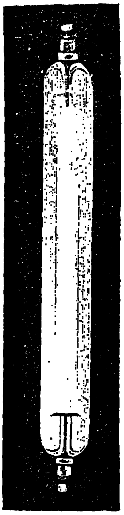

Fig. 14.—Luminous effect of an electric current passing through a rarefied gas.

4. *Luminous Effects.* — Connect a fairly well evacuated glass tube containing a little residual air (fig. 14) to the poles of an influence machine. A current passes through the tube, which lights up brightly with only a moderate evolution of heat. The same phenomenon is observed when a so-called *neon lamp* (i.e. a glass globe filled with neon at low pressure and provided with suitable

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* Gr., *hodós*, way. *Anode*, way in; *cathode*, way out.

† Gr., *lysis*, a loosing; *lytós*, capable of being loosed. *Electrolyte*, the substance decomposed by the electric current. *Electrolysis*, the process of decomposition by the electric current.

terminals) is connected to the lighting mains. Thus the energy of the electric current can be made to produce light in a more direct way than by raising a body to incandescence, as in the ordinary filament lamp.

**Current Indicators.**—Each of the above effects can be used for the detection of a current and, after the indicating instrument has been calibrated in suitable units (p. 153), to measure the current. A very large number of different types of instruments have been developed, the more useful of which will be discussed in greater detail in their proper places. For the present it will suffice to describe a few of the simpler forms which can be used for the detection of a current and, if necessary, for the indication of its constancy (see further pp. 210–215).

The simplest of these devices is the *hot wire instrument.* In this use is made of the elongation of a wire when heated by the passage of the current. The elongation is either observed directly or else magnified by suitable lever transmission (figs. 6 and 7, p. 257). The thermal expansion of an enclosed volume of air surrounding the wire can also be utilized.

Instruments depending upon the *magnetic* effects of the current can be made more sensitive. A device for the detection of electric currents with the help of a magnet is called a *galvanoscope.* If it permits also of a quantitative measurement of the current, it is called a *galvanometer.* The most primitive form of galvanoscope is that shown in fig. 15. It consists of a box enclosing a magnetic needle, which is balanced upon a fine point by means of a small agate cone. The box is provided on opposite sides with two binding screws, which are connected by a single straight copper wire placed close beneath the needle. The instrument is turned till this wire lies in the magnetic meridian, and the current is passed through it. The deflection of the needle is in this case produced by use of a single conductor.

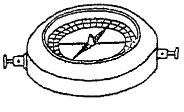
Fig. 15.—Simple galvanoscope

It is of advantage by using a conductor of suitable shape to give the magnetic field such a form that the action upon the magnetic needle is as great as possible. An especially good way of achieving this is to bend the wire into a loop; for then, as follows from the nature of the field (fig. 12, p. 139) and as will be explained fully later, every part of the conductor acts in the same sense upon the magnetic needle. In the same way we see that the individual turns of a coil all act in the same sense. The action of a current upon a magnetic needle can therefore be increased by sending the current through a coil with many turns.*

The deflecting action of a given current increases as the distance of the conductor from the needle is decreased. Hence we see that the essential constituents of a sensitive galvanoscope are (i) a magnetic needle capable of rotation and (ii) a coil with many turns surrounding the needle and as near to it as possible.

In the vertical galvanoscope shown in fig. 16 the conducting wire is wound upon a narrow horizontal bobbin. The magnetic needle (shown dotted) lies horizontally within the coil, and can turn about a horizontal axis at right angles to the plane of the figure. There are two independent windings upon the bobbin: the first consists of a few turns of thick wire and the second of many turns of thin wire. These windings end in respective pairs of binding screws on opposite sides of the bobbin, and can be used independently. The magnetic needle bears a vertical upward pointer, which moves over an easily read scale. Vertical galvanoscopes of this type are frequently used for lecture experiments and in other cases in which the deflections are to be visible from a distance.

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* Schweigger (1779–1857) showed at Halle in 1820 that the deflecting action of an electric current upon a magnetic needle could be increased or "multiplied" by leading the wire a number of times around the needle. Seebeck (1770–1831) gave the name "multiplier" to Schweigger's apparatus. The name galvanometer is due to Poggendorff (1796–1877). It is not very aptly chosen, as Galvani was unaware of the magnetic action of electric currents.

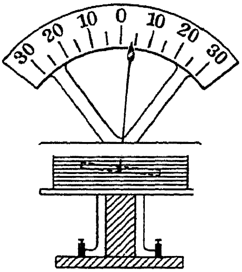

Fig. 16.—Vertical galvanoscope

Another convenient form of current indicator depends upon the attraction of a piece of soft iron into a coil through which a current is passing (fig. 81, p. 215).

Concerning other types of instrument see p. 214.

Galvanometers with a magnetic needle as movable part are little used nowadays, because they are influenced by the earth's field (p. 128) and can therefore only be employed in one particular position, namely that in which the magnetic needle is parallel to the wire when no current is flowing. Most modern instruments depend upon the motion of a conductor bearing a current in a magnetic field. In order to make this magnetic field practically independent of the earth's field, it is produced either by a strong permanent magnet or else by the current itself.

**Equivalence of the Different Types of Conduction.**—Every electric current, whether in a solid, liquid or gaseous conductor, is accompanied by a magnetic field and generally also by a development of heat.*

*Experiment.*—A U-shaped glass tube about 10 cm. in length is filled with dilute sulphuric acid and the terminals of the direct-current mains connected to two platinum plates dipping in the liquid as shown in fig. 17. A current flows through the liquid and gas is evolved copiously at the electrodes. Soon the liquid becomes noticeably warm. A magnetic needle placed above the tube is deflected in exactly the same way as by an equal current flowing through a wire in the place of the tube.

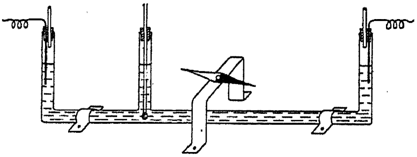

Fig. 17.—Thermal and magnetic effects of a current in an electrolyte

Similar observations can be made upon a glass tube filled with air under reduced pressure. In this case, however, greater E.M.F.s. must be applied in order to obtain currents of sufficient strength. The E.M.F. of the lighting mains suffices for a tube filled with neon and fitted with specially prepared electrodes (p. 340). Here again it is possible to observe, in addition to the luminous glow, a (slight) warming and also a magnetic effect.

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* Concerning the absence of heat development under special circumstances in solids and rarefied gases see pp. 246 and 312.

**Current Direction (or Sense).**—It has been decided to choose the direction (or sense) of flow of the positive electricity as the direction (or sense) of the electric current in a conductor. Thus:

**The direction of the current is that in which the positive electricity flows or would flow.***

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* This decision is not a happy one, because in metallic conductors (the most widely employed type of conductor) it is only the negative electricity which moves (§ 1, p. 361). Hence in metals the direction of flow of electricity, i.e. the motion of the electrons, is just opposite to the " direction of the current ", as defined above.

## CHAPTER IV

# Quantitative Determination of Current Strength

## 1. The Products of Decomposition due to the Passage of a Current through an Electrolyte *

As was stated on p. 140, only certain liquids, known as electrolytes, have the power to conduct an electric current. The current is accompanied by chemical changes at the electrodes. We will now consider the nature of these changes and their quantitative relation to the strength of the current.

In order to become acquainted with the phenomena, we will first take a few examples.

1. *Electrolysis of Fused Lead Chloride.*—With the help of a Bunsen burner we melt some lead chloride in a U-shaped tube (see fig. 1) made of hard glass. The quantity of lead chloride should be sufficient to fill the bend of the tube and a fraction of both limbs. We then dip two thin rods of carbon into the fused substance, one in each limb as shown in the figure, and connect them with the terminals of a source of current. The rods are carried in corks, that bearing the anode being fitted with a bent glass tube as shown. When the current is switched on a gas is seen to be evolved from the anode; it is recognized at once by its smell to be chlorine. It may be collected for further examination in a vessel placed under the bent tube. Small globules of metal are formed at the cathode, from which they separate when they have attained a certain size, falling and collecting in the bend of the U-tube. After 10

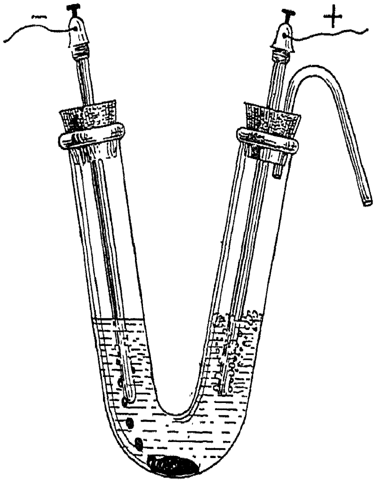

Fig. 1.—Electrolysis of fused lead chloride

minutes the contents of the tube are tipped out and the metallic globules can be shown to consist of lead. Thus the current has decomposed a part of the lead chloride in accordance with the equation

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* In the course of the year 1800, Nicholson and Carlisle were able to give a complete explanation of the electrolysis of water. Previous to this, various investigators had made random observations, but had usually interpreted them wrongly. In the same year Johann W. Ritter, then at Jena, carried out independent experiments and made equally comprehensive discoveries. He was also the first to decompose a metallic salt, namely, copper sulphate.

$$PbCl_2 \rightarrow Pb + Cl_2.$$

2. *Electrolysis of an Aqueous Solution of Lead Acetate.*—We fill a suitable vessel (fig. 2) with an aqueous solution of lead acetate and pass an electric current through it between two lead plates. When the current is switched on, lead is deposited in beautiful crystals upon the cathode, giving the impression of a growing tree. As may be shown by weighing the electrodes before and after the experiment, an equal amount of lead is dissolved off from the anode. In this case, therefore, the action of the current is to remove lead from the anode and deposit lead upon the cathode.

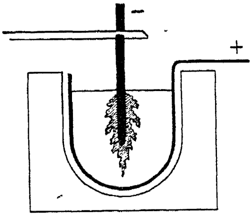

Fig. 2.—Lead tree

3. *Electrolysis of an Aqueous Solution of Common Salt (Sodium Chloride).*—This can be carried out conveniently in the apparatus shown in fig. 3. The electrodes are of carbon. When the current is switched on, gases are evolved at the two electrodes. The gas appearing at the anode can be recognized by its smell to be chlorine. At the cathode a colourless and odourless gas is evolved; it may be collected in a separate vessel with the help of a pneumatic trough, the side-tube of the electrolysis apparatus being fitted with a rubber tube connected to a convenient glass tube dipping in the trough. The gas is found to be inflammable, the product of combustion being water. The gas is therefore hydrogen. This cannot have been derived from the sodium chloride, but can only have come from the water in which the salt was dissolved.

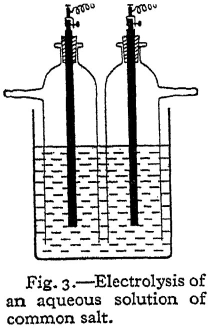
Fig. 3.—Electrolysis of an aqueous solution of common salt.

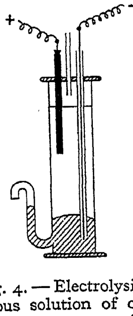

Fig. 4.—Electrolysis of an aqueous solution of common salt with mercury cathode.

If the electrolytic process in this case were analogous to that for lead chloride, it would occur in accordance with the equation

$$2NaCl \rightarrow 2Na + Cl_2.$$

We have seen, however, that no sodium is deposited, hydrogen being evolved instead from the water. We shall go into the reason for this in detail later (p. 287). The liquid in the neighbourhood of the cathode has an *alkaline* reaction, as can easily be proved by the addition of litmus (see p. 146). If a cathode of mercury (instead of carbon or platinum) be used, say in an apparatus of the kind shown in fig. 4, then there is no evolution of hydrogen; sodium is actually deposited and forms sodium amalgam with the mercury. The amalgam can be removed and the sodium in it detected, for example, by means of the copious evolution of hydrogen on warming with water.

4. *Electrolysis of a Solution of Sodium Nitrate.*—If sodium nitrate ($NaNO_3$) be used in place of sodium chloride, hydrogen is evolved at the cathode and oxygen at the anode. A little litmus solution added to the liquid gives it a violet colour. Passage of the current turns the colour to red near the anode and blue near the cathode. Thus near the anode the liquid has become acid and near the cathode alkaline.

Use is made of this in pole-testing papers, in which, however, the litmus is replaced by phenolphthalein, a substance which is colourless in acid solution but intensely *red* in alkaline solution. The pole-testing paper consists of a strip of absorbent paper impregnated with phenolphthalein and potassium nitrate. It is stored in a dry state. Before use it is moistened slightly and then the wires from the poles of the source of current are placed in contact with it at a short distance apart. A red coloration appears at once at the point of contact of the wire connected to the negative pole.

5. *Electrolysis of Dilute Sulphuric Acid.*—The apparatus shown in fig. 5 is filled with dilute sulphuric acid. The side tubes are equipped with small platinum electrodes, which are connected to the poles of the source of current. The circuit being made, bubbles of gas are evolved from both electrodes, and are collected in the upper parts of the side limbs beneath the closed taps. A certain amount of the acid is thereby displaced and driven up the central tube. The volumes of the two gases evolved are in the ratio 2 : 1. Chemical investigation shows them to be hydrogen (evolved at the cathode) and oxygen (at the anode).

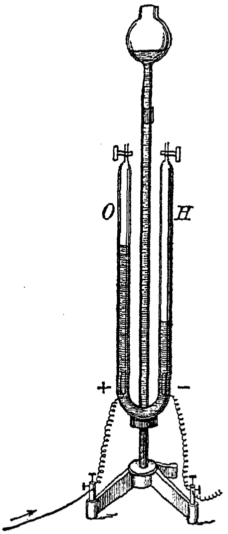

Fig. 5.—Electrolysis of dilute sulphuric acid

In this process the hydrogen has come from the sulphuric acid, but the oxygen has been formed by a secondary reaction (see below).

*Conclusion from the above Experiments.*—The results of the experiments 3 to 5 can be taken to indicate that, as directly observed in the case of fused lead chloride, the electrolyte is separated into two constituents; these may, however, in certain circumstances react with the water, thus giving rise to secondary products.

*Note.*—According to this view the primary electrolysis of sodium chloride solution takes place in accordance with the equation

$$2NaCl \rightarrow 2Na + Cl_2.$$

The metallic sodium formed in this primary process then reacts with the water, decomposing it at the cathode in accordance with the equation

$$2Na + 2H_2O \rightarrow 2NaOH + H_2.$$

The hydrogen thus formed escapes at the cathode as observable end-product of the electrolysis, while the sodium hydroxide dissolves in the water and can be detected by its alkaline reaction.

The electrolysis of sodium nitrate takes a similar course. The secondary reaction at the cathode is the same as that given above. At the anode the nitrate radical reacts with the water,

$$4NO_3 + 2H_2O \rightarrow 4HNO_3 + O_2,$$

evolving oxygen and forming nitric acid.

In the case of dilute sulphuric acid the whole process is as follows:

primary electrolysis: $H_2SO_4 \rightarrow H_2 + SO_4$;
secondary chemical action: $2SO_4 + 2H_2O \rightarrow 2H_2SO_4 + O_2$.

Thus the hydrogen evolved at the cathode is produced in the primary electrolysis, whereas the oxygen evolved at the anode is a secondary product due to the reaction of the acid radical, which cannot exist in the free state, with the water.

Although such secondary chemical processes play a part in many cases, the evolution of hydrogen or of oxygen (the latter from the water radical OH) is very often primary (for further details see p. 287). Thus the formulæ given in the above note, though they give the right end-result of the whole process, do not always provide a correct account of its detailed course.

We can draw the following general conclusion from the experiments described above:

*When an electric current passes through an electrolyte, either the metal or hydrogen separates at the cathode and either the acid radical or oxygen at the anode.*

When hydrogen is evolved at the cathode, the liquid in this region becomes alkaline: when oxygen is evolved at the anode, the liquid there becomes acid.

Concerning the nature of the chemical processes involved, we can say in general that *at the cathode there is always a chemical reduction, at the anode a chemical oxidation.*

## 2. Relation between Quantity of Electricity and Quantity of Substance decomposed. Faraday's First Law of Electrolysis

The next question we have to consider is the following. What is the relation between the quantity of electricity transported through the electrolyte and the quantity of substance deposited or evolved? *

To obtain the answer we perform the following experiment.

*Experiment.*—In an electric circuit we include several electrolysis tubes containing dilute sulphuric acid. The arrangement of the tubes is shown in fig. 6. Each tube is so constructed that the liquid can be let off through the tap, thus allowing of equalization of the levels in both limbs after a certain quantity of gas has been collected in the closed limb. This having been done for all the tubes, all the different amounts of gas are at atmospheric pressure and their volumes are therefore comparable. The arrangement of the tubes in the circuit (fig. 6) is such that the current coming from one pole of the source passes first through two tubes connected in parallel. The two branches of the circuit then unite and the whole current passes through a single tube. The wire then divides again into three branches, leading to three more tubes arranged in parallel. The current thus passes through these three tubes simultaneously. The three branches of the wire then unite again and lead to the other pole of the source of current. A current measurer (galvanometer, p. 141) is also included in the circuit. When a current flows, electrolysis occurs in all six tubes simultaneously, an explosive mixture of hydrogen and oxygen being evolved in each.

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* The complete answer to this question was provided by FARADAY (1833).

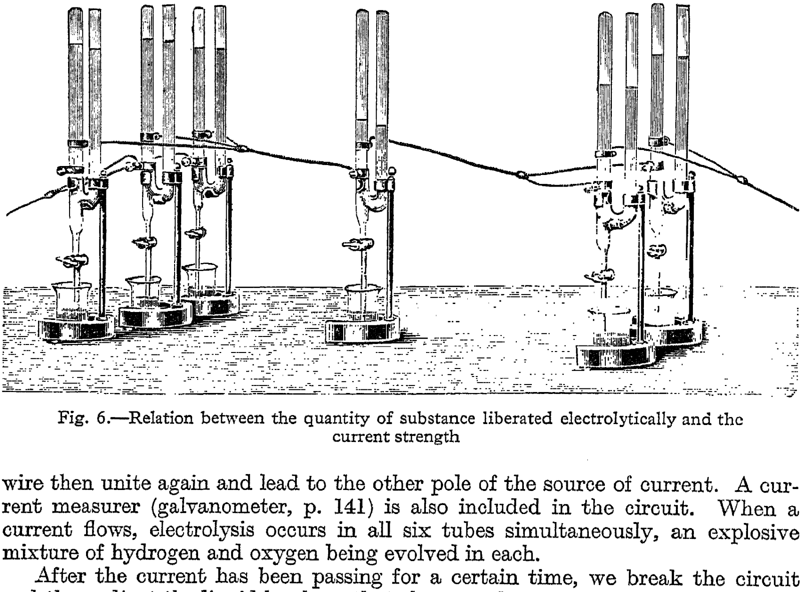

Fig. 6.—Relation between the quantity of substance liberated electrolytically and the current strength

After the current has been passing for a certain time, we break the circuit and then adjust the liquid levels so that they are the same in both limbs of each of the tubes.

We thus obtain the following result. The volume of gas mixture evolved in the middle single tube is equal to the sum of the volumes of the gas mixtures in the two tubes connected in parallel. Furthermore this volume is also equal to the sum of the volumes in the three tubes connected in parallel.

Now equal quantities of electricity pass through every section of a circuit carrying an electric current; for if this were not true, electricity would be dammed up somewhere and the corresponding portions of the conductor would become charged. These charges would eventually stop the current (p. 154). Hence the amount of electricity flowing in a certain time through the middle single tube must be equal to the total amount flowing in the same time either through the two tubes in parallel or through the three tubes in parallel. It does not matter whether the tubes which are connected in parallel are of the same or different sizes; for each group the sum of the volumes of gas mixture developed is found to be equal to the volume developed in the middle single tube. It is found further that the temperature has no influence upon the amount of substance decomposed. We are therefore led to the following conclusion:

*The quantity of substance decomposed by the passage of an electric current through an electrolyte is dependent only upon the quantity of electricity which passes.*

We can obtain a more precise relationship between the quantities of electricity and of substance electrolysed. While the current is passing through the tubes, we measure the quantities of substance evolved at different times. By means of a suitable instrument connected in the circuit we can easily see whether the current strength is constant. If this is the case, we find that the quantity of substance liberated is proportional to the time the current has been flowing. Thus, for example, twice the volume of gas is evolved in twice the time. Since now the quantity of electricity which has passed is also proportional to the time, it follows that twice the quantity of electricity decomposes twice the quantity of electrolyte, i.e. the amount of electrolysis is directly proportional to the quantity of electricity which has passed.

*Experiment.*—We can also obtain this result in the following way. If the three tubes connected in parallel (to the left in fig. 6) are exactly identical and if the branch wires leading to and from them are equal, then it follows from the symmetry of the arrangement that, when the tubes all contain the same liquid, the current flowing through each is the same i.e. $\frac{1}{3}$ of the total current flowing through the middle single tube. It is found, in fact, that equal volumes of gas mixture are evolved in each of the three tubes connected in parallel, and also that each of these volumes is $\frac{1}{3}$ of that evolved in the middle single tube.

The following relationship, **the first of Faraday's laws of electrolysis,** therefore holds:

**The quantity of the decomposition products formed by passage of an electric current through an electrolyte is directly proportional to the quantity of electricity which passes.**

## 3. The Application of Faraday's First Law of Electrolysis for the Measurement of Charge and Current. Coulombmeters (so-called Voltameters)

Faraday's first law of electrolysis can obviously be applied for measuring quantities of electricity. The volumes of hydrogen-oxygen mixture in the experiments of the last section are direct measures of the quantities of electricity which have passed through the respective tubes. It only remains to decide what volume of gas corresponds to unit quantity of electricity (see below).

Instruments in which the electrolytic action of an electric current is employed for the measurement of quantity of electricity are called **coulombmeters.***

*The Hydrogen-Oxygen Coulombmeter.*—This is shown in fig. 7. Instead of collecting the explosive mixture of gases evolved at both electrodes, the hydrogen generated at the cathode and the oxygen generated at the anode may be collected separately and their volumes added (see, for example, fig. 5, p. 146).

This type of coulombmeter has several disadvantages. The volume of oxygen, which ought to be exactly half that of the hydrogen simultaneously evolved, is always somewhat too small, because oxygen is rather more soluble in water than hydrogen. Further, a small fraction of this oxygen is transformed by the electrolysis into ozone.

The first source of error can be removed or at least diminished by not using the apparatus for measurement until the electrolysis has run for some time, i.e. until the liquid can be assumed to be saturated with oxygen. The second source of error can never be completely removed. Since one molecule of ozone contains three atoms of oxygen, whereas one molecule of oxygen contains only two atoms, the volume occupied by the gas evolved at the anode is always somewhat smaller than is to be expected theoretically.

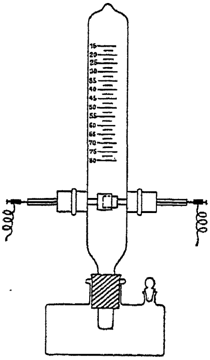

Fig. 7.—Hydrogen-oxygen coulombmeter

For these reasons, the more practical procedure is to measure only the volume of hydrogen evolved, and then to calculate from this the theoretical volume of the explosive mixture by multiplying by $\frac{3}{2}$.

In the instrument shown in fig. 7 and also in the tubes used in the experiment of fig. 6, p. 148, both gases are collected and measured together. This type of coulombmeter is specially suitable for measurements with strong currents, since the two electrodes are very close together, i.e. the internal resistance of the instrument is only small. The upper part of the apparatus is usually provided with a thermometer, in order that the measured gas volume may be corrected to 0° C. The volume must also be corrected to 760 mm. pressure. To obtain the actual pressure of the gas at the measured volume, the atmospheric pressure (as read off from a barometer) must be diminished (i) by the height of the column of mercury equivalent to the column of liquid between the level of the liquid in the container and that of the liquid in the measuring tube, and (ii) by the vapour pressure of the water vapour in the measuring tube. Since the gas comes directly out of the liquid, it is saturated with water vapour. The corresponding vapour pressure can therefore be obtained from Table V, Vol. II. The values given there are for vapour over water. In the coulombmeter, however, the vapour is over dilute sulphuric acid (about 15 per cent); the values taken from the table must therefore be multiplied by 0·9, the ratio of the saturated vapour pressure over 15 per cent sulphuric acid to that over pure water. The following Table III gives the volumes of hydrogen-oxygen mixture (in cm.$^3$) generated by 1 coulomb at the temperature $t°$ C. and pressure $p = (b \pm \frac{1}{12}h)$ mm., where $b$ is the external barometric pressure, and $h$ the height of the dilute sulphuric acid in the measuring tube above that in the container.

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* Faraday called these instruments volta-electrometers, as he used a voltaic pile for his experiments. The name *voltameter* is derived from Faraday's term. On account of the fact, however, that the name voltmeter is now generally used for instruments measuring potential difference, it is better to use the name *coulombmeter* for instruments measuring quantity of electricity, seeing that the unit of quantity is called a *coulomb* (p. 153).

TABLE III

| ↓ $t°$ C. | $p$ → 700 mm. | 705 | 710 | 715 | 720 | 725 | 730 | 735 | 740 | 745 | 750 | 755 | 760 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0·1982 | 0·1967 | 0·1953 | 0·1940 | 0·1927 | 0·1913 | 0·1899 | 0·1886 | 0·1872 | 0·1860 | 0·1848 | 0·1836 | 0·1824 |
| 12 | 0·2000 | 0·1984 | 0·1969 | 0·1956 | 0·1943 | 0·1929 | 0·1915 | 0·1902 | 0·1889 | 0·1877 | 0·1864 | 0·1852 | 0·1840 |
| 14 | 0·2017 | 0·2002 | 0·1987 | 0·1974 | 0·1960 | 0·1947 | 0·1933 | 0·1920 | 0·1907 | 0·1894 | 0·1880 | 0·1868 | 0·1856 |
| 16 | 0·2035 | 0·2020 | 0·2005 | 0·1992 | 0·1978 | 0·1964 | 0·1950 | 0·1937 | 0·1924 | 0·1911 | 0·1897 | 0·1885 | 0·1873 |
| 18 | 0·2053 | 0·2038 | 0·2023 | 0·2010 | 0·1996 | 0·1982 | 0·1968 | 0·1955 | 0·1941 | 0·1928 | 0·1914 | 0·1902 | 0·1889 |
| 20 | 0·2073 | 0·2058 | 0·2043 | 0·2029 | 0·2014 | 0·2000 | 0·1986 | 0·1973 | 0·1959 | 0·1946 | 0·1932 | 0·1920 | 0·1907 |
| 22 | 0·2093 | 0·2078 | 0·2063 | 0·2049 | 0·2034 | 0·2019 | 0·2004 | 0·1992 | 0·1977 | 0·1964 | 0·1951 | 0·1938 | 0·1925 |
| 24 | 0·2113 | 0·2098 | 0·2083 | 0·2069 | 0·2054 | 0·2039 | 0·2024 | 0·2011 | 0·1997 | 0·1984 | 0·1971 | 0·1958 | 0·1945 |
| 26 | 0·2133 | 0·2118 | 0·2104 | 0·2089 | 0·2074 | 0·2059 | 0·2044 | 0·2031 | 0·2017 | 0·2004 | 0·1991 | 0·1978 | 0·1965 |

In order to avoid the disadvantages of the above type of coulombmeter, instruments have been devised in which the subsidiary effects are less disturbing, and in which the determination is by weighing, a very accurate operation, instead of by volume measurement.

*The Copper Coulombmeter.*—The electrolyte used is a concentrated solution of copper sulphate. In order to repress secondary effects it is made somewhat acid and some alcohol is added. A thin, carefully weighed copper plate is used as cathode, the copper deposited upon it being weighed. The anode is a thicker plate of the same area made of chemically pure copper. The copper coulombmeter can be used with advantage with strong currents, since large copper plates can be employed.

Owing to secondary reactions occurring at the anode (p. 291), a fine sludge of copper is precipitated there (anode sludge). For this reason the loss of weight of the anode cannot be used as an additional measure of the quantity of electricity

*The Mercury Coulombmeter.*—This is sometimes used in electrical engineering for the indication of quantities of current electricity or, at constant potential difference, for the measurement of electrical energy (p. 252).

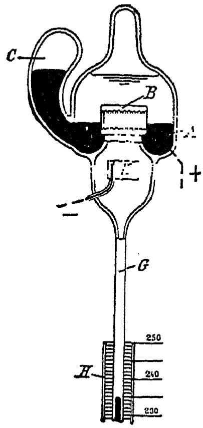

Fig. 8.—Mercury coulombmeter

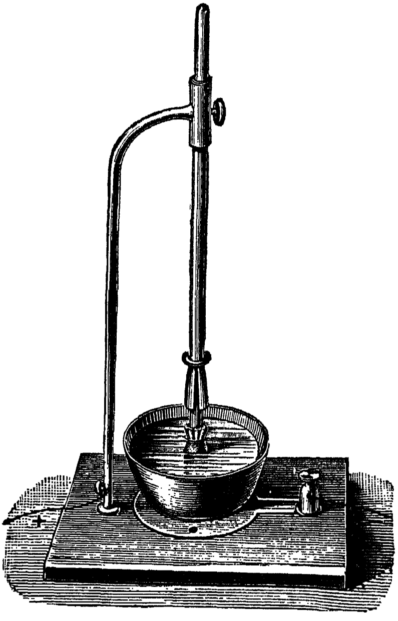

Fig. 9.—Silver coulombmeter

It consists (fig. 8) of a ring-shaped mercury anode A, an electrolyte containing mercury salts and a carbon cathode K. The latter is so arranged that the mercury liberated at it falls down into a glass tube G, in which its level can be read off upon a scale H as in a thermometer. The scale is calibrated so that it indicates amounts of electricity which flow through the instrument. When the level in H gets too high, the mercury can be run back into the anode mercury A by tilting the whole apparatus. The instrument reads correct to about 2 per cent, an accuracy sufficient for practical purposes.

*The Silver Coulombmeter.*—A practical form of silver coulombmeter is shown in fig. 9. The electrolyte is an aqueous solution of silver nitrate. The anode is a rod of pure silver clamped at its upper end in a suitable stand. Its lower end is enclosed in a small bag of clean linen, to prevent the falling away of particles which may become mechanically detached. This anode dips into the electrolyte, which is contained in a platinum vessel standing upon a metal base connected to the negative pole of the source of current. The platinum vessel thus acts as cathode. It is first cleaned and weighed, then filled with silver nitrate solution, and the silver anode dipped in. The quantity of electricity to be measured is passed through the instrument and the platinum vessel is once again emptied, carefully rinsed out, dried and weighed.

When correctly used, the silver coulombmeter gives results of extreme precision; it has therefore been used internationally for the definition of the unit quantity of current electricity.

## 4. Definition of the Units of Charge and of Current Strength

**The Unit of Electric Charge.**—On account of its easy and very accurate reproducibility the following quantity of electricity has been decided upon internationally as the unit:

**The unit quantity of electricity is that quantity which deposits 1·118 mgm. of silver at the cathode when passed through a silver coulombmeter.***

This unit bears the name **1 coulomb.**

**Comparison with the Quantities of Electricity measured in Electrostatics.**—In order to get an idea of electrostatic effect of the quantity of electricity passing through an electrolysis apparatus, we will measure with the help of a coulombmeter the quantity of electricity stored up in a condenser. For this purpose we employ the circuit shown in fig. 10 and allow the discharge of the condenser to pass through the coulombmeter. The change-over switch enables us first to connect the condenser with the mains, thus charging it up, and then to short-circuit it through the coulombmeter. Since, as we shall see, the volume of gas evolved is only small even if the capacity of the condenser is very great, we employ a hydrogen-oxygen coulombmeter with a very narrow collecting tube graduated in mm.$^3$. The best arrangement (fig. 11) is to have a capillary tube with a funnel-shaped lower end, closed above by an exactly fitting movable wire. By sliding this wire down, any bubbles of gas which may remain clinging to the walls can be detached and made to enter the capillary. As condenser we employ ten paper condensers each of 2 μf. (telephone condensers, p. 85) connected in parallel, the whole therefore having a capacity of 20 μf. To this battery of condensers we apply a potential difference of 220 volts by connecting it, for example, to the direct-current lighting mains. We then discharge it through the coulombmeter. This produces a noticeable evolution of gas. In order to be able to read the volume more exactly, we repeat the process twenty times. We then obtain 16·7 mm.$^3$ of hydrogen-oxgyen mixture. Since now 0·19 mm.$^3$ of the mixture corresponds to $1.10^{-3}$ coulombs of electricity (at room temperature and mean barometric pressure), the quantity of electricity which has passed through our instrument is $(16{\cdot}7/0{\cdot}19) \,.\, 10^{-3} = 88 \,.\, 10^{-3}$ coulomb. Each discharge therefore involved $(88 \,.\, 10^{-3})/20 = 4{\cdot}4 \,.\, 10^{-3}$ coulomb. This quantity of electricity was stored up in the condenser battery at a potential difference of 220 volts; the quantity stored up at a potential difference of 1 volt would therefore be $20 \,.\, 10^{-6}$ coulomb = 20 microcoulombs. These figures hold on the assumption that the condenser arrangement employed has a capacity of 20 μf. (see p. 73). This is a quite unusually great capacity, however; ordinary arrangements (such as leads, electrometers, &c.) have capacities which are very much smaller, so that even at large pressures only fractions of a microcoulomb can be stored up statically in them. In contrast to this, a pocket flash-lamp battery can furnish some 10,000 coulombs before it is exhausted. We see, therefore, how very much greater are the quantities of electricity which we use as currents than the quantities which the conductors can bear statically (see also p. 487).

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* For the reasons for choosing this quantity see p. 193.

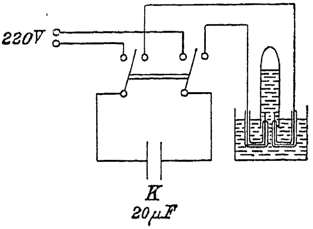

Fig. 10.—Measurement of the quantity of electricity stored up statically in a condenser.

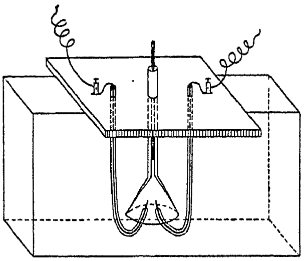

Fig. 11.—Microcoulombmeter (after Mie)

In this connexion it is instructive to remark that the quantity of electricity discharged in the course of a severe thunderstorm is about equal to that furnished by an ordinary bell battery during its useful life. The difference lies in the potentials, and in the extremely short time taken by the discharge in the case of lightning as compared with the time over which the bell battery discharge extends.

**The Unit of Current Strength.**—An electric current can be compared in a certain sense with a current in an incompressible liquid (e.g. water), since the same quantity of electricity (or liquid respectively) passes through every cross-section of the wire (or pipe). Even neglecting the magnetic field which is always associated with an electric current (p. 138), there is, however, an important difference between the two kinds of current, namely, the fact that in certain circumstances the electric current may be made up of two parts, the current of positive electricity and the current of negative electricity. These two partial currents are in opposite senses, and therefore add together. It is important to note that only the *sum* of the positive and negative currents is constant for all cross-sections of the conductor (p. 270). If the current strength were not the same at all cross-sections of the conductor, then in the case of the *stationary* or *steady currents* (i.e. constant in time) considered here there would necessarily be a damming up of electricity at some points and a corresponding charging up of certain parts of the conductor. But in consequence of the small capacity of the conductor and the comparatively enormous quantities of current electricity (see above), this would cause such high potential differences that the current would necessarily very soon cease to flow; for the potential differences giving rise to the motion of the electricity would be neutralized.* Thus, in the case of steady currents, "electricity" behaves like an incompressible liquid driven along through a system of pipes.

We have already (p. 137) defined "current strength" as the quantity of electricity passing through any cross-section of the conductor in a second. As pointed out above, we have to understand quantity of electricity to mean the sum of the quantities of positive and negative electricity.

The international *unit of current strength* is called **1 ampere.**†

**The strength of a current is 1 ampere when, flowing uniformly through a silver coulombmeter, it deposits 1·118 mgm. of silver per second.**

Thus in the case of a current of 1 ampere, 1 coulomb of electricity passes every cross-section of the conductor per second.

Quantity of electricity can therefore be expressed as the product of current strength and time. Since current strength and time can be readily measured, this method is often of advantage. 1 coulomb being equal to **1 ampere-second,** the name ampere-second is often used for the unit quantity of electricity. We shall keep to the name coulomb, however, since it is better to use a proper name for a concept, like "quantity of electricity", of which we can form some kind of direct mental picture.

**Current Density.**—The *current density* $G$ is the total quantity of electricity passing per second through *unit area* at right angles to the current. It is given therefore by the quotient of current strength $I$ divided by the area $A$ through which it flows, i.e. $G = I/A$.

## 5. The Relation between the Masses of Different Substances electrolysed by the same Quantity of Current Electricity. Faraday's Second Law of Electrolysis

We will now consider the following question. What is the relation between the quantities of different substances liberated from different solutions by the passage of the same quantity of electricity?

*Experiment.*—To obtain an answer to this question, we connect a copper coulombmeter, a silver coulombmeter and a hydrogen-oxygen coulombmeter in series in the same circuit (fig. 12). We allow the current to pass for a few minutes and then determine the amounts of the different substances liberated. Supposing that the mass of silver deposited in 10 min. is 0·108 gm., then the mass of copper deposited by the same quantity of electricity is 0·032 gm. and the (corrected) volume of hydrogen-oxygen mixture is 16·8 cm.$^3$, i.e. 0·009 gm. (hydrogen + oxygen).

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* These considerations do not necessarily hold for currents which are not steady. (See Chap. XVI, p. 567).

† Abbreviated to amp. after numerical data.

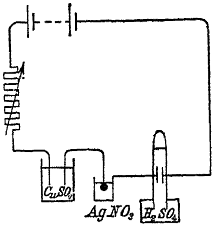

Fig. 12. — Determination of the quantities of different substances deposited by the same quantity of electricity.

Inspection of the numbers so obtained leads to the following law, known as **Faraday's second law of electrolysis:**

**The weights of different substances liberated in equal times by currents of the same strength (i.e. by equal quantities of electricity) are in the ratio of the corresponding chemical equivalent weights,** i.e. in the ratio of the quotients of the atomic weight divided by the valency of the respective elements, or the molecular weight divided by the valency in the case of radicals.

As can be seen from the chemical formulæ $HCl$, $NaCl$, $AgNO_3$, $H_2O$, $ZnCl_2$, $H_2SO_4$, $CuSO_4$, $HgCl_2$, $FeCl_3$, the following quantities of different elements and radicals are chemically equivalent: H, Cl, Na, Ag, $NO_3$, $\frac{1}{2}$O, $\frac{1}{2}$Zn, $\frac{1}{2}$Cu, $\frac{1}{2}$Hg, $\frac{1}{2}SO_4$, $\frac{1}{3}$Fe.

Hence the same quantity of electricity which liberates 1·008 gm. of hydrogen will liberate 35·46 gm. of chlorine, 107·88 gm. of silver, 62·01 gm. of nitrate radical, 8 gm. of oxygen, 32·7 gm. of zinc, 31·8 gm. of copper, 100·3 gm. of mercury, and 18·6 gm. of (ferric) iron.

It is to be noted that the electrochemical equivalent (see below) of an element changes with the valency. Thus, for example, if a solution of cupric sulphate ($CuSO_4$) and a solution of (chemically pure) cuprous chloride ($CuCl$) be connected in series in the same circuit, then in the time required for the deposition of 31·8 mgm. of copper from the cupric sulphate twice this mass (namely 63·6 mgm.) of copper will be deposited from the cuprous chloride. In cupric sulphate the copper is divalent, in cuprous chloride univalent.

The mass of a substance liberated by the passage of 1 coulomb of current electricity is called the **electrochemical equivalent** of the substance. The values for a few substances are contained in the following Table IV (top row of figures).

TABLE IV

| | A current of 1 amp. decomposes or deposits | | | | | |
|---|---|---|---|---|---|---|
| | mgm. equivalents | mgm. silver | mgm. copper | mgm. mercury | mgm. nickel | mgm. water |
| in 1 sec. | 0·01036 | 1·118 | 0·3294 | 1·040 | 0·3050 | 0·0933 |
| in 1 min. | 0·6215 | 67·08 | 19·76 | 62·40 | 18·30 | 5·60 |
| in 1 hr. | 37·29 | 4025 | 1186 | 3744 | 1098 | 335·9 |

Thus a current of 1 ampere deposits

$$1{\cdot}118 \text{ mgm.} = 0{\cdot}001118 \text{ gm.}$$

of silver in one second. The atomic weight of silver and hence also its equivalent weight (since silver is univalent) is 107·88. By division we obtain $0{\cdot}001118/107{\cdot}88 = 0{\cdot}000010363$. This is the number of chemical gramme-equivalents deposited in one second by a current of 1 ampere. The corresponding number of milligram-equivalents per second is 0·010363 (see Table IV).

These numbers have simply to be multiplied by the quotient of atomic weight by valency in order to obtain the mass of substance liberated by 1 ampere-second (= 1 coulomb), i.e. the electrochemical equivalent of any substance.

The mass of hydrogen liberated per second by a current of 1 ampere is $1{\cdot}008 . 0{\cdot}01036$ mgm. = 0·01045 mgm. This mass occupies at 0° C. and 760 mm. pressure a volume of $0{\cdot}01045/0{\cdot}00008995 = 116$ mm.$^3$.

In the electrolysis of dilute sulphuric acid both hydrogen and oxygen are evolved, and may be collected together. Since water is composed of $2H_2$ and $O_2$, the volume of the mixture is $1\frac{1}{2}$ times greater than that of the corresponding hydrogen alone. Hence a current of 1 ampere liberates in a second

$$1\tfrac{1}{2} . 116 = 174 \text{ mm.}^3 = 0{\cdot}174 \text{ cm.}^3$$

of hydrogen-oxygen mixture. In one minute the same current liberates $60 . 0{\cdot}174 = 10{\cdot}44$ cm.$^3$, in one hour 626 cm.$^3$ of the mixture. (All these volumes are corrected to 0° C. and 760 mm. pressure.)

**The Quantity of Electricity transported by 1 Gramme-equivalent.**— It is of importance to calculate the quantity of electricity transported by one gramme-equivalent of a substance, i.e. by a number of grammes of the substance equal to the quotient of its atomic weight divided by its valency. Using the data given above for silver we obtain the value $107{\cdot}88/0{\cdot}001118 = 96{,}494$ coulombs.

*Each gramme-equivalent transports* 96,494 *coulombs of electricity.*

This quantity of electricity is called *one faraday.*

Two faradays or three faradays are transported respectively by one gramme-atom of a divalent or trivalent substance. In the application of the fundamental laws of electrochemistry to chemical reactions it has been found very convenient to associate these characteristic quantities of electricity with the symbol of the corresponding element or radical. The notation employed is illustrated by the following examples: $H^+$ or $H^{\bullet}$ stands for 1·008 gm. of hydrogen + 96,494 coulombs (= 1 faraday) of positive electricity; $Cl^-$ or $Cl'$ stands for 35·46 gm. of chlorine + 96,494 coulombs (= 1 faraday) of negative electricity; $Fe^{+++}$ or $Fe^{\bullet\bullet\bullet}$ stands for 55·8 gm. of trivalent (ferric) iron + 3 . 96,494 coulombs (= 3 faradays) of positive electricity; and $Fe^{++}$ or $Fe^{\bullet\bullet}$ stands for 55·8 gm. of divalent (ferrous) iron + 2 . 96,494 coulombs (= 2 faradays) of positive electricity.

*Behaviour of fused and solid electrolytes.*—Faraday showed that the above laws hold not only for aqueous solutions but also for fused salts. In the latter case, however, the conditions are so masked by secondary effects that the exact determination of the regularities presents very serious experimental difficulties. The measurements of quantities of metal deposited are rendered especially difficult by the fact that at the high temperatures employed the metal distributes itself in finely divided form throughout the whole of the fused electrolyte (LORENZ). When secondary reactions are avoided, complete agreement with Faraday's laws is found. Glass conducts an electric current electrolytically even in the solid state (WARBURG), especially at high temperatures (p. 355). Using mercury electrodes, for example, it is possible with a sufficiently high E.M.F. to remove four-fifths of the sodium from a vessel of glass by electrolysis. This fact can be used as an artifice for introducing very pure sodium into a closed space (for example, into a high vacuum for the removal of oxygen) without opening the enclosing walls. Solid crystals also show electrolytic conductivity under certain conditions. The experimental difficulty met with in investigating this behaviour is the formation of a fine thread of the metal deposited at the cathode; this thread extends through the crystal and soon forms a conducting bridge between the electrodes. This can be prevented by covering the cathode with a layer of compressed $\alpha$-silver iodide, a salt in which the bridge-formation does not occur. The crystal under investigation is placed against this layer and the anode against the crystal. Concerning the remarkable phenomena of current transport observed in this way see p. 273.

**The Quantity of Electricity Transported by one Atom.**—The gramme-atom (or gramme-molecule) of any substance, i.e. a number of grammes of the substance equal to the atomic (or molecular) weight, contains the same number of atoms (or molecules). The value of this number, the so-called Loschmidt or Avogadro number, is $N = 6{\cdot}06 \,.\, 10^{23}$ (see Vol. II, p. 53).

Hence we can easily calculate the quantity of electricity (i.e. the charge) associated with each atom of a univalent element. We obtain the value

$$96{,}494/(6{\cdot}06 \,.\, 10^{23}) = \mathbf{1{\cdot}59 \,.\, 10^{-19}\ coulombs/atom.}^{*}$$

But this is the same value as was obtained (p. 51) for the **elementary electric charge.** In Millikan's experiment (p. 48) it was found that, when the charge upon the particle is changed (e.g. by X-rays), *only* integral multiples of the elementary charge occur. We conclude, therefore, that in electrolysis the above value is not only an average per atom, but that every atom transporting the current is indeed charged with exactly this amount of electricity (or with an integral multiple of it).

*Every atom of a univalent element carries one elementary charge of (positive or negative) electricity in electrolysis. Divalent, trivalent, &c., atoms or radicals carry a corresponding number of elementary charges.*

*Atoms or radicals charged in this way (i.e. corresponding to their valency) are called* **ions.**

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* Or $1{\cdot}59 \,.\, 10^{-20}$ electromagnetic C.G.S. units (p. 192) = $4{\cdot}77 \,.\, 10^{-10}$ electrostatic C.G.S. units.

We may mention here in advance that all the experimental evidence proves that only the negative elementary charge, the **electron**, is capable of existing independent of matter. The corresponding positive charge cannot be separated from the matter with which it is associated (p. 345). This means that addition of electricity (i.e. electrons) can only charge a body negatively. In order to charge the body positively, we must remove electrons (i.e. negative electricity) from it. A negative ion is therefore an atom or radical which has taken up one or more electrons; a positive ion is an atom or radical which has lost one or more of its electrons. (See, however, footnote on p. 345.)

The detailed nature of the current transport and the molecular processes involved are treated in Chapter VIII (p. 266).

## CHAPTER V

# The Relation between Current and Magnetic Field

## 1. The Magnetic Lines of Force associated with an Electric Current

**Field of Force of a Straight Conductor.**—We have already become acquainted (p. 138) with the fundamental fact that a straight conductor carrying a current is surrounded by a magnetic field. We will now consider the nature of this field in more detail.

We saw in figs. 10 and 11 (p. 138) that the magnetic lines of force in a plane at right angles to the conductor are concentric *circles* with their centre upon the axis of the conductor.

In order now to find out whether these circles represent components of forces acting in a different direction, we make use of a freely movable magnetic needle. With its help we explore the field in the neighbourhood of a vertical wire through which a current is flowing. The result is shown in fig. 1, from which it is seen that the needle always sets itself perpendicular to the wire, remaining always in a horizontal plane and forming the tangent to a circle described about a point upon the axis of the wire. Hence:

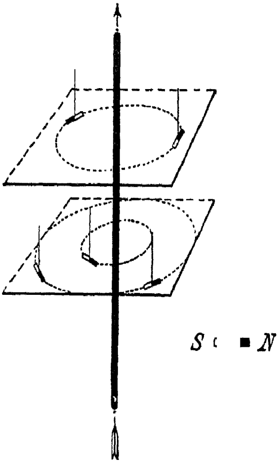

Fig. 1.—Magnetic field of a straight conductor carrying a current

**The magnetic lines of force are circles with their planes perpendicular to the straight conductor and with their centres upon its axis.**

**Sense of the Lines of Force.**—On p. 143 we defined the positive sense of an electric current as that in which the positive electricity moves or would move. Taking account now of the south-north sense in which the magnetic needle points in the above experiment (fig. 1), we arrive at the following important result:

*Looking along the conducting wire in the positive direction or sense of the current, the north pole of the needle always points in the clockwise sense.*

Since we have defined the direction or sense of the magnetic lines of force as that from the south pole to the north pole of a test needle, therefore:

**Looking along the conductor in the positive direction or sense of the current, the magnetic lines of force are circles and their sense is clockwise** (fig. 2 *).

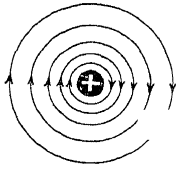

Fig. 2. — Magnetic field of force around a conductor carrying a current. Direction of current is away from the observer.

As an aid to the memory, this result may be expressed in the following form:

*The sense of the circular lines of force is that in which a corkscrew must be turned in order to drive it in the positive direction of the current.* (**Maxwell's Screw Rule, Corkscrew Rule**) (see fig. 3).

**Historical.**—The connexion between the direction of the current and the direction of deviation of a magnetic needle was discovered in 1820 by the French physicist Ampère. He expressed the result as follows (**Swimmer rule**):

*Imagine a man swimming in the positive direction of the current (so that the sense of the positive current is from his feet towards his head) and looking at the magnetic needle. Then the direction of deflection of the north pole of the needle is that of his outstretched left arm.*

This rule is often stated nowadays in the following form, known as the **right-hand rule**:

*Lay the right hand upon the conductor in such a way that the positive direction of the current is from the wrist towards the finger-tips. The palm of the hand must face towards the magnetic needle, as though the hand were about to grasp its pole. Then the north pole of the needle is deflected in the direction of the extended thumb* (fig. 4).

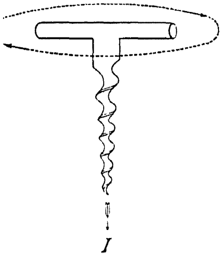

Fig. 3.—Corkscrew rule

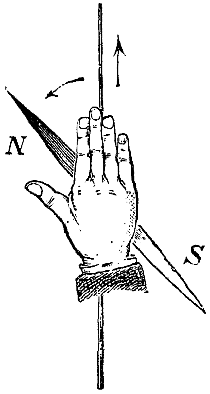

Fig. 4.—Right-hand rule

The best way is always to think out problems of this kind in terms of the corkscrew rule. With its help the nature of the magnetic field can easily be discovered even in the more difficult cases.

**Relation between Electric and Magnetic Lines of Force.**—As has already been shown on p. 134, the electric lines of force are at right angles to the direction of the current. As the current flows they move at right angles to their own direction. Fig. 5 shows the electric lines of force (dotted) and the magnetic lines of force (continuous) of a single conductor, which is to be imagined as passing at right angles through the plane of the paper and carrying a current whose positive sense is away from the observer.

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* The white cross in the middle of the conductor, which is printed black, represents the feathered end of an arrow seen end-on, and indicates that the current is flowing *away* from the observer.

The magnetic lines of force are seen to be at right angles both to the direction of the current and also to the electric lines of force.

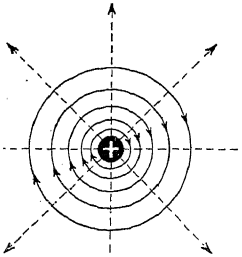

Fig. 5.—Relation between electric and magnetic lines of force

This can also be seen very well in the more complicated case of a double conductor, when the two parts, in which the current flows away from and towards the observer respectively, are fairly close together. Figs. 6 *a* and *b* show the course of the electric lines of force (compare also fig. 9, p. 10), figs. 7 *a* and *b* the course of the magnetic lines of force. The current in the right-hand conductor is assumed to be flowing towards the observer, that in the left-hand conductor

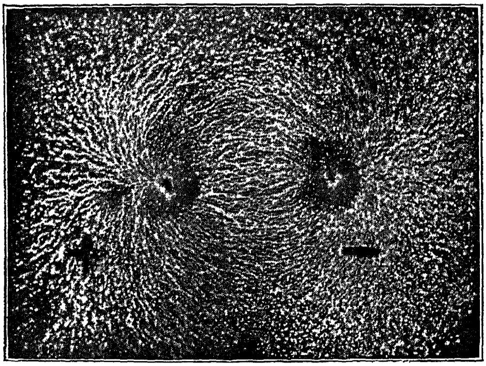

Fig. 6*a*.—Electric field of a double conductor in a plane at right angles to the current direction

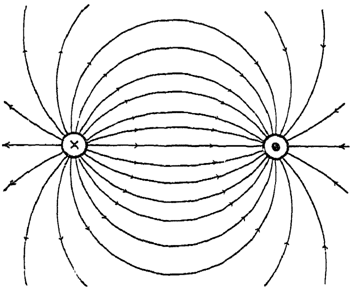

Fig. 6*b*.—Electric field of a double conductor

away from the observer. We see here very clearly, especially in the central region where the lines of force are nearly straight, how the three directions, namely the direction of motion of the electricity (vector **v**), the direction of the electric force **E** and the direction of the magnetic force **H** are mutually perpendicular. We arrive therefore at the following result:

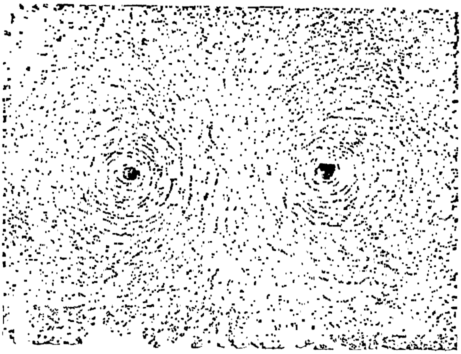

Fig. 7*a*.—Magnetic field of a double conductor in a plane at right angles to the current direction

*Electric lines of force moving at right angles to their direction are always associated with magnetic lines of force, the latter being perpendicular both to the electric lines of force and to their direction of motion.*

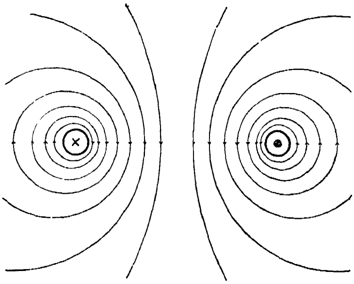

Fig. 7*b*.—Magnetic field of a double conductor

We have defined the positive sense of the lines of force as that from a positive charge to a negative charge. The relation between the directions and senses of the three vectors **v**, **E** and **H** is therefore that shown in the diagram of fig. 8.

A more general quantitative treatment of these facts will be given later (Chap. XVI, p. 567 *et seq.*).

The following experiment gives a hydrodynamical analogy to the above conditions. When a rotating cylinder is moved at right angles to its axis of rotation, a transverse current is produced. This is the inverse of the Magnus effect (Vol. I, p. 388), appearing in accordance with the principle of reaction. A stream of air is directed by means of a fan (fig. 9) against a rotating wooden cylinder mounted upon the axis of an electric motor. The transverse current can be demonstrated by the deflection of two candle flames placed one on each side of the cylinder (see fig.). The air stream from the fan representing the electric current, the rotating cylinder thus corresponds to an electric line of force, the transverse current to a magnetic line of force (see also p. 50).

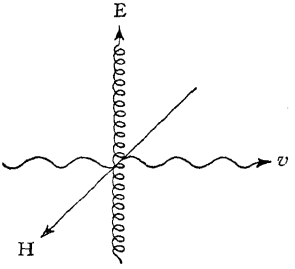

Fig. 8.—Relation between **E**, **H**, and *v*

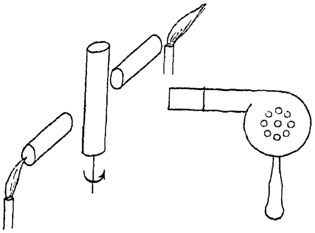

Fig. 9.—Hydrodynamical analogy to the magnetic field of a moving electric field

## 2. The Magnetic Field as a Property of Moving Electric Lines of Force

As we have already learned on p. 135, an electric current is nothing else than electric charge in motion. We shall expect, therefore, that every electric charge in motion, no matter by what means it is moving, will have a magnetic field. This is indeed the case, as experiments (especially those of ROWLAND, RÖNTGEN, HIMSTEDT, EICHENWALD, and TOLMAN) have shown.

**Convection Current.**—A moving electrified body, for example a moving charged ball or a moving condenser, constitutes what is called a *convection current*. In contrast to a current flowing through a conductor, a convection current can flow without electromotive force (i.e. potential difference) and without any energy transformations; thus it produces no heat. In accordance with the laws of mechanics, such an electric charge, when once it is set in motion, can go on moving uniformly in a straight line without the agency of any force.

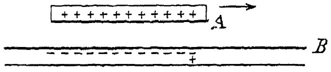

Fig. 10.—Conduction current induced by a convection current

**Motion of the Lines of Force in the Case of a Convection Current.**—It is important to consider first the course of the lines of force when a charge is moving slowly.

Take the case of a charged conducting surface A (fig. 10) moving along over an uncharged conducting surface B. There will be a displacement of the electricity in the second conductor, a charge of opposite sign from that on A being induced upon the surface B facing A. As A moves along, this induced charge will also move along correspondingly, and an electric current, the so-called **conduction current,** will be formed in B. The question now arises whether, as might at first be supposed, the lines of force pull the opposite charges along with them, or whether they remain always at right angles to the conducting surfaces and the displacement of charge occurs anew at every successive point. In the latter case the displacement process would always be just beginning in the lower conductor immediately beneath the forward edge of A, whereas beneath the rear edge of A the displaced charges would always be reuniting, i.e. the induced charge on B would be disappearing.

If the process takes place in this way, a potential difference will be produced in consequence of the pull of the surroundings upon the charge (of like sign to that of A) which has become free in the lower conductor. This potential difference will cause a motion of the electricity in the lower conductor in accordance with Ohm's law (p. 219). Experiments carried out by EICHENWALD have shown that the process does actually take place in this way.

It follows that if the motion causes no alteration of the charge distribution relative to the surroundings, there will be no potential difference and hence also no conduction current. Since the experiments upon this point are of fundamental importance for the understanding of the behaviour of moving lines of force, we will go into them in some detail. The case illustrated in fig. 10 may be modified as shown in fig. 11, namely by rotating a semicircular charged disc A over an uncharged disc B. Discs of glass with very thin coatings of tinfoil may be used. The resulting conduction current from $v$ to $r$ flows in accordance with Ohm's law (p. 219), both by way of $z$ and also by way of $n$. If, on the other hand, the disc A is not semicircular but in the form of a complete circle, then the field between the two discs remains constant during the rotation, and *no conduction currents whatever are produced in B.*

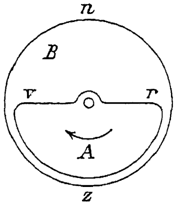

Fig. 11.—Production of a conduction current

Thus if the tinfoil coating of the disc B has a radial slit cut in it, and if the ends of this be connected to a sensitive galvanometer, no current is indicated when the *complete circular disc* A is rotated uniformly* over B. Hence the electric lines of force remain at right angles to the coatings of the discs even during the relative motion, and are not subjected to torsional deformation.

**Magnetic Action of a Convection Current** (ROWLAND, 1876; A. EICHENWALD).—In order that the conditions may be quite definite, we will use a condenser consisting of two plates capable of rotation.

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* When the rotation is non-uniform (e.g. at the beginning and end of the motion) feeble induced currents are observed (p. 369).

Fig. 12 shows the arrangement in section. A small magnetic needle M with mirror is placed above the charged plate K so as to indicate its magnetic action. As a protection against electrostatic disturbances, the needle is enclosed in an earthed metal case. The condenser is charged and the plate K set in rotation. The magnetic needle is observed to be deflected, an effect that can only be due to the motion of the charges upon the rotating plate K, because we have seen above that no conduction currents are produced in this case. Experiments of this kind have shown that in all cases, no matter whether the plates are rotated singly or both in the same direction or in opposite directions, the magnitude and direction of the resulting magnetic field are the same as would be produced by the corresponding currents flowing in the ordinary manner by conduction through the discs at rest.

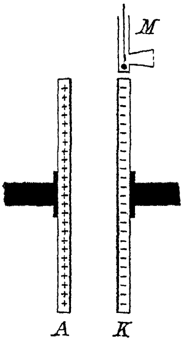

Fig. 12. — Magnetic effect of moving electric charges.

*Hence a convection current is completely equivalent to a conduction current as regards magnetic action.*

**Magnetic Action of a Moving Polarized Dielectric.**—The experiments upon the magnetic action of a moving polarized dielectric are also very instructive and important. The effect of filling the space between the plates of a condenser with a dielectric of dielectric constant K is to make the charges upon the plates K times as great as before for the same potential difference between them (p. 82). We shall expect, therefore, that the convection current produced by rotating the plates, as measured by its magnetic action, will also be K times greater than before. Experiment shows that this is actually the case.

A magnetic field can also be observed (Röntgen 1885, Eichenwald) when the condenser is kept at rest and the dielectric is rotated (see fig. 13). This effect is due to the "apparent" charges (p. 89) upon the surfaces of the dielectric. Their surface density (p. 101) is (K — 1)/K times that of the true charges of the plates. Since both surfaces of the dielectric necessarily rotate together, only a difference effect can be observed. Concerning the magnetic field in the case when condenser and dielectric are moved together see Vol. V.

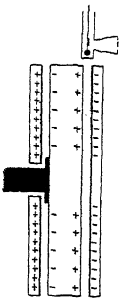

Fig. 13.—Magnetic effect of a moving polarized dielectric.

**The Magnetic Field of a Displacement Current.**—We have already seen (p. 134) that during the discharge of a condenser the electric lines of force move at right angles to their own direction. We must expect, therefore, that the increase (or decrease) of the electric field during the charging up (or discharge) of a condenser will be accompanied by a magnetic field (fig. 14).

This expectation, which is actually fulfilled in experiment, originated in the mind of Maxwell, who made it the basis of his calculations. Since magnetism was only known in association with an electric current, Maxwell gave the name **displacement current** to this kind of motion of the electric lines of force. Throughout our discussion we have directed attention to the fact that moving electric fields in the ether are associated with current phenomena, so that the concept of a current corresponding to the motion of lines of force alone and not of charges will probably seem natural enough.

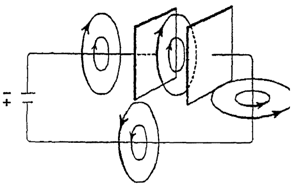

Fig. 14.—Magnetic fields of current flowing in a conductor and of displacement current.

If the condenser is filled with a dielectric of dielectric constant K, the increase of the field during charging up produces a polarization (p. 87). This process involves movements of the electric charges within the dielectric, the opposite charges of the molecules moving apart to a slight extent in opposite directions. In this case therefore we have both the magnetic effect of the displacement current corresponding to vacuum and also the magnetic effect due to these molecular displacements. These two effects are superimposed, and the total magnetic action is therefore increased. The displacement of the molecular charges out of their normal position of equilibrium is (like an elastic extension) proportional to the force producing it, i.e. proportional to the electric field (p. 100).

Like every movement of electric charge, this displacement of molecular charges corresponds to an electric current. In this case we may call it the **dielectric polarization current.**

The magnetic field produced by the polarization of the dielectric can be demonstrated as follows:

We employ a rotating dielectric (as in fig. 13), but the condenser plates are each in the form of two semicircles insulated from one another and charged as shown in fig. 15 *a* and *b*.

So long as the rotating dielectric is between 1 and 2, it is polarized in the manner indicated in the lower half of fig. 15*a*. But when it is between 3 and 4 it is oppositely polarized (top half of fig. 15*a*). Hence in passing from the first position into the second the polarization must be reversed. As can be easily seen from the figure, this gives rise to a positive current in the direction $I_p$. The magnetic action of this polarization current can be measured by means of a very sensitive magnetometer placed at M. The magnitude of the apparent charge is $(K - 1)Q$, where Q is the charge upon the condenser without dielectric. If $n$ be the number of revolutions of the dielectric per second, the polarization current in the above case is equivalent to an ordinary current of strength $I_p = 2(D - 1)\,nQ$.

The factor 2 is due to the fact that the polarization is not merely reduced to zero (as in our imagined experiment upon the discharge of a condenser, p. 87) but is at once reversed.

The first to succeed in proving the existence of these currents was Röntgen.* Their quantitative equivalence with ordinary conduction currents of corresponding strength was established by A. Eichenwald.

The existence of displacement currents of the kind assumed by Maxwell (i.e. the magnetic action due only to the motion of electric lines of force, an effect which must also occur in the absence of material dielectric) was first proved by H. Hertz. We must, however, leave the treatment of this until later (Chap. XVI, p. 567).

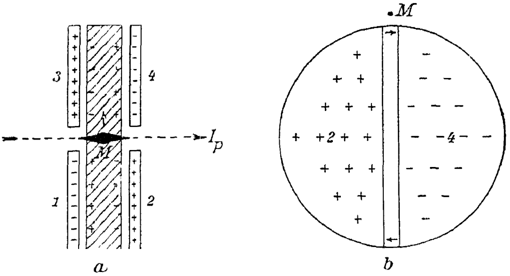

Fig. 15.—Magnetic effect of polarization current

As we shall gradually recognize in what follows, the introduction of the concept of displacement currents into the treatment of the discharge of a condenser has led, through Maxwell's inspired work, to very far-reaching results.

## 3. The Form of the Magnetic Fields of Currents flowing in Conductors of different Shapes

**Field in the case of a Circular Ring.**—The form of the magnetic lines of force around a circular conductor carrying a current can be most easily seen with the help of iron filings. If the conductor be laid upon the table, a piece of paper laid upon it and iron filings strewn upon the paper, then when the current is switched on the filings arrange themselves as in fig. 16. The lines of force are at right angles to the conductor. When the current is flowing in a clockwise direction round the ring, a magnetic needle placed within the ring is deflected so that its north pole points downwards.

We now set up the conductor with the plane of the ring vertical, and support a piece of paper horizontally in such a way that its plane passes through a diameter of the ring. (The conductor passes through holes in the paper, see fig. 18.) When the current is switched on, iron filings strewn upon the paper arrange themselves in the manner shown in fig. 17.

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* For this reason they are sometimes called Röntgen currents.

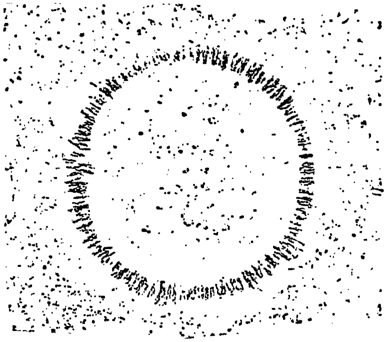

Fig. 16.—Diagram of the magnetic lines of force in the plane of a ring conductor carrying a current

Fig. 18 shows in perspective the field in a plane through the centre of the circular ring and perpendicular to its plane. Application of the Maxwell screw rule for the different parts of the conductor (fig. 19) enables us at once to determine the sense of the field. In agreement with the previous experiment with the magnetic needle we obtain the following result, which is obvious from fig. 18:

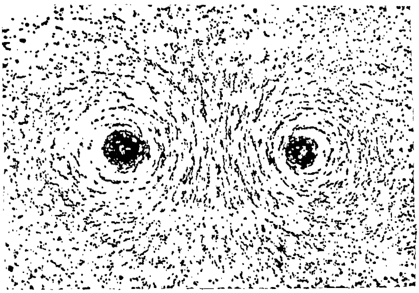

Fig. 17.—Lines of force in a plane at right angles to that of a ring conductor carrying a current

*As looked at in the direction and sense of the lines of force, the current in the ring is clockwise.*

The same result can be obtained with the help of the corkscrew rule. If the direction and sense of the lines of force are those of the motion of a corkscrew being driven in, the corresponding current must flow round in the same sense as that in which the corkscrew is being turned (fig. 20).

**Field of a Solenoid.**—In the case of a conducting wire covered with an insulating substance and wound in the form of a close spiral upon a cylinder (e.g. upon a glass tube), each individual turn may be regarded as a circular ring. Such a spiral coil is called a *solenoid* * (fig. 21).

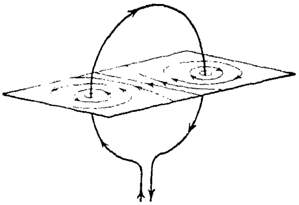

Fig. 18.—Magnetic field of a current flowing round a ring conductor

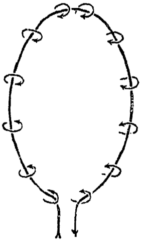

Fig. 19.—Sense of the field due to a current in a ring conductor

The form of the magnetic field, as made visible by iron filings, is shown in fig. 22. The black strips indicate the position of the solenoid, being the holes in the paper through which the turns of wire passed when the experiment was being performed.

As is seen from a comparison with fig. 11, p. 116, the magnetic field outside a solenoid corresponds completely to that of a bar magnet. Within the solenoid the field is seen to be approximately homogeneous.

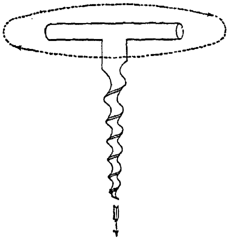

Fig. 20.—Relation between current sense and sense of magnetic field for a current flowing round a ring conductor.

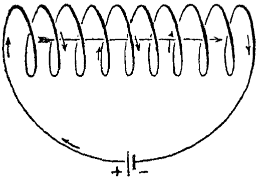

Fig. 21 —Solenoid

As is shown by figs. 23 and 24, the departures from homogeneity are due firstly to the fact that the lines of force circulate round the individual turns owing to the distance between them, and secondly, to the fact that near the ends the lines of force diverge. If, however, the turns are wound very close together (as, for example, in fig. 12, p. 139), the field within the solenoid becomes quite homogeneous, the extent of the homogeneous region increasing with the length of the solenoid. (See fig. 25.) At the same time the field *outside* the solenoid becomes very intense at the ends and practically zero near the middle. This can be demonstrated by means of the deflection of a small magnetic needle.

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* Gr., *sōlēn*, tube. The name solenoid is due to Ampère (1822).

Since the external fields are the same, any solenoid through which a current is flowing is equivalent to a bar magnet. If the current direction is as shown by the arrow in fig. 21, then the north pole of the magnet will be at the right hand end and the south pole at the end on the left.

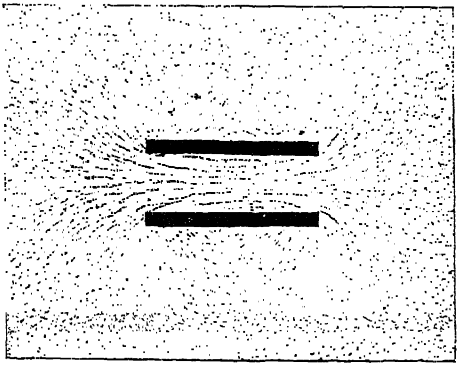

Fig. 22.—Lines of force of a solenoid

The shorter the solenoid, the shorter also the equivalent magnet. Hence if the solenoid be reduced to one circular turn, the plane of this turn must behave as a very short thick magnet, i.e. as a thin plate with northern polarity on one face and southern polarity on the other. A system of this kind is called a **magnetic shell.**

A circular conductor carrying a current is equivalent to a magnetic shell whose surface of southern polarity is on the side from which the current appears to flow in the clockwise sense.

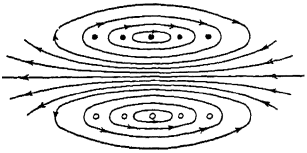

Fig. 23.—Field of a solenoid with widely spaced turns

[• Represents section of turn where the current is flowing downwards through the plane of the figure. o Represents section of turn where the current is flowing upwards through the plane of the figure.]

The interaction between a circular conductor bearing a current and a magnet placed near to it may be measured by suspending the conductor, as in fig. 26, so as to form one arm of a balance. The current enters and leaves at the axis of the balance beam. In the case in which the current is flowing in the anticlockwise sense viewed from above and in which the north pole of the magnet is uppermost, there is an attraction, the magnitude of which can be measured by means of suitable rider weights.

Another arrangement is to suspend the conductor, in the manner shown in fig. 27, upon a long thread so as to be capable of rotation about a vertical axis. The current enters and leaves through small cups of mercury arranged one above the other upon two small columns a small distance apart. When the current is flowing the conductor sets itself with its plane at right angles to the magnetic meridian. The deflecting action of a magnet can also be observed in this case.

The same behaviour, namely setting in the magnetic meridian and deflection by a magnet, can also be shown for a solenoid with a similar arrangement (see fig. 28).*

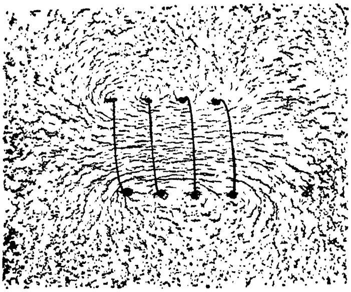

Fig. 24.—Field of a solenoid with widely-spaced turns

**Magnetic Field of a Closed Ring-shaped Solenoid.**—If a solenoid be bent round upon itself as in fig. 29, we obtain a field of the type shown in fig. 30, which is quite analogous to that of the toroidal magnet or toroid described on p. 120. The lines of force run almost exclusively within the solenoid; the external field is practically zero. The closer the turns, the more nearly are these field properties realized.

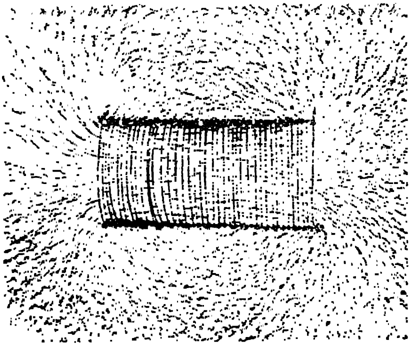

Fig. 25.—Field of a solenoid with closely-spaced turns

**Electrical Theory of Molecular Magnets.**—In accordance with the facts outlined on p. 115, we can imagine a magnetized body to consist of small directed magnets (so-called *molecular magnets*), much as a tourmaline crystal exhibits permanent electric polarization by virtue of its directed molecular dipoles (p. 95). According to a hypothesis first put forward by Ampère and developed later by Weber (1852), a small current circulates around each of the elementary

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* AMPÈRE was the first (about 1820) to investigate thoroughly the interaction between a magnet and a conductor carrying a current. He constructed a special apparatus in which the conductor could be suspended so as to be capable of rotation. This is generally known as *Ampère's stand.*

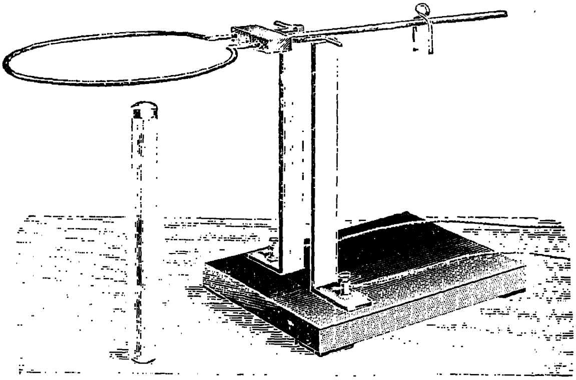

Fig. 26.—Magnetic behaviour of a ring conductor carrying a current

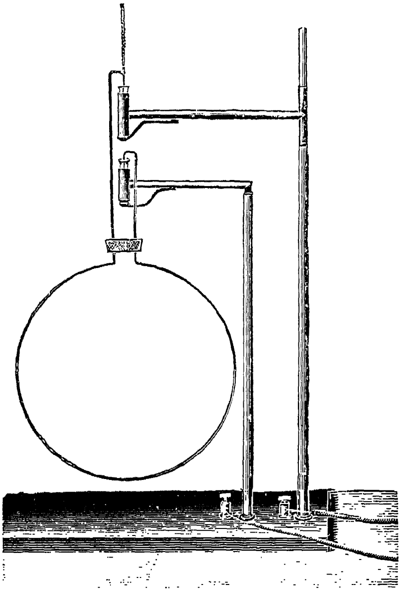

Fig. 27.—Ring conductor carrying a current in the earth's magnetic field

particles of a magnet (**elementary currents**) (see fig. 31). In Ampère's theory the molecular magnets are to be regarded as such elementary currents.*

* In this way Ampère achieved a simplification of the physical concepts involved, in that he made it unnecessary to regard magnetism as a special " substance ".

As is seen from fig. 32, the elementary currents of neighbouring molecular magnets neutralize each other, and we are left simply with a current circulating right round the magnet. Hence a bar magnet may be regarded as a solenoid, so far as its external field is concerned.

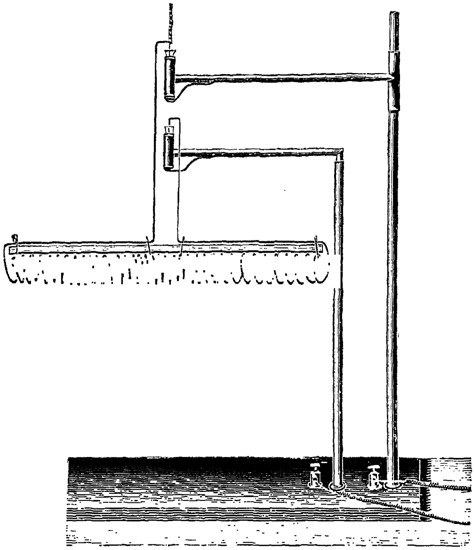

Fig. 28.—Solenoid capable of rotation

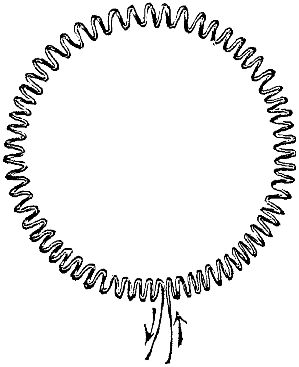

Fig. 29.—Ring solenoid

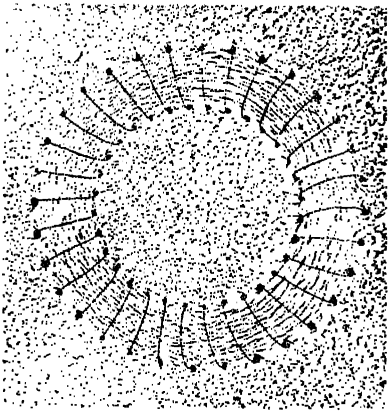

Fig. 30.—Field of a ring solenoid carrying a current

Thus from this point of view the magnetic interaction of two or more magnets can be completely explained in terms of the attraction and repulsion of the elementary currents. The greatest difficulty of the theory lay in the assumption of electric currents flowing perpetually in a body without resistance. For if there were a resistance to the currents in the interior of the body, the magnetization would necessarily be accompanied by development of heat. This would, of course, involve a loss of energy and would make the existence of permanent magnets quite impossible. Current conduction without resistance has, however, been observed recently in closed metal wires at very low temperatures (KAMERLINGH-ONNES), so that it is now possible to imitate Ampère's elementary currents.

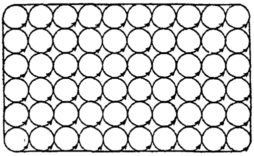

Fig. 31.—Diagrammatic representation of the elementary currents

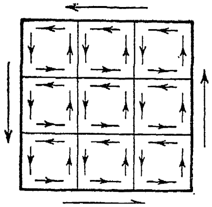

Fig. 32.—Partial mutual compensation of the elementary currents

Modern investigations of the structure of matter have also led to new views which are in harmony with Ampère's and Weber's hypothesis of the existence of enduring currents (i.e. movements of electric charges) in the elementary particles (see p. 444 and Vol. V).

## 4. The Quantitative Relation between Magnetic Field Strength and Electric Current Strength.

**Quantities characterizing the Magnetic Field.**—We have already become acquainted (p. 121) with a quantity, the magnetic induction **B**, and a corresponding measurement of a magnetic field by means of the force exerted upon magnetic needles. The recognition (p. 186) that electric current (in the widest sense) and magnetic field are only different aspects of the same fundamental process has led to the measurement of magnetic fields by means of the associated currents, and the consequent introduction of a second quantity characterizing the magnetic field, namely the **magnetic intensity** or **magnetic field strength H.**

This description of a magnetic field by two quantities may seem at first unnecessarily complicated; but it is justified by the great simplification which it brings into the treatment of magnetic fields in space filled with matter. The case is completely analogous to the description of electric fields by means of the two vectors **E** and **D**, the former derived from the force effects and the latter from the charges.

**Strength of the Magnetic Field within a Solenoid.**—As with electric fields, the simplest relationships are to be expected in the case of homogeneous magnetic fields. Now we have already met with homogeneous fields within solenoids, and will therefore consider first the properties of solenoids.

As we have seen on p. 172, the external field of an infinitely long solenoid is zero. From this we can at once derive a fundamental property of the field in the interior. Imagine two very long similar solenoids with equally spaced turns to be placed side by side as in fig. 33, and to be carrying currents of similar sense and equal strength. The field within each of the solenoids is the same as if the other were not present; for their external fields are negligible. Now since the currents in the contiguous sides are exactly equal and opposite, the conditions are not affected by the removal of these sides. This gives a solenoid of double the cross-sectional area, but *with exactly the same field* as before. As may be seen from a diagram analogous to that of fig. 32, this consideration may be generalized to any desired extent. We see further that the essential condition is that every turn of the one solenoid shall correspond to one turn of the other solenoid carrying a current of exactly the same strength. That is to say, the product of the number $z$ of turns and the current strength I must be the same for the same length (e.g. for 1 cm.) of each of the solenoids, measured in the direction of their axes. The current strength I being measured in amperes, the product I$z$ may be called the number of **ampere-turns.**

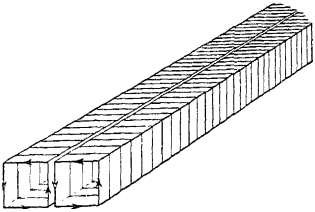

Fig. 33.—Diagram showing that the field strength within a very long solenoid is independent of the cross-sectional area.

*For a given number of ampere-turns per cm., the field within a solenoid is independent of the cross-sectional area.*

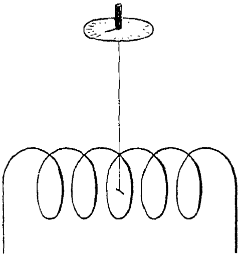

Fig. 34.— Diagram of the measurement of the field strength within a solenoid.

It may be shown experimentally that the magnetic field within a solenoid depends only upon the value of the product I$z$ per cm., and not upon the values of the individual factors. For this purpose a small magnetic needle may be suspended within the solenoid in such a way that its position of equilibrium when no current is flowing is determined by some elastic device, say by the torsion of a wire (as in fig. 34) or by means of a spring. Since the tendency of the needle is to set itself along the lines of force, it is adjusted so as to be at right angles to the axis of the solenoid when no current is flowing. When now a current is sent through the solenoid, the needle is deflected towards the axis and a certain turning moment is required to bring it back to its original direction. In the arrangement shown in the figure this moment is applied by twisting the wire, and its magnitude is obtained from the angle of torsion (Vol. I, p. 207). This gives a measure of the turning moment exerted upon the needle by the homogeneous field (p. 122). Since the magnetic moment of the needle remains constant, the turning moment is proportional to the magnetic induction. Hence the fields are identical in two cases, if the turning moments are equal.

Using various solenoids with different numbers of turns, different lengths, and different distances between the turns, we find that, in order to get the same turning moment upon the needle, the current strengths must be regulated so that in every case the product of the current strength and the number of turns per cm. length of solenoid is the same. This involves the assumption that the lengths of the solenoids are large and the distances between their turns small in comparison with the length of the magnetic needle; for otherwise the fields are not sufficiently homogeneous. Moreover, the current strengths employed must be so great that the influence of the earth's magnetic field can be neglected.

By varying the current strength I for one and the same solenoid and measuring the turning moment exerted upon the needle, we obtain the following result: For a given solenoid the turning moment is proportional to the current strength.

It is found that the turning moment exerted upon the needle is the same for different solenoids, provided that the number of ampere-turns per cm. is the same in each case. Thus a solenoid with few turns per cm. but a correspondingly strong current shows the same effect upon the needle as a solenoid with many turns per cm. but with a correspondingly weak current: the one condition which must be fulfilled is that the product of the current strength I and the number $z$ of turns per cm. must be the same in both cases.

*Thus the intensity of the magnetic field within a solenoid is dependent only upon the number of ampere-turns per cm.*

**Definition of the Unit of Magnetic Field Strength.**—The magnetic field within a solenoid is characterized by the number of ampere-turns per cm. We will call this measure of the magnetic field the **magnetic field strength.** It is a vector, and its direction in space devoid of matter coincides with that of the magnetic lines of force, i.e. with that of the magnetic induction **B**. The above result for a solenoid can be used to measure the magnetic field strength. Thus we can define the strength (or the intensity) of a magnetic field in terms of the number of ampere-turns per cm. of a solenoid whose internal field is identical with the field to be measured at the particular point in question. If I is the current strength in amperes and $z$ the number of turns per cm., then we have

$$H = |\mathbf{H}| = C \, . \, I \, . \, z,$$

where C is a constant. We have thus obtained an extremely simple relation for a *homogeneous* magnetic field. It is of fundamental significance, in that it shows clearly that the strength of the homogeneous magnetic field is dependent only upon the product of current strength and number of turns per unit length. Just as a conductor carrying a current is surrounded by circular magnetic lines of force, so also does a magnetic field have an electric current flowing round it. On account of its simplicity and its clear relation to the fundamental connexion between current and magnetic field (see further p. 186), the above equation is used to define the unit of field strength. The constant factor C vanishes from the equation (i.e. becomes equal to unity), if we adopt the following definition of the unit of magnetic field strength:

**The unit of magnetic field strength H is that field strength which exists within, and parallel to the length of, a long solenoid through which is passing a current of 1 ampere per centimetre of length.**

The unit is thus 1 ampere-turn per cm. In future we shall express numerical data in these units, except where otherwise stated.

The definition of field strength in ampere-turns per cm. is not dependent upon the force exerted upon a unit pole, but is based upon the direct connexion between the circulating current and the magnetic field associated with it. Magnetic field strength is defined as the effect of the circulation of a certain current (p. 186).

**Relation between Magnetic Induction and Magnetic Field Strength.**—The result of the above experiments, namely that the force effects within a solenoid are proportional to the number of ampere-turns per cm., indicates that (in space devoid of matter, to which the quantitative relationships so far obtained all relate) the magnetic induction (p. 121) is proportional to the field strength. Thus we have:

$$\mathbf{B} \text{ volt sec./cm.}^2 = \mu_0 \, \mathbf{H} \text{ ampere-turns/cm.}$$

where $\mu_0$ is a constant.

By measuring the induction (by the methods given on p. 124) and also the magnetic field strength of a homogeneous field, e.g. within a sufficiently long solenoid, we can obtain the numerical relationship between these quantities. For another method depending upon another definition of induction, see p. 379.

The value of $\mu_0$ determined in this way is

$$\mu_0 = 1{\cdot}257 \,.\, 10^{-8} \text{ (volt sec./cm.}^2\text{)/(amp.-turn/cm.)}.$$

Hence we have

$$\begin{aligned} 1 \text{ gauss} &= 10^{-8} \text{ volt sec./cm.}^2 \\ &= 0{\cdot}796 \text{ amp.-turn/cm.} = 5/(2\pi) \text{ amp. turn/cm.} \\ 1 \text{ amp.-turn/cm.} &= 2\pi/5 \text{ gauss} \\ &= 1{\cdot}257 \text{ gauss} = 1{\cdot}257 \,.\, 10^{-8} \text{ volt sec./cm.}^2 \end{aligned}$$

The factor $\mu_0$ has the same significance for magnetic fields as $K_0$ for electric fields, giving the numerical relation between the measure of the field itself and the measure of the force effects which the field produces.

The unit ampere-turn per cm. is widely used nowadays, especially in engineering. But in scientific work and tables magnetic fields are still very often measured in gauss. The reasons for this are historical. The system of measurement originally employed was obtained by making $\mu_0 = 1$, just as in the so-called absolute electrostatic system $K_0$ is made equal to unity. This made the numerical measures of induction and field strength equal in space devoid of matter: thus the number expressing the field in gauss was also the number expressing the field strength. We shall therefore have to make frequent reference to the unit 1 gauss, although from a purely scientific point of view the unit 1 ampere-turn/cm. is the more natural; for even in permanent magnets the magnetism is due to circulating currents.

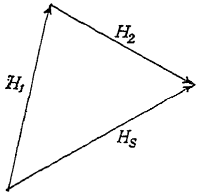

Fig. 35.—Vector sum of two magnetic fields

**Measurement of Magnetic Fields by Superposition.**—Since the magnetic field strength **H** is a vector and since magnetic fields add together without affecting each other, the combined field strength $\mathbf{H}_s$ of two fields $\mathbf{H}_1$ and $\mathbf{H}_2$ is equal to the vector sum $\mathbf{H}_1 + \mathbf{H}_2$ (see fig. 35).

This can be made use of for the measurement of magnetic fields. A field of known strength $\mathbf{H}_K$ is superimposed upon the unknown field of strength $\mathbf{H}_x$, exactly at right angles to it. The direction of the resultant field strength **H** is determined. Then, as can be seen from fig. 36, we have the equation

$$H_x = H_K \cot (\mathbf{H}_x, \mathbf{H}).$$

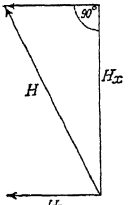

Fig. 36.—Measurement of magnetic field strength by superposition.

This method is applied in order to measure the *field-strength of the earth's magnetic field.* The known field may be produced by means of a current of I amperes in a circle of radius $r$, the field strength at the centre of the circle being $H_K = \frac{1}{2}I/r$ ampere-turns per cm. (p. 193). A magnetic needle is placed at the centre of the circle and the plane of the circle adjusted so as to have the same direction as the needle (i.e. to be in the magnetic meridian) when no current is flowing. When now the current is allowed to flow, the magnetic field associated with it is superimposed at right angles upon the earth's field (fig. 37) and the magnetic needle is deflected through a certain angle $\alpha$. The magnitude H of the horizontal component of the earth's field is then given by $H = H_K \cot \alpha$. The actual value obtained at London is about 0·12 ampere-turns per cm. (For more exact data see Vol. V.)

Conversely, if the strength of the earth's field be known, the value of $H_K$ can be obtained from the equation $H_K = H \tan \alpha$. But $H_K$ is proportional to the current strength in the circle, so that the current strength is also proportional to the tangent of the angle of deflection. This form of apparatus is therefore known as a **tangent galvanometer** (fig. 38), and was formerly used to measure electric currents. Since the above equation $I = 2rH_K$ holds, the instrument can easily be calibrated when the strength of the earth's field is known—a very useful method in earlier times before the present convenient and more precise methods of electrolytic calibration.

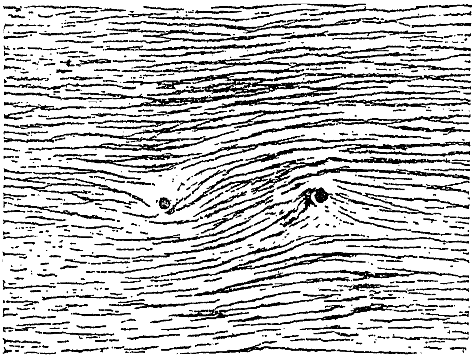

Fig. 37.—Deformation of earth's field due to a current flowing round a ring conductor

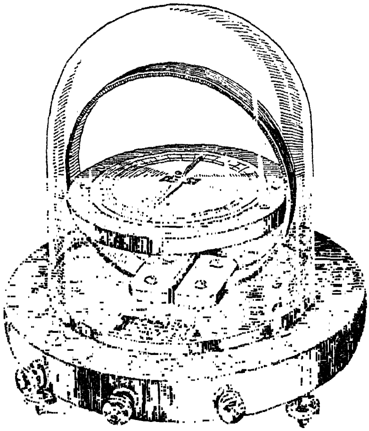

Fig. 38.—Galvanometer

*Note*.—If field strengths be measured in gauss instead of in ampere-turns per cm. we have $H_K = \frac{1}{5}\pi I/r$ at the centre of the circle and hence $I = (5rH \tan \alpha)/\pi$, where H is the value of the earth's horizontal intensity expressed in gauss (p. 192).

The factor $H \,.\, 2r$ (when H is expressed in ampere-turns per cm.) or $5rH/\pi$ (when H is expressed in gauss) is called the *constant* (or *reduction factor*) of the instrument; its value depends only upon the dimensions of the instrument and upon the earth's horizontal intensity. When once the constant has been determined, current strengths can be found (at one place) by simple multiplication of the tangent of the angle of deflection by the constant.

## 5. The Form of the Magnetic Field of a Straight Conductor carrying a Current

**Motion of " Magnetic Poles " around the Conductor.**—The magnetic lines of force run in circles around the straight conductor and exert forces upon magnetized bodies in its neighbourhood. We have already become acquainted with these force effects as turning moments exerted upon magnetic needles. We will now consider translational motion due to the magnetic forces.

According to the definition given on p. 127, the magnetic lines of force are the paths along which a massless north pole, free from constraint, would move. To realize a north pole as nearly free from constraint as possible, we suspend a magnetized knitting needle NS (fig. 39) from the ceiling by means of a long thin thread, so that its north pole hangs downwards. We then set up a brass rod with its lower end clamped in a massive foot and its upper end opposite the middle of the magnetized needle. We now connect one pole of a large accumulator to the foot of the rod and the other pole to a flexible wire. By touching the top of the brass rod with the free end of this wire the circuit is made; at the same moment the north pole of the suspended needle experiences an impulse, as the result of which the needle moves in a circle around the brass rod. Current sense and sense of rotation of the north pole are as shown in fig. 39. When the current sense is reversed, the needle moves in the opposite sense around the rod. When, further, the needle is suspended the other way up (i.e. with its *south* pole downwards), the sense of rotation is once more reversed in accordance with the corkscrew rule.

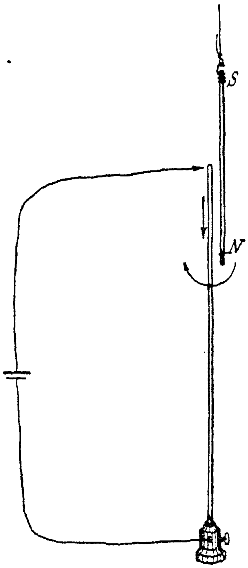

Fig. 39.—Motion of a magnet in the field of a current

**Dependence of Field Strength upon Distance from the Conductor.**—By means of a modification of the above arrangement we can investigate how the field strength depends upon the distance measured radially from the conductor.

We hang a long conducting wire from the ceiling and load it at its lower end so as to keep it vertical (see fig. 40). The upper end of the wire we connect to one pole of a source of current; the lower end dips into a cup filled with mercury, to which the other pole is connected. We now suspend a small circular plate by means of three threads joined above (see figure), so that the conducting wire passes centrally through a hole in the middle of it. The plate is provided with three small sockets to take three magnetized knitting needles arranged with their north poles all pointing towards the wire. When now a current is passed through the wire, we observe *no rotation or displacement* of the whole system of magnets, no matter how great the current strength is made. Upon each of the north poles acts a force of magnitude $P_1$ with an arm $r_1$ so as to tend to cause motion in the clockwise sense. Similarly upon each of the south poles acts a force of magnitude $P_2$ with an arm $r_2$ so as to tend to cause motion in the opposite sense. Since there is no resultant effect the two turning moments must be equal and opposite, i.e. we must have

$$P_1 r_1 = P_2 r_2$$

or

$$P_1 : P_2 = r_2 : r_1.$$

In words: The force exerted upon a magnetic pole by a current flowing in a straight conductor is inversely proportional to the distance of the pole from the conductor.

The derivation of this result is completely independent of the concept of " magnetic pole ", although we have made use of this term on account of the resultant simplicity of the mathematical formulation. As can be readily appreciated, the essential condition is that as many lines of force shall enter the needle as leave it—a condition which is actually fulfilled (see the diagrams of lines of force) and which is indeed identical with the principle that all magnetic lines of force are closed.

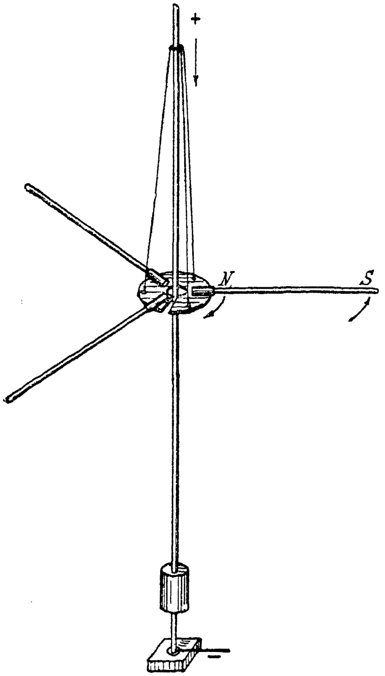

Fig. 40.—Experiment proving the magnetic field strength to be inversely proportional to the distance from a straight conductor carrying a current.

Since the force is proportional to the field strength, i.e. $\mathbf{P} = m\mu_0\mathbf{H}$, and since both the poles are of the same strength, i.e. $m$ has the same value for both, it follows that:

*The magnetic field strength in the neighbourhood of a straight conductor carrying a current is inversely proportional to the distance from the conductor.*

This result can also be investigated directly as follows:

At some distance from the wire D (see fig. 41) place a solenoid so that its axis is tangential to the lines of force. Within the solenoid a magnetic needle is suspended by a torsion wire as in fig. 34 (p. 176). When now a fairly strong current is passed through the wire, the magnetic needle is deflected. This deflection can be just compensated by sending a current of suitable strength through the solenoid in the correct direction.

In order to avoid disturbing effects due to the earth's field, the axis of the solenoid must be set in the east-west direction. Solenoid and needle must also be small in comparison with the length of the wire and their distance from it.

By placing solenoid and magnet at different distances from the wire carrying the current, we find that the strengths of the currents required in the solenoid to compensate the needle deflections are inversely proportional to these distances, whence the above law follows at once.

**Dependence of Field Strength upon Current Strength.**—It has already been shown (p. 177) that the magnetic field strength within a solenoid is proportional to the current strength. Since magnetic fields add together as vectors without affecting each other, the field may be regarded as the superposition of the partial fields from the different elements of current, and hence we are led to assume the same proportionality between field strength and current strength in the case of a single conductor.

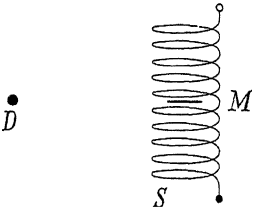

Fig. 41. — Arrangement for measuring the magnetic field strength due to a current in a wire D by means of a solenoid S and magnetic needle M.

This can also be proved directly, using the arrangement shown in fig. 41 above. We send a known current through the wire D, and compensate the consequent deflection of the magnetic needle M by passing a suitable, measured current through the solenoid. On varying the current in the wire, we find that in order to obtain compensation we must vary the current in the solenoid in the same ratio. An instructive modification of the experiment consists in connecting the solenoid (with a suitable resistance) in parallel with the wire. The resistance is adjusted so that for one particular current strength the needle deflection is compensated. It is then found that the compensation is unaffected (i.e. the needle remains undisturbed) when the current in the whole circuit (wire and solenoid together) is altered.

Thus we have the following result:

*The magnetic field strength at any point in the neighbourhood of a straight conductor is directly proportional to the strength of the current flowing through the conductor.*

This can be expressed by means of the equation $H = CI/r$, where C is a constant and $r$ is the distance from the conductor.

Before we can measure field strengths in this way, we must first determine the value of the factor C in terms of our system of measurement (i.e. H in ampere-turns per cm.). For this purpose we proceed as follows:

## 6. Line-Integral of Magnetic Field Strength

In order to obtain a measure of the difference of the magnetic state at two points, we may try the course already adopted in the case of electric fields, where the desired measure (called the potential difference between the two points) was arrived at by forming the line-integral of the electric field strength. The line-integral V of the magnetic field strength over a length $l$ measured along a line of force is equal to H$l$, where H is the magnitude of the field strength, considered as constant throughout. The unit in which the line-integral is expressed is the ampere-turn. When H is not constant, we have to form the sum of all the elementary products H$dl$ for all the elements of path $dl$. Thus we obtain the line-integral $V = \int H dl$. In analogy to the corresponding quantity in the case of electric fields, viz. the pressure, potential difference, or electromotive force, we may call this the *magnetic pressure, magnetic potential difference*, or *magnetomotive force* between the two points.

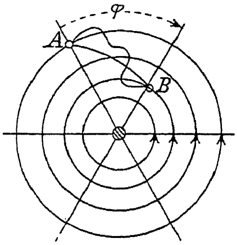

Fig. 42.—Diagram to show that the value of the line-integral of the magnetic field strength between two points is independent of the path, provided that this does not pass right round the conductor.

If the element of path $dl$ is inclined at an angle to the direction of the field, only the resolved part of the field strength in the direction of the element is effective. In general therefore

$$V = \int H \cos(\mathbf{H}, \mathbf{l})\, dl.$$

**Line-Integral for a Closed Path round the Conductor.**—The value of the line-integral taken round the whole circumference of a circle of radius $r$ around the conductor as centre is

$$V = (CI/r) \,.\, 2\pi r = 2\pi CI.$$

Thus the value of this line-integral is *quite independent of the distance* at which we pass round the conductor; for the field strength diminishes in the same ratio as the length of the path increases. For a given fraction of the total circuit round the conductor (measured by the angle $\phi$ in fig. 42), the value of the line-integral of the field strength is always $\phi$CI, no matter what the radius of the path. Moreover, it can be seen from the figure that this value is also independent of the shape of the path. Between the two points A and B, for instance, the value of the line-integral is always the same, whatever the shape of the path (provided, however, that the path does not pass right round the conductor, see p. 187). For every path can be considered as made up of radial elements and elements in the direction of the circular lines of force. The radial elements contribute nothing to the value of the integral, since they are at right angles to the direction of the field strength. For the elements which coincide with the lines of force the value of the integral is independent of the radius, and therefore the total result is always equal to $\phi$CI.

We can also express this as follows:

*The number of ampere-turns corresponding to passage once round the conductor is constant and independent of the path.*

**Absolute Value of the Line-Integral.**—The absolute value of the line-integral is expressed as the number of ampere-turns corresponding to the path. Hence we have to determine the total number of ampere-turns corresponding to passage once round the conductor. For this purpose we divide up the space around the conductor into closed ring-shaped solenoids of rectangular cross-section (see fig. 43), and imagine the magnetic field to be produced by their combined action.

Since no current is flowing in the space surrounding the conductor, the currents in the solenoids must exactly compensate each other *.

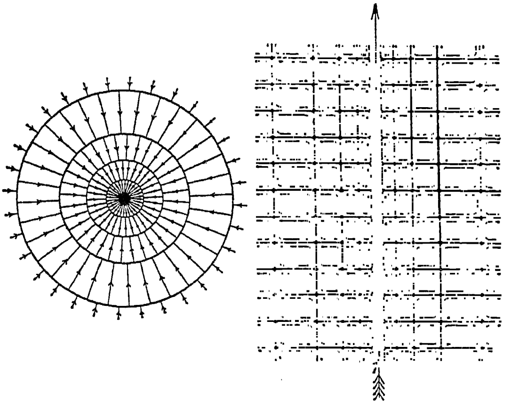

Fig. 43.—Division of the magnetic field around a straight conductor into solenoids (after Mie). [From Mie's *Lehrbuch der Elecktrizität und des Magnetismus*, published by F. Enke, Stuttgart.]

As can be seen clearly from the longitudinal section shown in fig. 43, the solenoid currents flow in opposite directions at every point. If the currents are to compensate each other, the number of ampere-turns per unit length of the outer ring-shaped solenoids must clearly be smaller than that of the inner ones; more precisely, the number of ampere-turns per unit length must be inversely proportional to the distance from the conductor. This means simply that the magnetic field strength must be inversely proportional to the distance from the conductor—the relation we have already derived above.

The total number of ampere-turns is the same for all the solenoids. In particular it is equal to that of the innermost solenoid. But this solenoid is bounded on the inside by the conductor itself, so that its total number of ampere-turns is simply equal to the strength of the current in the conductor, expressed in amperes. Hence:

---

* The outermost turns are not compensated, but give a total current which is equal to that flowing through the conductor but in the opposite direction. This current corresponds to the return flow of the current through the conductor; for every current is necessarily *closed.* Thus the fact that every circuit through which a current is flowing is necessarily closed is an essential condition of this treatment.

*The line-integral V (in ampere-turns) for one circuit round the conductor is equal to the strength I of the current in the conductor (in amperes), i.e.* $V = I$.

The value of the line-integral for one circuit round the conductor may be called the **circuital potential** (German, *Randspannung*).

**Thus the circuital potential is numerically equal to the strength of the current to which it is due.**

From the equation $V = 2\pi CI$ it follows now that in our system of measurement the constant C has the value $1/(2\pi)$.

For a partial circuit of the conductor, measured by the angle $\phi$ (see fig. 42), the absolute value of the line-integral is $V = \phi I/(2\pi)$. The magnetic field strength or intensity (in ampere-turns/cm.) at the distance *r* cm. from a *straight* conductor is therefore

$$\mathbf{H} = \mathbf{I}/(2\pi\mathbf{r}).$$

**Magnetic Field and Electric Current.**—It is the above simple connexion between circuital potential and current strength that makes the system of measurement we have employed so lucid. We perceive that there is even a quantitative identity between magnetic field and electric current. It does not matter what material the conductor is made of, or what is the source of the current. There is no electric current without a magnetic field. We must conclude, therefore, that the magnetic field is only another aspect of that single process in the ether which we call an electric current.

As a help in forming a mental picture of the above relationships it is useful to remember the hydrodynamical analogy previously put forward on p. 163. As soon as the rotating cylinder is moved, the cross-current is at once produced. Thus this cross-current is nothing but another manifestation of the state of motion of the spinning cylinder. As is shown by the production of a magnetic cross-field by the motion of an electric field, the electric field has properties which justify the above analogy. Conversely, we must conclude that there can be no magnetic field without an electric field in motion, i.e. without a current. No such magnetic field has been found; even permanent magnets are regarded as due to small circulating electric currents. Magnetic field and electric current are two inseparable phenomena, i.e. *one* process in the ether. The term electric current is to be taken here in its widest sense, namely as meaning an electric field in motion.

**Potential in the case of the Magnetic Field of a Current in a Straight Conductor.**—As we have seen, the line-integral of the magnetic field strength between two points has a definite value, provided that the path does not pass right round the conductor. Hence for any closed path not enclosing the conductor the line-integral vanishes.

This can be seen by fig. 44. The value of the line-integral for the path from A to B is $\phi I/(2\pi)$, from B to C zero, from C to D $-\phi I/(2\pi)$, and from D to A zero. The value for the closed path is therefore

$$V = \phi I/(2\pi) + 0 - \phi I/(2\pi) + 0 = 0.$$

Since now the surface enclosed by any closed curve can be divided up into elements of the same shape as ABCD, it follows quite generally that:

*The line-integral of the magnetic field strength vanishes for any closed path not enclosing the conductor.*

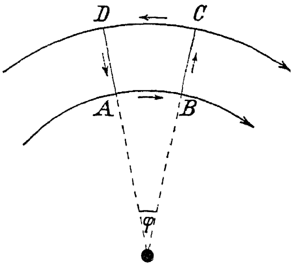

Fig. 44.—Closed path in the magnetic field of a current

The value of the line-integral increases by I for every complete circuit of the conductor. Hence the general expression for the value between two points at an angular distance $\phi$ apart is

$$V = \phi I/(2\pi) + nI,$$

where $n$ is the number of times that the path passes right round the conductor.

Thus the magnetic potential at a point, i.e. the work required to bring a unit north pole from infinity up to the point (cf. Vol. I, p. 185), is not uniquely defined. The unit pole can always be brought up to the point in such a way as to make the work performed zero; this is the case when the path is radial, so that the magnetic lines of force are intersected at right angles. But if a path be chosen which makes circuits round the conductor, the value of the potential alters by $\pm$ I for each circuit. We see, therefore, that the potential itself is quite *indeterminate*, and that the potential difference V between any two points is *many-valued*. The magnetic field strength, however, as gradient of the potential difference (Vol. I, p. 191), is uniquely determined at every point. In forming the gradient of V by differentiating with respect to the path length $l = \phi r$, the constant term $nI$ vanishes and a unique value is obtained.

**General Equation for Circuital Potential.**—In determining the circuital potential we select a certain closed path such that the current producing the magnetic field flows through the surface enclosed by the path. In general, when the current is not flowing in a single conductor but is passing in any manner through the selected surface, we have to consider the sum $\Sigma I$ of all the currents concerned. Then

$$\oint \mathbf{H}\, dl = \Sigma \mathbf{I}.$$

For example, if a current of I amperes is flowing through the solenoid shown in section in fig. 45, then the total current passing through the surface enclosed by the dotted path is 6I amperes.

**Field within a Hollow Conductor.**—There must be a magnetic field within a conductor carrying a current; for we may regard every conductor as composed of a very large number of thin parallel conductors, each of which must give rise to a magnetic field.

Consider first the case of a hollow conductor, i.e. a tube with very thin walls. The arrangement of iron filings (see fig. 46) shows that:

*There is no field within a tube carrying a steady current in the direction of its axis.*

In the external space the form of the lines of force is the same as for a solid wire of the same cross-section. Thus the magnetic fields of the elementary current filaments compensate each other in the interior of the tube.

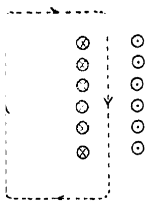

Fig. 45.—Total current enclosed by a closed path.

**Field in the case of a Solid Conductor.**—We may regard the solid conductor as composed of concentric tubes, each of which has an external field but no internal field.

It follows, therefore, that the field strength is zero in the axis of the solid conductor. Hence the formula given on p. 186 *can only hold for the external space.* Within the conductor the field strength increases as we pass outwards from the axis, becoming at the surface of the conductor equal to the corresponding field strength as calculated for the external space. The variations of the field strength, both within the conductor and outside it, are shown graphically in fig. 47. Later on (p. 475) we shall meet with a phenomenon which is due to the internal magnetic field.

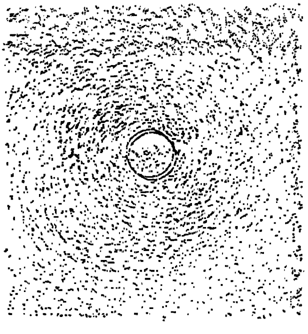

Fig. 46.—Magnetic field within a hollow conductor (after König)

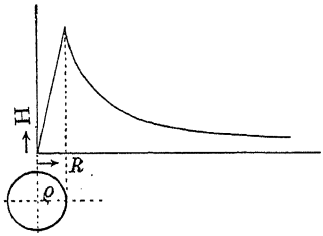

Fig. 47.—Magnetic field in the interior of a solid conductor

The field in the interior of a conductor can be investigated qualitatively by means of a small magnetic needle submerged in mercury contained in a wide glass cylinder. The electric current enters and leaves at the ends of the cylinder by suitable coaxial leads.

As we have seen on p. 187, there exists a many-valued potential difference between any two points in the external field. Defined as the value of the line-integral of the field strength, the potential difference between two points is the same for any two paths which do not pass right round the conductor, provided that the closed path formed by combining the two paths does not pass round the conductor. If this condition is not fulfilled, i.e. if the closed path formed by combining the two paths passes round the conductor (or a part of the total current in a system of conductors), then the two values of the potential difference corresponding to the two paths differ by an amount which depends upon the strength of the current enclosed. Now in the interior of the conductor every closed path must pass round a part of the total current. Therefore the potential difference between two points in the interior of the conductor must depend entirely upon the nature of the integration path.

In the interior of a region through which an electric current is passing there is no such thing as a determinate magnetic potential.

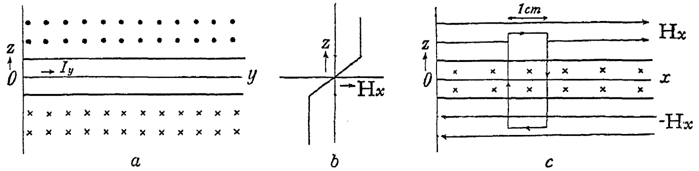

Fig. 48.—*a*, *b*, and *c*. Magnetic field of a current flowing through an infinite plate

**Such a field is called a rotational or vortex field.**

*The indeterminate nature of the potential in the external field of a current flowing in a straight conductor is due to the fact that this field is produced by a vortex of magnetic force.*

**Hydrodynamical Analogy.**—We have already learned the significance of the line-integral in the case of circulatory flow (Vol. I, p. 378). The circulation or vortex strength was found in this case to be $Z = 2\pi\, r_0 v_0$, the velocity of flow $v_0$ being assumed to be inversely proportional to the distance $r_0$ from the axis of circulation. Thus we obtain the hydrodynamical analogue of the above electromagnetic field by replacing H by $v_0$. Since in the analogy Z corresponds to I, it becomes particularly clear that the current passing through the surface considered is nothing else than the vortex strength of the magnetic vortex.

**Magnetic Field of a Plate.**—The following is a very instructive example in which the relation between electric current and magnetic field comes out with especial clarity. Imagine a plate of infinite area and finite thickness. The $z$-direction (see fig. 48) is normal to the plate; the $x$-direction and $y$-direction are in the plane of the plate; the origin of co-ordinates is mid-way between the two faces. Imagine an electric current to be flowing through the plate in the $y$-direction, the lines of flow being straight and parallel. Then the magnetic lines of force of the field of this current are also parallel and straight; above the plate they run in the positive $x$-direction (towards the observer in fig. 48*a*), below the plate in the negative $x$-direction. Both above and below the plate the field is homogeneous, i.e. the field strength is independent of the distance from the plate. This is due to the assumed infinite extent of the plate in the $x$- and $y$-directions.

In the interior of the plate, however, the field strength falls off linearly, becoming zero half-way through. The thickness of the plate being $2z$ and the external field strength $H_x$, the internal field strength decreases from $H_x$ to zero in the distance $z$ (see fig. 48*b*). The connexion between the magnetic field strength and the strength of the current in the plate can be found easily. We determine the magnetic potential for a closed curve, e.g. the rectangle shown in fig. 48*c*. We choose this rectangle so that the side which is parallel to the magnetic lines of force is 1 cm. long. According to p. 186 the circuital potential is equal to the strength of the current encircled. In our case this is the current which flows through a strip of the plate 1 cm. wide. If $G_y$ is the current density in amp./cm.$^2$, the strength of this current is $G_yA = G_y \cdot 2z \cdot 1$, where A is the area enclosed by the rectangle. Therefore $V = 2H_x = 2G_yz$, whence $H_x = G_yz$.

Thus as we pass through the plate from the upper surface to the lower we have a change of the magnetic field strength in a direction at right angles to the magnetic field. But such a change corresponds to a vortex (cf. fig. 31, p. 31). In the example under consideration we have a vortex of the simplest kind. It follows from above that the magnitude $H_x/z$ of the vortex is equal to $G_y$. It is thus the same at all points and equal to the current density. This example provides the simplest case of the rotational or vortex field of a current.

## 7. The so-called Absolute Electromagnetic Unit of Current Strength

Before the relationships discussed in the foregoing paragraphs had been expressed in the clear form which Maxwell gave to Faraday's intuitive mental pictures, efforts were directed towards the description of electromagnetic phenomena in terms of laws formulated on the pattern of Newton's law of gravitation. Since the first attempts to fix magnetic and electric units were based upon this type of formulation, we must for historical reasons consider them in more detail.

**Biot-Savart Law.**—BIOT * and SAVART † were the first (1820) to investigate thoroughly the magnetic field of a current flowing in a conductor. They came to the conclusion that the total magnetic effect could be considered as the combination of the partial effects of the current elements.

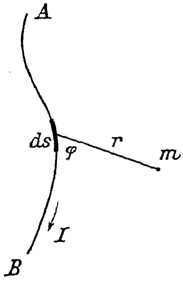

Fig. 49.—Biot-Savart law

In fig. 49 let AB be a very thin conductor carrying a current of strength I. The small element $ds$ of the conductor can be regarded as straight. A point magnetic north pole of unit strength is situated at $m$, its distance from $ds$ being $r$. Let the line joining $ds$ to $m$ make an angle $\phi$ with the element $ds$. Then according to the Biot-Savart law the force exerted by the current in the element upon the unit pole is

$$dH = C' \cdot \frac{I \sin\phi}{r^2} \cdot ds.$$

---

* JEAN BAPTISTE BIOT (1774–1862), celebrated French savant, announced the law in 1820.

† SAVART (1791–1841).

This force is normal to the plane through the element *ds* and the line joining it to the pole, and the sense of the force is determined by the sense of rotation of a corkscrew boring in the direction of the current. The factor C′ is a constant whose value depends upon the choice of units (pp. 192 and 193).

The Biot-Savart law is formulated on the pattern of Newton's law of gravitation and Coulomb's law; like these it is by nature a law of action at a distance. With its help it is possible to calculate the field of a current in a conductor of any shape, the partial field strengths corresponding to the different elements being compounded to give a resultant by the parallelogram law.

The actual calculation usually presents very great difficulties, and can only be performed in the case of conductors of simple shapes. The value of the above formula is also restricted by the fact that it only holds for homogeneously filled space.*

The Biot-Savart law can also be formulated for conductors of finite cross-section. In this case we have to consider volume elements *dv* instead of elements of length *ds*, and current density G instead of current strength I. The total effect of all the elements is obtained by integration and has the value

$$\mathrm{H} = \mathrm{C}'\int \frac{\mathrm{G}}{r^2}\sin\phi\, dv,$$

where $\phi$ is the angle between the direction of flow of the current and the line joining the volume element to the point under consideration.

The Biot-Savart law also holds for points in the *interior* of the conductor.

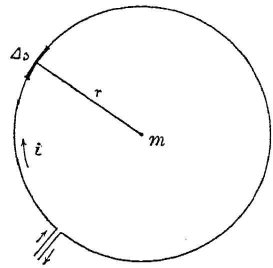

Fig. 50.—Field at the centre of a ring conductor due to a current flowing round it. Application of Biot-Savart law.

**Field Strength at the Centre of a Circular Ring round which a Current is Flowing.**—We will calculate this as an example of the application of the Biot-Savart law. Let the strength of the current in the ring be I. Consider a magnetic pole of strength *m* to be situated at the centre of the ring (fig. 50). We divide up the conductor into elements *ds*, each of which can be regarded as straight. Since each of these elements is at right angles to the corresponding radius, the factor sin $\phi$ in the formula of the Biot-Savart law is equal to unity for every element. The force exerted by each element upon the magnetic pole at the centre is therefore equal to $\mathrm{C'I}m\,ds/r^2$. The total force P is obtained in this case by simple addition of all the elements of force, all of which are in the same direction. Now the factors C′, I, and $r^2$ are constant, and we obtain

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* More strictly, for space of constant permeability (p. 404).

$$P = \frac{C'Im}{r^2}\int ds = \frac{C'Im}{r^2} \cdot 2\pi r = \frac{2\pi C'Im}{r}.$$

The magnetic field strength is therefore

$$H = P/m = 2\pi C'I/r.$$

The same calculation can be performed for a circular arc of length $b$ (fig. 51). In this case we have $\int ds = b$ and the magnetic field strength is therefore $H = C'bI/r^2$.

**Definition of the Electromagnetic Unit of Current Strength.**—The result just obtained was made use of by Gauss and Weber to define the unit of current strength in terms of purely mechanical quantities measurable in cm., gm., and sec. This involves the definition of H by force effects (see p. 125 and 178), i.e. in purely mechanical units. Then H being measured in gauss, the unit of current strength is defined so as to make the factor C′ in the last equation above equal to unity, i.e. the unit of current strength is defined as that strength for which $H = 1$ gauss when $b = 1$ cm. and $r = 1$ cm.

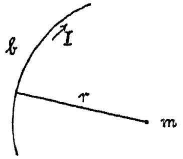

Fig. 51.—Field at the centre of a circular arc through which a current is flowing

**The electromagnetic C.G.S. unit of current strength is the strength of a current which, when flowing round a circular arc of 1 cm. length and 1 cm. radius, produces at the centre of the circle a magnetic field strength of 1 C.G.S. unit, i.e. 1 gauss.**

Employing this unit, the expression for the field strength at the centre of a circular ring carrying a current of strength I simplifies to $H = 2\pi I/r$ (H in gauss, I in electromagnetic C.G.S. units).

The **electromagnetic C.G.S. unit** of current strength is equal to **10 amperes.***

Measuring I in amperes and H in gauss, the formula for the field strength at the centre of a circular ring becomes

$$H = \tfrac{1}{5}\pi I/r.$$

It can be shown with the help of the Biot-Savart law (the details of the calculation need not be given here) that the field strength at a distance $r$ from a very long *straight conductor* carrying a current is $H = 2C'I/r$. If we express H in gauss and I in electromagnetic C.G.S. units, then $C' = 1$ and we obtain $H = 2I/r$. Or expressing H in gauss and I in amperes, $H = \tfrac{1}{5}I/r$. But when H was expressed in ampere-turns/cm. we had (p. 186) $H = \frac{1}{2}I/(\pi r)$. Therefore 1 ampere-turn/cm. is equal to $2\pi/5 = 1{\cdot}257$ gauss. From this it follows that when H is expressed in ampere-turns per cm. and I in amperes the factor $C'$ has the value $1/(4\pi)$. (See also p. 178.)

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* See the historical account on p. 193.

The derivation of the Biot-Savart law involves a point magnetic pole, such as can never be realized in practice. Moreover, the formulæ for a ring conductor apply only to the central point, whereas every magnetic needle is of finite size and necessarily extends to other points. These considerations show that the actual realization of the unit of current defined by means of the law is beset with very great practical difficulties and theoretical doubts, such as required all the genius and acumen of GAUSS and WEBER to overcome them. On this account it has been decided to define the unit of current strength electrolytically. In order to simplify the change, the unit defined electrolytically was made as nearly as possible equal to $\frac{1}{10}$ of the earlier electromagnetic C.G.S. unit. The electrolytically defined unit is the **ampere,** to which reference has already been made on p. 155.

**Historical.**—GAUSS and WEBER* were the first to attempt to build up a rigid quantitative system of electrodynamics. For this purpose it was necessary to fix a system of measurement. Their fundamental aim was to refer all physical measurements to three basic units (nowadays called the gramme, centimetre, and second). At this time (the first half of the 19th century) there existed upon the Continent a mathematical-physical ideal, namely to reduce all natural laws to the form of Newton's law of gravitation and wherever possible to explain them in terms of this law. In the hands of the celebrated French mathematicians this law had shortly before (towards the end of the 18th century) led to surprising successes in the mechanics of heavenly bodies. Moreover Coulomb's laws, which were formulated after the same pattern, had just at this time become of epoch-making importance for the shaping of the concepts and the construction of quantitative systems in the sciences of magnetism and electricity. In the equations of Coulomb's laws only mechanical quantities (force, distance) occur, except for a quantity characteristic of the phenomena concerned (quantity of electricity or magnetic pole strength). Thus it is possible (see p. 125) by means of these equations to fix units for electrical and magnetic quantities in the C.G.S. system. This method seemed particularly suitable at that time, because the measurement of forces, upon which the method ultimately depends, had been developed to a high degree of precision. Thus it was possible in this way to achieve a previously unattained clarity and reproducibility in the measurements. When therefore it proved necessary to introduce international legal units, reference was naturally made to the fundamental system of GAUSS and WEBER. At the Paris Congress in 1884 (at which WEBER was not present) the units still in general use nowadays were defined as equal to the electromagnetic C.G.S. units multiplied in each case by a power of ten, as suited the requirements of the science at that time. Since the electromagnetic unit of current strength was considered to be too great, the legal unit was defined as $\frac{1}{10}$ of the electromagnetic unit. In spite of the distinguished work of WEBER, whose name was suggested by the English delegates as a name for the new unit, this was called the ampere.†

**Comparison of the Electrostatic and Electromagnetic Units of**

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* See footnote, p. 195.

† See further, pp. 155 and 376.

**Current Strength.**—In addition to the legal system, we have become acquainted with two so-called absolute systems of measurement, the first of which is based upon the forces between point electric charges and the second upon forces between point magnetic poles. Now an electric current is associated with a magnetic field, and therefore it must be possible to find a connexion between the two systems. Since the magnetic field is a property of electric charges in motion, we shall expect that the concept of motion will play a part in the connexion between the two systems. The comparison of the two systems has been of great importance historically and therefore we will go into it in some detail.

In order to obtain the ratio of the measures of current strength (or quantity of electricity) in the two systems, all that is necessary is to measure the magnetic field produced by a current strength of one electrostatic unit when flowing round a circular conductor.

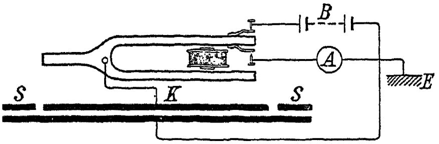

Fig. 52.—Determination of the ratio of the electrostatic and electromagnetic units of current strength

Having regard to the historical development of the subject, we may use the absolute electrometer of Thomson (fig. 86, p. 72) to carry out this measurement. In this method the electrical measurement is performed by means of a weighing and a measurement of length. In this way we determine the electromotive force (potential difference) of a galvanic battery B (fig. 52) in electrostatic units, basing the measurement (in accordance with the definition of the electrostatic unit) upon the force between static charges.*

We then connect the one plate of a measuring condenser K to earth. The other plate, which is surrounded by a guard ring, we connect to the tuning fork shown. This is so arranged that during each vibration its prong makes contact once with a point which is connected to one end of the battery B and once with another point which is connected via a galvanometer A to earth. The other end of the battery is also connected to earth (see figure). In accordance with the historical development of the ideas involved we use a tangent galvanometer of high sensitivity, i.e. with a low constant. (Instead of using the earth's field in the measurement, as is done in the case of the tangent galvanometer, it is much better to make use of the much more accurately determined electrolytic effects, i.e. to use some form of sensitive ampere meter and, applying the equation 1 electromagnetic unit = 10 amperes, read off the current strength directly in electromagnetic units.) Naturally the circuit must be so arranged that the guard ring is charged up by the battery simultaneously with the condenser proper, and that condenser and guard ring are also discharged simultaneously. The discharge of the guard ring must not, however, pass through the galvanometer. After each half vibration of the tuning fork there is alternately a discharge and recharging of the condenser to the potential difference of the battery. Suppose that the condenser takes up $Q_0$ electrostatic units of electricity each time it is charged. Let $\nu$ be the frequency of the fork. Then $\nu Q_0$ electrostatic units pass through the galvanometer per second, i.e. the mean current strength $I_0$ in electrostatic units is $I_0 = \nu Q_0$. The capacity $C_0$ of the condenser is given by $C_0 = A/(4\pi d)$ (see pp. 73 and 81). Hence

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* Nowadays the potential difference would be determined more simply by means of a standard cell.

$$Q_0 = \frac{AU_0}{4\pi d},$$

where $U_0$ is the potential difference of the battery B, and

$$I_0 = \frac{\nu AU_0}{4\pi d}.$$

*Example.*—Suppose the battery B to consist of 1000 cells. In this case the potential difference as measured with the absolute electrometer would be 6·7 electrostatic units (e.s.u.). Let the diameter of the circular condenser plate be 20 cm., the distance $d$ between the condenser plates 0·5 cm. and the frequency of the fork $\nu = 250$. We should then have

$$I_0 = \frac{250 \,.\, 6{\cdot}7 \,.\, 100\pi}{4\pi \,.\, 0{\cdot}5} = 8{\cdot}4 \,.\, 10^4 \text{ e.s.u.}$$

Suppose that a tangent galvanometer with $n = 3000$ turns of very fine wire is used. Let the measured value of the earth's horizontal intensity be $H = 0{\cdot}18$ gauss, and let the mean radius of the turns of the galvanometer coil be $r = 5$ cm. The galvanometer constant would then be

$$R = \frac{5 \,.\, 0{\cdot}18}{2\pi \,.\, 3000} = 4{\cdot}77 \,.\, 10^{-5}.$$

The galvanometer deflection, measured with POGGENDORFF's mirror device, would be $\alpha = 3° \, 20'$, i.e. $\tan\alpha = 0{\cdot}0583$. Therefore

$$I = R \tan\alpha = 4{\cdot}79 \,.\, 10^{-5} \,.\, 5{\cdot}83 \,.\, 10^{-2} = 2{\cdot}78 \,.\, 10^{-6} \text{ electromagnetic units.}$$

(Instead of using the tangent galvanometer, the current strength might have been read off directly from a suitable calibrated instrument as $2{\cdot}78 \,.\, 10^{-5}$ amperes.) Now the current strength $I_0$ in electrostatic units is the same as I in electromagnetic units (e.m.u.). Therefore we have

$$27{\cdot}8 \,.\, 10^{-7} \text{ e.m.u.} = 8{\cdot}4 \,.\, 10^4 \text{ e.s.u.},$$

whence

$$1 \text{ e.m.u.} = \frac{8{\cdot}4 \,.\, 10^4}{27{\cdot}8 \,.\, 10^{-7}} = 3 \,.\, 10^{10} \text{ e.s.u.}$$

or

$$c = \frac{1 \text{ e.m.u. of current strength}}{1 \text{ e.s.u. of current strength}} = \frac{1 \text{ e.m.u. of charge}}{1 \text{ e.s.u. of charge}}$$
$$= \mathbf{3 \,.\, 10^{10}}.$$

**Weber's Constant.**—Experimental investigations with the object of determining the ratio of the two fundamental units of charge (or current strength) were first undertaken in 1856 by R. KOHLRAUSCH and WILHELM WEBER *, after WEBER had already indicated in 1852 how the numerical value of $c$ could be measured. Our knowledge of this constant is thus primarily due to WEBER, and in this book we will therefore refer to it in future as *Weber's constant.* Repeated determinations of its value have been made with the greatest possible accuracy right up to the most recent times.

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* WILHELM EDUARD WEBER (1804–91), Professor of Physics in Göttingen, one of the greatest of physicists. In collaboration with his brother ERNST HEINRICH (1795–1878), who later became Professor of Physiology at Leipzig, he published in 1825 fundamental treatises upon the theory of waves. Working with KARL FRIEDRICH GAUSS (1777–1855), he made the fundamental magnetic and electrical observations for the construction of the so-called absolute system of measurement. Amongst all investigators GAUSS and WEBER were the first to attempt to base the laws of electromagnetism upon precise quantitative measurements.

The magnitude of Weber's constant agrees with surprising accuracy with that of the velocity of light. The discovery of this fact was the most far-reaching outcome of these investigations, which therefore showed that *light* must undoubtedly be related in some way to the magnetic field of electricity in motion—a fact of fundamental importance for an understanding of the nature of light.

From the most recent determinations * which give the international ohm as 1·00051 absolute ohms (p. 221), we deduce the value $c = 2{\cdot}9979 \,.\, 10^{10}$ for Weber's constant. The most reliable value for the velocity of light (according to MICHELSON) is also $2{\cdot}9979 \,.\, 10^{10}$ cm./sec.

*Weber's Constant as a Velocity.*—From considerations of the magnetic field of a charged body in motion (convection current) it can be shown that Weber's constant has the dimensions of a velocity.

Imagine an electrically charged ring of tinfoil to rotate about an axis through its centre and at right angles to its plane. The lines of force may be supposed to end upon very distant surroundings; they therefore move at right angles to themselves. If the mean radius of the ring of tinfoil is $r$ and its breadth $b$, the total area of both its surfaces is $4\pi rb$. If its total charge is $Q_0$ e.s.u., the surface density $\sigma_0$ is $Q_0/(4\pi rb)$. Now there are $4\pi\sigma_0$ electrostatic lines of force per unit area. Hence the electric field strength near the ring in air is $E_0 = 4\pi\sigma_0 = Q_0/(rb)$ e.s.u.

The rotating metal ring behaves like an electric current of strength I e.m.u. The circuital potential of the magnetic field (p. 192) is $4\pi I$. Let the magnetic field strength near the ring be H gauss. Then we can also calculate the circuital potential as the work performed in taking a unit pole against the field H around the cross-section of the ring. This cross-section has the form of a rectangle with two long sides each of length $b$ and two very short sides whose length can be neglected when the tinfoil is very thin. The work done during the circuit is then $2bH$. Hence $2bH = 4\pi I$ or $I = \frac{1}{2}bH/\pi$ e.m.u. But $I_0 = \nu Q_0$ e.s.u., where $\nu$ is the number of revolutions per second. Also $I_0/I = c$. Therefore $\nu Q_0 = \frac{1}{2}cbH/\pi$. Substituting $Q_0 = E_0 rb$ from above, we obtain

$$E_0 rb\nu = \tfrac{1}{2}cbH/\pi$$

or

$$cH = 2\pi r\nu E_0.$$

Here $2\pi r\nu$ is equal to $v$, the velocity of translation of the electric charge or of the field associated with it. Hence we have

$$\boldsymbol{c}\mathbf{H} = \boldsymbol{v}\mathbf{E}_0.$$

This equation, which we will call the **convection equation,** states that:

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* Made in 1921 by E. GRÜNEISEN and E. GIEBE at the German *Phys.-techn. Reichsanstalt.*

*In the case of an electric field in motion there is also a magnetic field. The direction of the magnetic field is at right angles both to the direction of the electric field and to its direction of motion. The field strength of the magnetic field is proportional both to the field strength of the electric field and to its velocity of motion. For a velocity* $v = c = 3 \cdot 10^{10}$ *cm./sec. the value of the magnetic field strength expressed in magnetic units (gauss) is equal to that of the electric field strength expressed in electrostatic units.*

*The significance of this relationship* is as follows: Both systems, the electrostatic and the electromagnetic, have a common foundation in that the units of field strength, upon which they are respectively based, are defined respectively by means of an electric field and a magnetic field, in both of which the force effects are of the same absolute magnitude. The convection equation therefore expresses the fact that a magnetic field is absolutely equal, as regards force effects, to the moving electric field producing it, i.e. both fields have the same energy density, when the electric field is moving with the velocity $c$ (velocity of light) at right angles to the electric and magnetic lines of force. This is the reason for the importance of the constant $c$ in connecting the two systems of measurement, both of which are based upon the same unit of force.

This historical significance of the comparison of the two systems of measurement lies in the fact that it stimulated the comparison of electric and magnetic fields of the same absolute strength. The discovery that the constant $c$ is identical with the velocity of light was of special importance in that it forged a connecting link between two previously quite separate branches of physics—light and electromagnetism.

**Generalized Convection Equation.**—In the case where the motion of the electric field is not at right angles to its lines of force, i.e. where the velocity vector $\mathbf{v}$ makes an angle $(\mathbf{E}, \mathbf{v})$ with the field strength vector $\mathbf{E}$, we have to take the component of the field strength at right angles to $\mathbf{v}$ instead of the whole field strength as above. Thus in place of E we have $E \sin(\mathbf{E}, \mathbf{v})$.

Writing the equation in the legal system of measurement, and remembering that 1 volt/cm. = 1/300 e.s.u. and 1 ampere-turn per cm. = $2\pi/5$ gauss, we obtain

$$\frac{2\pi}{5} cH = \frac{1}{300} vE \sin(\mathbf{E}, \mathbf{v}).$$

or

$$H = \frac{1}{120\pi} \cdot \frac{v}{c} \cdot E \sin(\mathbf{E}, \mathbf{v}) = \frac{1}{377} \cdot \frac{v}{c} \cdot E \sin(\mathbf{E}, \mathbf{v}).$$

Writing $E_n$ for $E \sin(\mathbf{E}, \mathbf{v})$, we have finally

$$\mathbf{H} = \frac{1}{377} \frac{v}{c} \mathbf{E}_n.$$

where H is measured in ampere-turns per cm. and $E_n$ in volts per cm.

The direction of the magnetic field may be found by imagining the vector $\mathbf{v}$ to be turned towards the positive direction of the vector $\mathbf{E}_n$. The direction of $\mathbf{H}$ is then the direction in which a right-handed screw would advance when turned in the same sense as $\mathbf{v}$ (see fig. 8, p. 164).

Although a static electric field ($v = 0$) has no magnetic field, a magnetic field is produced as soon as the electric field moves in a direction at right angles to its lines of force. The strength of the magnetic field increases in direct proportion to the magnitude of the velocity. The significance of the velocity $c$ is that an electric field moving with this velocity at right angles to its lines of force produces a magnetic field of such a strength that the energy densities of both fields are equal at every point.

In this particular case, i.e. when the electric and magnetic energy densities are everywhere equal, we have $E = 377H$, E being expressed in volts per cm. and H in ampere-turns per cm.

*Note.*—It should be noted that, though a static electric field cannot be detected when an equal and opposite static field is superimposed upon it, yet as soon as one of the fields is set in motion its magnetic effect can be observed. This case occurred, for example, in the experiments upon convection currents discussed on p. 165. The magnetic effect of the rotating plate was detected by means of a magnetometer totally enclosed in a metal case to shield it from electrostatic influences. This leads us to the remarkable conclusion that at a point at which two fields neutralize each other the corresponding equal and opposite excitations of the ether must still be present.

## 8. Maxwell's First Equation

**Calculation of the Strength of the Magnetic Field of a Displacement Current.**—Maxwell's assumption that a condenser in the process of being charged or discharged must be surrounded by a magnetic field due to the displacement current has already been discussed on p. 167.

During charging, the electric lines of force of the field in the condenser move inwards from the periphery in all directions. The magnetic lines of force must therefore be circular and at right angles both to the direction of motion of the electric lines of force and to the electric lines of force themselves. Thus there must be a rotational magnetic field (p. 189 *et seq.*) within the condenser, just as though the charging current passed through the condenser by metallic conduction (fig. 14, p. 167).

We may make use of the convection equation of the last paragraph to estimate the strength of the *conduction current* which, passing by *metallic conduction* through the condenser, would produce the same magnetic field there as is produced, according to Maxwell's theory, by the displacement current during the charging process.

If A is the area and $r$ the radius of the circular condenser plates, and if E is the magnitude of the electric field strength within the condenser, then the total number of lines (or flux) of force in the condenser is $K_0AE$. According to p. 70 the corresponding charge is

$$Q = K_0AE.$$

In the element of time $dt$ the quantity $dQ$ of charge flows into the condenser and increases the flux of lines of force by

This increase in the number of lines of force can also be calculated in another way, for it is due to the passage of lines of force into the condenser from the edge of length $2\pi r$. If the velocity of motion of the lines of force passing inwards at the edge is $v$, then in the time $dt$ the lines of force initially associated with a peripheral ring element of area $2\pi rv\,dt$ will have been completely replaced by new lines entering from outside. Now in the time $dt$ the field strength within the condenser does not alter appreciably. Hence the newly arrived lines of force must produce the same field strength E. The number of lines of force entering in the time $dt$ must therefore be $2\pi rv\,dt\,.\,E$.

Thus
$$dQ = 2\pi r K_0 v E\,dt,$$

whence
$$\frac{1}{2\pi r K_0}\,\frac{dQ}{dt} = vE.$$

Here $dQ/dt$ is the rate of addition of electricity to the condenser, i.e. the current strength I. Putting $vE = 377cH$ in accordance with the convection equation obtained on p. 197, and remembering that $377K_0c = 1$, we have

$$H = \frac{I}{2\pi r}\;\frac{1}{377K_0c} = \frac{I}{2\pi r}.$$

But according to the formula of p. 186 this is identical with the magnetic field strength of a current flowing in a straight conductor, I being the current strength and $r$ the distance from the conductor. Thus the magnetic field of our condenser during the process of charging is the same as if the charging current of strength I were flowing through it by metallic conduction. We arrive therefore at the following important result:

*The magnetic effect of a variable electric field between condenser plates is exactly the same as if the charging current were flowing through a metallic conductor joining the plates.*

**Closed Character of Electric Currents.**—It follows from the above result that, neglecting the processes in the interior of a conductor through which a current is flowing, and judging the current only by means of its observable effects in the surrounding space (i.e. imagining the whole arrangement, conductors, condenser, &c., enclosed in an opaque cardboard box), we have no means whatever of distinguishing the displacement current within the condenser from the conduction current in the metal wires. A circuit including a condenser in the process of being charged is indistinguishable from a circuit consisting only of metallic conductors without any breaks. Following MAXWELL, we ascribe the magnetic field of the varying electric field in the condenser (either in vacuum or in a dielectric) to a so-called *displacement current* (see p. 167). According to MAXWELL's view, therefore, the circuit is "closed" in the sense of being completed by the displacement current at the places where conduction is impossible.

If the space between the plates of the condenser is filled with a dielectric of dielectric constant K, then we have to replace $K_0E$ by $K_0KE$ in the above treatment. The result of the calculation is independent of E and is thus not affected. For a given electric field strength E, however, the charge Q and hence also the strength I of the charging current are increased by the factor K. Hence the displacement current must also be increased in the same ratio.

*For a given field strength, varying in a given manner, the displacement current is K times greater in a dielectric of dielectric constant K than in a vacuum.*

The magnitude of the strength of the displacement current passing normally through an area A in a dielectric of dielectric constant K is therefore

$$I = K_0KA\frac{dE}{dt} = \frac{d(AD)}{dt},$$

where E and D are the magnitudes of the resolved parts of the electric field strength and displacement respectively at right angles to the area A.

Now we can regard every electrically charged body as part of a condenser. Hence the process of charging must always produce a displacement current in the space surrounding the body. During the charging of a metal ball supported upon a glass column, for example, there is a displacement current in the surrounding space. This current is between the ball and those conductors upon which, after the charging process is over, the equal and opposite charge is to be found, i.e. the displacement current is produced in the region occupied by the electric field necessarily accompanying the charging process. During this process the whole of this region is filled with a rotational magnetic field, the invariable sign showing the presence of a current, whether conduction current or displacement current. In this sense, therefore, the following statements are seen to hold:

*All electric currents are necessarily closed*; and:

*Every charging current flowing to an insulated conductor is continued into the insulator as a displacement current without alteration of total current strength.*

**Maxwell's First Equation.**—The equation of p. 187, which connects the value of the line-integral of the magnetic field strength around a closed path (circuital potential) with the total current strength flowing through the surface bounded by the closed path, can now be written in a more general form. Let the strengths of the conduction currents through the surface bounded by the closed path be $I_\nu$, and those of the convection currents $Q_\pi v_\pi$ ($Q =$ charge, $v =$ velocity). In addition to these currents, suppose that the electric field strength is varying. Then in addition to the magnetic fields of the conduction and convection currents we shall have the magnetic fields of the displacement currents corresponding to the field variations. Suppose that the closed path encloses a number of surfaces of area $A_\mu$ over which the field strength $E_\mu$ is varying. The fluxes through these areas are $A_\mu D_\mu$ ($D_\mu$ = corresponding electric displacement) and the equivalent charges are also $A_\mu D_\mu$. The strengths of the displacement currents are equal to the rates of change of these charges, i.e. $d(A_\mu D_\mu)/dt$. Hence the magnitude of the circuital potential V is the sum of the strengths of the conduction, convection, and displacement currents. This is expressed by the general equation:

$$\mathbf{V} = \sum_{\nu,\pi,\mu}\left[\mathbf{I}_\nu + \mathbf{Q}_\pi v_\pi + \frac{d}{dt}(\mathbf{A}_\mu \mathbf{D}_\mu)\right].$$

This is the general form of the equation of p. 187, and is one of the fundamental equations first set up by MAXWELL.

*Note.*—When no closed metallic conductors penetrate the surface bounded by the closed path and when no moving charged bodies pass through it, we obtain an electromagnetic law for the dielectric alone. Supposing D to be constant over the whole surface bounded by the closed path, the total flux through this surface is

$$\varphi_e = AD$$

where A is the area. In this case Maxwell's equation assumes the form

$$\mathbf{V} = \frac{d\phi_e}{dt}.$$

*A varying flux of electric displacement is surrounded in space by a magnetic field whose direction is everywhere at right angles to the direction of the electric field and whose circuital potential is equal to the rate of change of the displacement flux.*

## 9. Ponderomotive* Force Effects of a Steady Magnetic Field

The tension and lateral repulsion of the lines of force are transmitted to the matter through which the current producing the magnetic field is flowing. This gives rise to ponderomotive effects, i.e. motions of weighable matter. In what follows we will only give a qualitative treatment of the forces tending to produce these motions. The quantitative treatment cannot be undertaken until we have considered the reaction of the motions upon the field (p. 440 *et seq.*). In the case of magnetic poles we have already become acquainted with a formula expressing the forces (Biot-Savart law).

**Mutual Action of Two Currents.**—The motions of a solenoid and a ring conductor carrying currents in a magnetic field have already been mentioned shortly on p. 171.

Since a ring conductor carrying a current is completely equivalent to a magnetic shell, it follows that two such ring conductors must attract each other when placed so that the north polar surface of the one equivalent magnetic shell faces the south polar surface of the other. In this case both currents flow round in the same sense. On the other hand, if the two rings are so placed that the adjacent surfaces of the equivalent magnetic shells are of similar polarity, then there is mutual repulsion. In this case the two currents flow round in opposite senses.

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* Lat., *pondus*, weight.

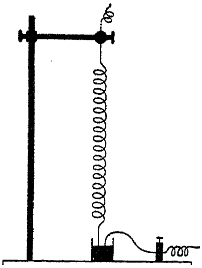

Fig. 53.—Roget's spiral

The action of Roget's spiral* (shown in fig. 53) depends upon the mutual attraction of two parallel ring conductors round which currents are flowing in the same sense. In the case of the spiral this attraction between the turns causes the spiral to shorten. As a result the lower end is lifted out of the cup of mercury and the current is interrupted. Thus the spiral alternately contracts and expands again, breaking and making the current circuit at regular intervals.

Conductors of other shapes also attract and repel each other when carrying currents.

Reference to fig. 7 (p. 163) and fig. 54 makes the following facts easy to understand:

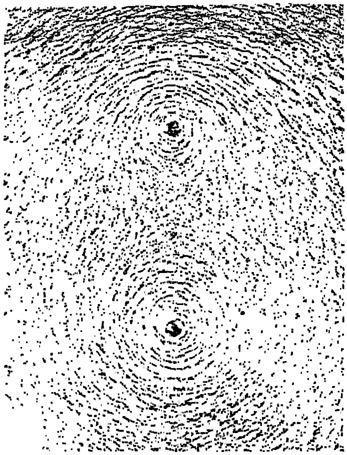

Fig. 54.—Lines of force around two parallel conductors carrying currents in the same sense

*Parallel currents attract one another when flowing in the same sense and repel one another when flowing in opposite senses. The forces between inclined currents are such as to tend to make them parallel.*†

The mutual attraction and repulsion between two parallel conductors carrying currents can be demonstrated conveniently as shown in figs. 55 and 56. Two copper wires of about 0·5 mm. diameter and about 60 cm. long are stretched vertically at a distance of about 3 mm. apart. For this purpose the wires are passed through two small corks which are then held in suitable clamps. When, as in fig. 55, the two wires are joined above and below by connecting sleeves, the currents in both are in the same sense. As soon as the current (about 10 amp.) is switched on, the two wires approach one another. On the other hand when, as in fig. 56, the wires are only joined at the top and the current is made to enter and leave the lower ends, the sense of flow in the one wire is opposite from that in the other. In this case the wires are forced apart when the current is switched on.

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* Peter Roget (1779–1869), physician in London.

† These laws were first expressed in this general form by Ampère.

Application of these principles to the case of a ring conductor shows that all the elements of the ring tend to move radially outwards. Thus when a current is passed through a flexible loop of thin metal foil or tinsel, the loop is at once opened out into a circle. This illustrates very clearly the lateral repulsion between the magnetic lines of force which thread the loop (p. 430).

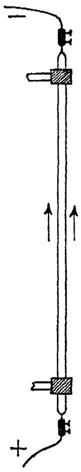

Fig. 55.—Attraction between parallel currents flowing in the same sense.

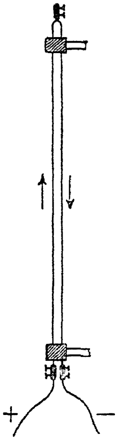

Fig. 56.—Repulsion between parallel currents flowing in opposite senses.

**Magnetic Pole in the Magnetic Field of a Current.**—The experiment given on p. 181 can be so modified as to give a continuous rotation. Fig. 57 shows the arrangement.

Two parallel bar magnets NS with like poles adjacent can rotate about an axis in the manner shown in the figure. The current flows from below up the axis, but branches off opposite the middle points of the magnets and passes by way of a curved copper arm to a circular ring-shaped trough of mercury, which bears the other terminal. When the current is flowing, the magnets rotate continuously.

This rotation is at once understandable in the light of what was said on p. 181. The energy of rotation is obviously derived from the magnetic field, i.e. from the current producing this field. The way in which this occurs cannot be discussed until later (cf. p. 543).

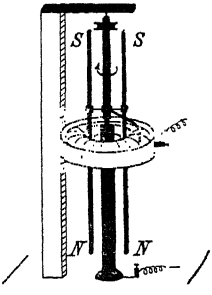

Fig. 57.—Rotation of a "magnetic pole" around a conductor carrying a current.

**Principle of Action and Reaction.**—This principle (see Vol. I, p. 70) also holds for the mutual forces between currents and magnets. A force is not only exerted by the current upon the magnet, but also by the magnet upon the current (i.e. upon the conductor carrying the current). This can easily be shown by the following experiments.

The electric circuit, consisting of an accumulator B and a loop of wire, is so arranged that the current flows round a magnetic needle. When the circuit is held fast (as in fig. 58), the magnetic needle is deflected in accordance with the considerations put forward on p. 138. But when the circuit is connected rigidly to the magnetic needle (fig. 59), the whole being free to move, no motion is observed in spite of the considerable turning moment exerted by the current upon the needle. This proves that the conductor carrying the current is subjected in the field of the magnet to forces which are exactly equal and opposite to those exerted by the current upon the magnet. This can also be demonstrated directly by clamping the magnet and suspending the whole electric circuit so that it is free to rotate (fig. 60). The electric circuit is deflected in the opposite direction from the previous deflection of the magnet.

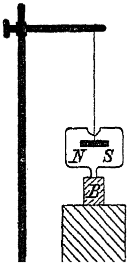

Fig. 58. — Magnet movable, electric circuit fixed: the magnet turns.

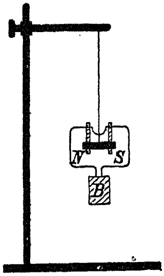

Fig. 59. — Magnet and electric circuit rigidly connected and movable as a whole: no result.

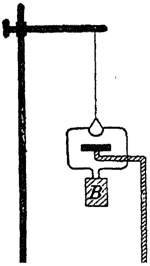

Fig. 60. — Magnet fixed, electric circuit movable: the circuit turns in the opposite sense from the magnet in fig. 58.

Figs. 58, 59, and 60.—Principle of action and reaction

**Conductor carrying a Current in a Magnetic Field.**—The motion of a conductor carrying a current in a magnetic field can be demonstrated elegantly by the following experiment.

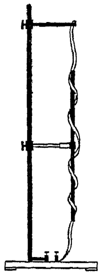

Fig. 61.—The wrapping of a flexible conductor around a bar magnet when a current is flowing.

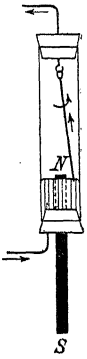

Fig. 62 Faraday's rotation experiment.

A flexible conductor composed of several strips of thin metal foil or several strands of tinsel is hung up loosely beside a vertical steel magnet (fig. 61). When a current is sent through it, the conductor winds itself in a spiral around the magnet (see figure). Reversal of the current causes the conductor to unwind itself and then wind itself up again in the opposite sense.

The experiment can also be varied by using a conductor suspended at the upper end and dipping at the lower end into mercury (see fig. 62). When carrying a current, the conductor rotates continuously around the pole of the magnet (Faraday's rotation experiment).

The sense of the rotation can easily be derived from a consideration of the nature of the magnetic field, which is governed by the corkscrew rule. In this connexion the following experiment is instructive.

We hang a flexible conductor loosely between two stands (see fig. 63), so that a part of it is between the poles of an electromagnet. We may imagine the poles of the magnet to be placed so that in the figure the lines of force are directed away from the observer. The stands holding the conductor are now connected to the poles of a battery and the current is switched on. In the case where the current is from left to right in the figure, the conductor is immediately jerked upwards from between the magnetic poles. Reversal of the current (as in the figure) or reversal of the poles of the electromagnet causes the conductor to move in the opposite sense.

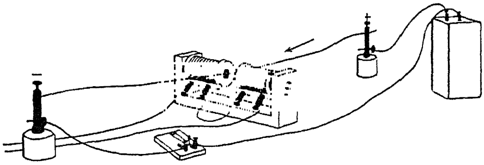

Fig. 63.—Force upon a conductor carrying a current in a homogeneous magnetic field

If we attempt to bring back the flexible conductor to its original position between the magnetic poles, we experience a marked resistance, just as if there were an invisible spring opposing us.

A modification of this experiment is the so-called Barlow's wheel (fig. 64). A small, easily rotating copper wheel is mounted between the poles of a magnet. Its rim dips into a small trough of mercury, which is connected to one pole of a battery. The other pole is connected to the axis of the wheel, so that the current flows radially. When the current is flowing, the wheel rotates continuously. A solid circular disc can be used instead of a wheel with spokes. Concerning the conclusions to be drawn from this see p. 363.

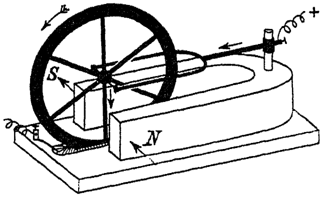

Fig. 64.—Barlow's wheel

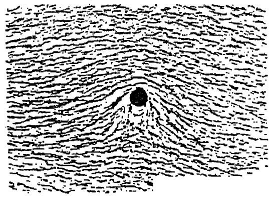

Fig. 65.—Lines of force around a conductor carrying a current at right angles to a magnetic field.

The sense of the motion can be readily derived from the diagram of the lines of force (fig. 65). We see from this that the conductor is driven at right angles to the magnetic field. The sense can be obtained in each case by application of the corkscrew rule. The relationships are shown in fig. 66.

*When the current flows from left to right, as viewed by an observer looking in the positive direction of the magnetic lines of force, the conductor tends to move upwards.*

These relationships can be remembered by means of the so-called **left-hand rule** (fig. 67).

*Imagine the left hand to be laid upon the conductor so that the direction and sense of the current is from wrist to finger tips, and so that the palm is turned towards the north pole as if the hand were about to grasp it (i.e. so that the lines of force are normal to the hand and directed from the palm towards the back). Then the direction in which the conductor is urged is the direction of the outstretched thumb* (see figure).

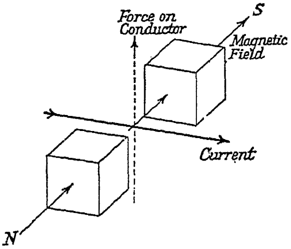

Fig. 66.—Direction and sense of the force upon a conductor carrying a current in a magnetic field

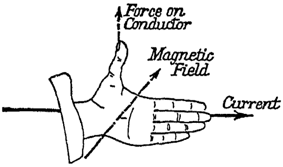

Fig. 67.—Left-hand rule

Concerning the *magnitude* of the forces acting see p. 440 *et seq.*

## 10. Electromagnets and their Applications

Although we cannot yet give a quantitative treatment of the influence of matter upon a magnetic field (see § 2, p. 402), the importance of the effects involved is so great that we must make ourselves acquainted with them qualitatively before going farther.

**Electromagnets.**—A long iron rod, placed within a solenoid so as completely to fill the space within it, becomes strongly magnetized when a current is passed through the solenoid. This arrangement is called an *electromagnet.* The north pole is at the end nearest to an observer, who, looking along the axis of the solenoid, sees the sense of the current as anticlockwise. To an observer looking at the south pole along the axis of the solenoid, the current appears to flow in the clockwise sense (fig. 68).

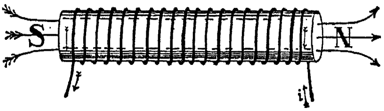

Fig. 68.—Electromagnet

After having withdrawn the iron core, we have considerably to increase the strength of the current in the solenoid in order to produce the same external magnetic effects as before.

Thus a solenoid with iron core behaves like a solenoid without iron core but with a *considerably greater* number of ampere-turns.

In the light of the considerations put forward on p. 172, there can be no doubt that this increase in the effective number of ampere-turns is due to the circulatory molecular currents. These are at first orientated at random, but are turned round by the field of the solenoid so as to send their lines of magnetic force in the same direction.

This effect is quite similar to that which occurs when a dielectric is introduced between the plates of a condenser. As fig. 109 (p. 89) shows, the density of the lines of force between the plates is increased by the factor K at constant potential difference, K being the dielectric constant of the substance introduced. The present case is quite analogous: the introduction of the iron core into the solenoid at constant current causes an increase in the density of the magnetic lines of force in the surrounding space. The magnitude of this increase is different for cores made of different substances. Just as in the electrical case this behaviour is described by means of the dielectric constant, so here we can describe the increase in the density of the magnetic lines of force by means of a number characteristic of the particular material used. This number is called the **permeability** of the material. It will be discussed more fully in § 2, p. 402 *et seq.* It may be remarked here, however, that there is an important difference between dielectric constant and magnetic permeability, in that the former hardly ever exceeds the value 10 whereas the latter may have a value running into thousands. Special mention must be made of iron and its alloys, which have very high permeabilities. Moreover, the dielectric constant is really a constant within wide limits, i.e. is independent of the magnitude of the electrical field strength; but the magnetic permeability varies very much with the magnetic field strength in many cases.

The forces exerted by a magnetic field upon a piece of iron are essentially analogous to those to which a substance of very high dielectric constant is subjected in an electric field (p. 94). Thus the iron is urged towards the region of higher field strength, &c.

We will now consider a few of the important applications of electromagnets.

**Electromagnetic Contact Breaker.**—Electromagnets can be used with advantage for breaking and making electric current circuits *, provided that these operations do not have to be performed very rapidly (cf. p. 438). The principle consists in sending the current through the coils of an electromagnet and utilizing the consequent motion of an attracted piece of soft iron for the purpose of breaking the circuit. (Roget's spiral works upon a similar principle, see p. 202).

*Electric Bell* (fig. 69).—The soft iron armature A is held by a suitable spring at a small distance from the poles of a horseshoe-shaped electromagnet M. At the lower end of the armature (and usually carried upon a steel spring F) is a small platinum disc, against which presses a contact screw U with a platinum tip. The one end of the electromagnet winding is connected to the one terminal

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* The automatic electromagnetic contact breaker was invented by J. P. WAGNER (1799–1879) in 1837 at Frankfort on the Main—hence the name *Wagner's hammer*. The common trembling electric bell, in which the device is used, was invented by JOHN MIRAND in 1850.

$K_1$, the other end to the armature. The contact screw U is connected to the other terminal $K_2$.

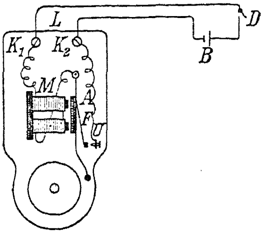

Fig. 69.—Electric bell

When the two terminals are connected to the poles of a source of current, the core of the electromagnet becomes magnetized and attracts the armature, thus automatically breaking the contact between the platinum disc and the tip of the screw U. This stops the current, the armature is no longer attracted and springs back, the platinum disc makes contact again with the tip of the screw U, and the current begins once more. The same process is repeated continually, so long as the terminals are connected to the source of current.

The contact spring F is important; if the contact were with a *rigid* part of the armature, the duration of the current would be too short for the magnetic force to overcome the inertia of the armature and the resistance of its spring sufficiently to impart a large enough amplitude to the vibrations.

This type of make and break is also used in *induction coils* (p. 516).

**Electromagnetic Relays.**—A *relay* is a device for controlling a current of any strength by means of a relatively weak current. The motion of the armature is used for the purpose of making or breaking the circuit of the stronger current (fig. 70). In this way a long telegraph line may be divided up into several shorter sections, requiring only relatively weak sources of current. A suitable relay is then included before the receiving apparatus. Electromagnetic relays are used very widely nowadays in electrical engineering for distant switching and automatic control of currents. The difficulties arising when the current strengths and potential differences are high form interesting engineering problems.

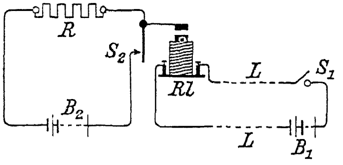

Fig. 70.—Relay circuit

B, Battery or other source of current; S, switch; Rl, relay; L, line; R, instrument to be operated by relay. Subscript 1 refers to primary circuit, subscript 2 to the relay-controlled circuit.

**Telegraph***.—Every telegraphic system consists of two parts, the **transmitter** and the **receiver.** These are situated at two different places and are connected together by a conductor (the *line*) in which is included a source of current. The transmitter generally consists of a key by means of which the current sent through the line to the receiving station can be started, stopped, or reversed. The receiver consists of some device or other to indicate the current. In **recording telegraph systems** the signals are automatically written out upon a strip of paper.

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* Gr., *téle*, distant; *grápheín*, to write. The first practical electromagnetic telegraph apparatus was set up in 1833 by Gauss and Weber at Göttingen.

A form of recording telegraph known as the Morse inker * is widely used. It consists of an electromagnet (fig. 71) with an armature. The armature is carried upon an arm which can turn about a pivot and which is held back by a suitable spring. When the electric current from the battery flows through the coil of the electromagnet, the core becomes magnetized and attracts the armature towards it. This causes a pen attached to the lever to press against a strip of paper which is being moved along by clockwork. If the transmitting key is depressed for a relatively long period, the pen writes a correspondingly long dash on the paper. If the key is only depressed for an instant, only a dot is recorded. The letters of the alphabet can be represented by combinations of dots and dashes according to a prearranged code (Morse code).

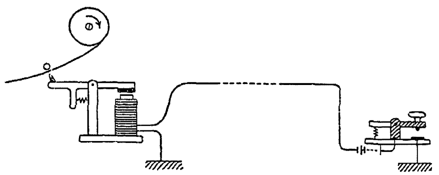

Fig. 71.—Morse inker circuit

With the Morse inker it is possible to telegraph about 100 letters per minute. As an example of a system allowing of greater speed, a short description may be given of the Siemens **high-speed telegraph.** Here the signals are not sent by hand, but by means of a perforated strip (see fig. 72, to the left). This strip passes over five contact levers. When a hole passes over a lever, the latter is displaced and makes contact with the positive pole of the local battery (see figure); in their normal positions, however, the levers are in contact with the negative pole. The perforations of the strip are made in a special kind of "typewriter". The signal for each letter consists of five short currents—of positive or negative sense—combined together in a certain way. In fig. 72 the letter Z has just been sent; it consists of two short currents in one sense and three in the other. These currents are sent in rapid succession through the line, the distributor brush sweeping round over the distributor plate at the moment that the contact levers are just beneath the perforation. The currents are sent along the line through the transmitting relay, whose armature moves in the one direc-

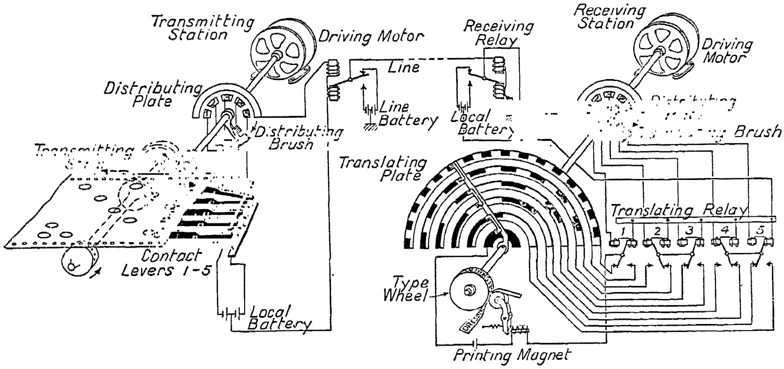

Fig. 72.—Diagram of Siemens high-speed telegraph system

tion or in the other according to the sense of the current. The receiver is similarly arranged. Its distributor brush must rotate in exactly the same way as (i.e. must be synchronized with) that of the transmitter. This is ensured by special devices. In this way the current corresponding to each of the contact levers is distributed to the proper member of a set of five translating relays at the receiving end of the line. These relays are connected through a special battery and a printing magnet to the translating disc, whose rotation is also synchronized. Upon the same shaft is the type wheel, which has the inked type letters upon its periphery. The whole is so arranged that the printing magnet presses the paper strip against the type wheel just at the moment when the letter corresponding to the position of the translating relays is opposite to it. In actual instruments a complete circular translating disc is used instead of the semicircular disc shown in the figure; this allows of the use of two sets of relays.

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* Samuel Morse (1781–1872), an American, invented the recording telegraph in 1837.

In addition to the so-called type-printing telegraphic systems, which print telegrams directly in type, other systems are in use in which the mirror deflections of sensitive galvanometers (p. 211) are registered photographically. By the latter method a still higher speed of transmission can be attained. With the most modern instruments it is possible to transmit up to 100,000 letters per hour. It may also be mentioned that as many as six telegrams can be sent simultaneously in the same or opposite directions through the same line without mutual interference (*muliplex telegraphy*).

*Note.*—On account of the relatively large currents which they require, electromagnets cannot be used for transatlantic cables. The reason for this lies in the electrostatic capacity of the cable, which acts as a cylindrical condenser whose inner plate is the metallic conductor, whose outer plate is the sea water, and whose dielectric is the insulating cover of the cable. Since the cable must be very long, its capacity is very great. Consequently a considerable time is required to charge up the condenser, i.e. to produce the potential difference necessary to actuate the instruments. Moreover, this potential difference rises gradually to its final value and dies away again gradually, so that the signals are blurred. For these reasons the smallest possible currents and the most sensitive receivers (mostly galvanometers) are used. Concerning further problems of telegraphy see p. 580.

**Johnson-Rahbeck Effect.**—This effect, which was discovered in 1920, presents the possibility of using the force of attraction of a charged plate condenser instead of an electromagnet for mechanical action. A semi-conductor (ground agate or slate) holds a metal plate very tightly when the two are placed together and connected to the poles of a battery of about 200 volts (fig. 73). When the E.M.F. is removed, the metal plate can be moved easily relative to the stone. The explanation is as follows. The metal plate and the stone are only in actual contact at a few points. At these points the potential difference is equalized by the flow of a *very* weak total current. Since now both the stone (because of its large extent) and also the metal have practically no resistance except at these points, whereas these points (being very small) have a very great resistance, therefore the whole potential drop occurs across a very small distance (about $10^{-3}$ cm.). Consequently the forces of attraction between the stone and the metal are very great (p. 70). The currents necessary to produce these forces are, however, vanishingly small.

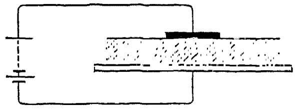

Fig. 73.—Johnson-Rahbeck effect

**Current-measuring Instruments.**—The magnetic effect of a current can be used to measure its strength. The instruments used may be divided into three main types: **galvanometers,** whose action depends upon the interaction of a magnet and a coil through which the current flows; **soft iron ammeters,** in which use is made of the attraction of a piece of soft iron into a coil or a region of higher magnetic field strength; and **electrodynamometers,** which are actuated by the mutual attraction of two coils carrying the current.

Each of these types can be realized as pointer instrument, torsion instrument or mirror instrument. For most purposes use is made nowadays of *pointer instruments,* in which the current can be read off directly upon a scale.

In *torsion instruments* the measurement is obtained by means of the turning moment which must be applied to a torsion suspension in order to bring back the moving part to its normal position (see, for example, fig. 34, p. 176).

*Mirror instruments* have a small mirror rigidly connected to the moving part. Its deflection is measured by the deviation of a beam of light (Gauss-Poggendorff mirror observation, see Vol. IV).

In all measuring instruments a good *damping* is important; the position of rest must be attained quickly and without oscillations of the moving part. This is usually attained either by means of Foucault current damping (p. 380) or else by means of an aluminium vane which is damped by air resistance.

**Galvanometers.**—The older instruments have a fixed coil and a moving magnet suspended within it so that when no current is flowing the magnet is parallel to the turns. When a current flows through the coil, the magnetic field tends to turn the magnet into a direction at right angles to its normal position (fig. 34, p. 176), the turning moment exerted upon the magnet being proportional to the flux density (p. 122) and therefore also to the current strength (p. 177). The restoring force may be that of the earth's field (e.g. in the tangent galvanometer), in which case the plane of the turns of the coil must lie in the magnetic meridian; or auxiliary magnets or any kind of mechanical force may be used.

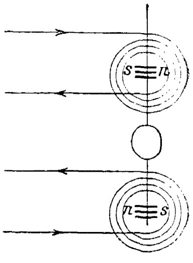

Fig. 74.—Astatic system with coils

A simple instrument of this type has already been mentioned (see fig. 15, p. 141).

**Astatic Systems.**—In order to eliminate the influence of the earth's magnetic field, two coils may be used and two systems of magnets. Fig. 74 shows an arrangement of this type. Within each of the two coils is situated a number of short, thin magnetic needles of steel, both sets of magnets being attached to the same rigid vertical axis. Half-way between the two sets of magnets is attached a small mirror. The axis is suspended from a thin thread of silk or metal. The two sets of magnets are parallel but the poles of the upper set point in the opposite direction from those of the lower set. The current passes in opposite senses through the two coils. If the magnetic moments of the two sets of magnets were exactly equal, then when no current was flowing there would be no moment whatever acting upon the system; for the earth's field would exert equal and opposite moments on the two sets. Thus the whole system would be in equilibrium in any position. Such a system is said to be astatic.* In reality one of the two sets of magnets always has a slightly greater magnetic moment than the other, so that the earth's field exerts a very small directing couple in addition to that of the suspension.

By means of two steel compensating magnets arranged above the apparatus so as to be adjustable both in direction and in height (see fig. 76) the external field may be varied within wide limits, and the strength of the directing couple acting upon the magnet system adjusted at will.

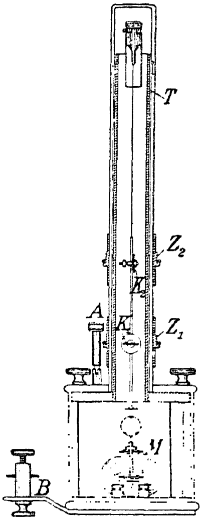

Fig. 75.—Nernst galvanometer

M = galvanometer magnets; $K_1$ and $K_2$ = compensating magnets upon aluminium discs, in the plane of which they can be turned; $Z_1$ and $Z_2$ = iron cylinders to improve the compensation; A, arresting knob; T, torsion head.

[From *Handbuch der Physik*, Vol. 16, published by J. Springer, Berlin.]

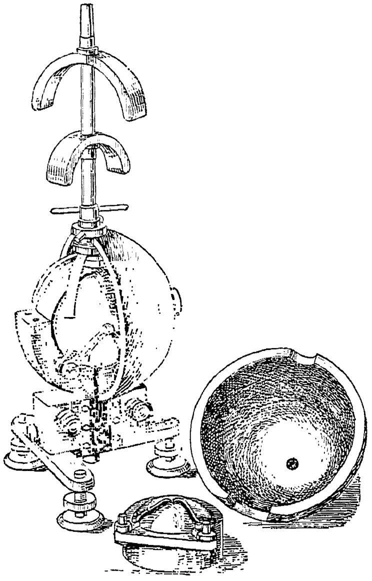

Fig. 76.—Galvanometer with spherical iron screening

[From Jaeger, *Elektrische Messtechnik*, published by J. A. Barth, Leipzig.]

As mentioned above, the current is sent through the two coils in opposite senses. The deflecting moments of both coils are thus in the same sense. Since now the restoring moment of the external field may be made as small as desired without altering the deflecting moment of the current in the coils, the instrument can obviously be made very sensitive and can be used to detect and measure very weak currents. Fig. 75 shows a very sensitive instrument in which the compensating magnets move with the astatic system. In this particular type the current flows round only one half of the astatic system.

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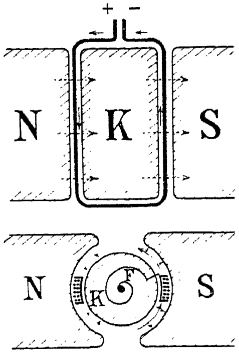
* Lat., *stare*, to stand; hence astatic = without definite equilibrium, i.e. in equilibrium in all positions.

In order further to weaken the effect of the earth's field and to protect the instrument from magnetic disturbances, the whole galvanometer may be surrounded with an iron casing (screened galvanometer of Bois and Rubens, fig. 76). Sensitive galvanometers with moving systems of magnets are hardly ever used nowadays without such an iron screen.

**Moving-coil Galvanometers.**—These are the most reliable and sensitive instruments. The magnet is fixed and the coil through which the current passes is movable. When a current is flowing through it, the coil tends to set itself with the plane of its turns at right angles to the magnetic field. The turning moment acting upon it is proportional to the current strength. In order to make this moment as great as possible, the coil must have a large number of turns and must be placed in a very strong field. In practice as close an approximation as possible to a magnetic circuit (§ 4, p. 421 *et seq.*) is employed, the space between the poles of a permanent magnet being made cylindrical and a cylindrical piece of iron being placed in this space so as nearly to fill it.



Figs. 77 and 78.—Principle of moving-coil galvanometer

This gives an arrangement like that shown in figs. 77 and 78, which represent side elevation and plan respectively. The pole pieces are marked N and S, and K is the iron cylinder between them. The lines of force run practically normally to the iron across the narrow separating spaces, i.e. as shown by the small dotted arrows in the figures. The coil is rectangular (see figures) and is suspended so as to be able to turn in the space between the magnetic poles and the iron cylinder. The longer sides of the rectangular coil are in a practically homogeneous magnetic field and, as the current flows in opposite senses in the two cases, one side is urged in one direction and the other in the opposite direction. The direction of the current being as indicated in fig. 77, the left-hand side of the coil would be subjected to a force tending to move it towards the observer, the right-hand side to a force tending to move it away from the observer. Thus the whole coil is subjected to a turning couple whose sense is anticlockwise as seen in the plan of fig. 78.

Since the field is homogeneous, the moment of this couple is proportional to the current strength. If the coil be suspended by a torsion wire or if its position of equilibrium when no current is flowing is determined by two small spiral springs, the deflecting moment and hence also the current strength may be measured. In both cases the displacement is proportional to the couple within the elastic limits. Hence the current strength can be read off upon a circular scale, whose divisions are all exactly equal. The current may be made to enter and leave the coil by way of the two springs (one of which is shown in fig. 78 and marked F).

The internal arrangement of moving-coil instruments can be seen from figs. 79 and 80.

In addition to having scales with equal divisions, moving-coil galvanometers possess two further advantages over other types. Firstly they are almost entirely independent of external magnetic fields—in particular of the earth's field; for the normal position of the coil is determined only by the elastic constraint of the suspension. For a given instrument the magnitude of the deflection depends only upon the strength of the current and the strength of the permanent magnet. Hence care must be taken to make this magnet as constant as possible. This is done by subjecting it to repeated mechanical shock and temperature change (*artificial aging process*).

The second advantage of moving-coil galvanometers is that they can easily be constructed so that the pointer takes up its final position almost immediately after the current has been switched on: the deflection is *aperiodic*. The reason for this cannot be given until the induction of electric currents has been discussed (p. 380).

Fig. 80 shows a *mirror galvanometer* of the moving-coil type.

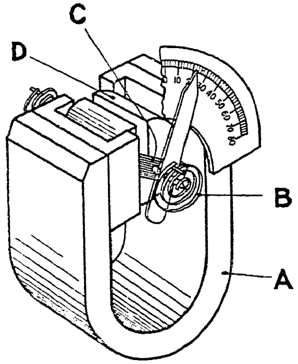


Fig. 79.—Moving-coil galvanometer
A, Magnet. B, Control spring. C, Moving coil. D, Core.
[Elliott Brothers (London), Ltd.]

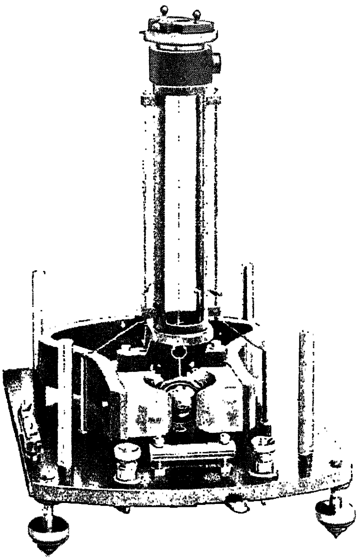


Fig. 80.—High sensitivity reflecting galvanometer
[Cambridge Instrument Co., Ltd.]

**Soft-iron Ammeters.**—The iron core is usually shaped so that the deflections are as nearly as possible proportional to the current strengths. But in spite of this the scale divisions are generally more crowded together at the beginning of the scale than in the range over which the instrument is actually to be used. Fig. 81 shows a practical form of moving iron (or soft-iron) ammeter.

**Dynamometers.**—When the planes of the turns of the two coils do not coincide, the current passing through the fixed coil (fig. 82) exerts a turning moment upon the movable coil, through which the current also passes. Since, as the current strength increases, the fields of *both* coils increase correspondingly, the deflection produced is proportional to the square of the current strength. It is thus independent of the sense of the current. This and their great reliability are the chief advantages of these instruments.

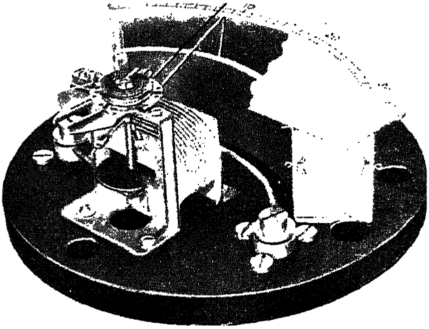


Fig. 81.—Moving iron ammeter

[Evershed and Vignoles, Ltd., London.]

The *Kelvin ampere balance* is a very accurate instrument, the action of which depends on the forces between currents whose planes are parallel. See p. 490.

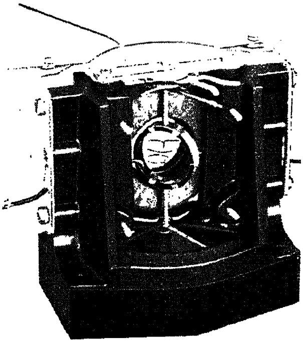


Fig. 82.—Dynamometer ammeter

[Evershed and Vignoles, Ltd., London]

**Specification of the Accuracy of Current-measuring Instruments.**—Modern manufactured instruments are sometimes marked with their type and accuracy. The symbols chosen to represent the different types are shown in fig. 83. Degrees of accuracy are specified by the letters E, F, G, and H, which indicate that the error in the reading is not greater than the following percentages of the maximum value of the range of measurement: E, ±0·4%; F, ±0·6% (for moving-coil instruments one half of these values); G, ±1·5%; and H, ±3%.

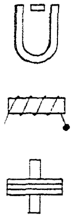


Fig. 83.—Symbols representing current-measuring instruments: viz., reading downwards, (1) moving-coil instrument with permanent magnet, (2) soft-iron instrument, (3) electrodynamometer.

**Oscillograph.***—This instrument depends upon the motion of a conductor carrying a current in an electric field. A single loop D (fig. 84) of very thin silver wire is stretched between the poles of a powerful electromagnet. A small mirror *s* (about 1 mm.$^2$) is attached between the wires of the loop where the magnetic field strength is greatest. A current flowing round the loop passes along the two wires in opposite senses. The two sides of the loop therefore move in opposite senses in the magnetic field, which thus exerts a couple upon the small mirror. A beam of light reflected from the mirror is deviated from its normal direction. Since the loop of wire and the mirror are of very small mass, they can move with practically no delay in obedience to the variations of the current. When the beam of light reflected from the small mirror *s* is allowed to fall upon a rotating mirror and thence upon a screen, the variations of the current become apparent as a continuous curve. To obtain a photographic record, the point image of a source of light is projected upon a strip of sensitive paper which is moved along by some suitable mechanism.

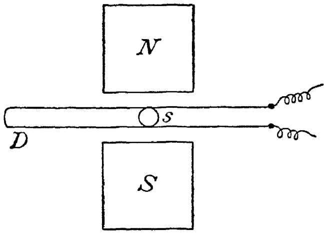


Fig. 84.—Principle of the loop oscillograph

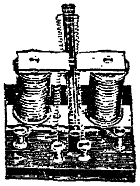


Fig. 85.—Simple loop oscillograph

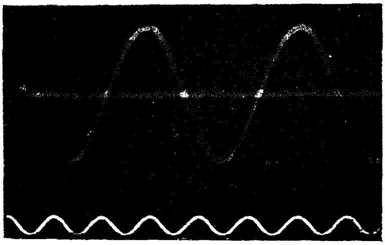


Fig. 86.—Oscillograph record

Fig. 85 shows a simple oscillograph. In fig. 86 we see the photographic record of the current variations from an alternating-current machine (p. 549). From the form of the curve certain conclusions may be drawn as to the nature of the current. The same figure also shows the record of the vibrations of a tuning fork of known frequency. By comparing the two curves the rate of alternation of the current can easily be determined. See also fig. 10, p. 575, and fig. 50, p. 600.

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* After Blondel (1893) in France, and Duddell (1897) in England.

The *string galvanometer* of W. EINTHOVEN [*] works upon the same basic principle. The movable conductor consists of a silvered quartz fibre, stretched like a violin string between the poles of an electromagnet. When a current is passed through the string, it is deflected at right angles to the magnetic lines of force. The arrangement is thus the same in principle as that of the experiment shown in fig. 63, p. 205. The magnitude of the displacement of the middle of the string is observed directly through holes in the pole-pieces of the magnet with the help of a microscope with micrometer eyepiece. The motion of the string can also be recorded photographically (*string oscillograph* of M. EDELMANN; compare figs. 41 and 42, pp. 497 and 498.

In order that an oscillograph may give reliable results, the natural period of the mirror or string must be very small compared with that of the alternating current under investigation; otherwise the inertia of the instrument will prevent its following the current variations sufficiently rapidly. Experiment has shown that satisfactory results can be obtained provided that the natural period of the moving part is at least about 30 times smaller than that of the current variations. Secondly, the natural oscillations of the instrument must be damped so as to be completely aperiodic (see Vol. II, p. 192); otherwise the record of the variations of the current will have a record of the natural vibrations of the instrument superimposed upon it. The damping must not be made too great, however; for this would prevent the instrument from following the current variations with sufficient rapidity. Hence the damping must be adjusted with great care. In the bifilar oscillograph of BLONDEL the correct degree of damping is attained by immersing the moving parts in a bath of oil of accurately adjusted viscosity.

**Ballistic [†] Galvanometer.**—A galvanometer only indicates a steady deflection provided that the current flowing through it is of constant strength. When a current of short duration (e.g. the discharge of a condenser) passes through the instrument, the movable part (needle or rotating coil) experiences only a sudden impulse. It is jerked out of its equilibrium position to an extent that depends upon the strength of the current. It then swings back to its original position, but overshoots this on account of its inertia and only comes to rest finally after performing numerous oscillations about its position of equilibrium. When the time $\tau$ during which the impulse acts upon an undamped system capable of performing oscillations is very small compared with a quarter of the period of these oscillations, i.e. when the impulse acts always in the same sense and only during the time that the moving system is very little displaced from its mean position, then the magnitude of the first displacement is independent of the duration of the impulse and directly proportional to the *quantity of electricity* passing through the galvanometer.

For if P is the force with which the system is displaced, the impulse theorem (Vol. I, p. 72) states that the velocity $v$ imparted to the system is proportional to the impulse $P\tau$. But the displacing force is proportional to the current strength I; therefore $v$ is proportional to $I\tau$. Now $I\tau$ is the quantity of electricity which passes through the galvanometer in the time $\tau$. Hence the velocity $v$ with which the system leaves its mean position is proportional to the total quantity of electricity which passes through the instrument, provided that the duration of the impulse is so short that the system may be regarded as practically undisplaced the whole time it is acting. But according to the laws of harmonic motion (Vol. I,

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[*] W. EINTHOVEN, Professor of Physiology in Leyden, published details of the string galvanometer in 1903.

[†] Gr., *bállein*, to throw.

§ 32, p. 146) the amplitude is always proportional to the velocity of passage through the mean position.

An instrument working upon this principle is called a *ballistic galvanometer.* Any galvanometer with a sufficiently long period can be employed in this way.

The proportionality of the amplitude of the first displacement to the quantity of electricity which has passed through a ballistic galvanometer can be demonstrated by the following experiment.

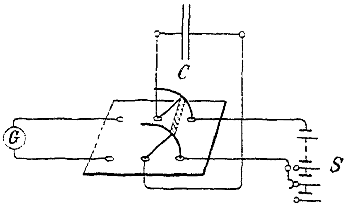

Fig. 87.—Calibration of a ballistic galvanometer

A condenser C (fig. 87) may be connected through a double-pole, double-throw switch to a source S of potential difference on the one hand and a ballistic galvanometer G on the other. To begin with, the condenser is put into connexion with the source of potential difference (position of switch as shown in the figure). The charging pressure can be read off on a suitable meter. The switch is now moved over to its other position and the condenser is discharged through the galvanometer. This is repeated for different pressures and it is observed that the amplitudes of the first displacements are proportional to the respective pressures, i.e. to the quantities of electricity discharged. By using a variable condenser and keeping the charging pressure constant, it can be shown that the amplitude of the first displacement is proportional to the capacity, i.e. once again to the quantity of electricity discharged. Thus a ballistic galvanometer may be used for measuring either quantities of electricity, capacities, or potential differences.

## CHAPTER VI

# Current and Potential Difference

### 1. Ohm's Law

When two points A and B which are at different electric potentials (i.e. between which there are electric lines of force) are joined by a conductor, the electric charges are enabled to follow the tension of the lines of force. As the opposite charges unite, the lines of force vanish; their density decreases and hence also the field strength and the potential difference. In order to maintain the original difference of potential, new lines of force must continually be produced, i.e. new charges equivalent to those which have united by passage through the conductor must continually be brought to the two points. Thus in accordance with our considerations on p. 148 (incompressibility condition) the current strength in the leads to the points A and B must be the same as that in the conductor joining them. The question now arises as to the nature of the possible dependence of this current strength (or the rate of combination of the opposite charges) upon the potential difference, and also upon the material, dimensions and shape of the conductor. We may investigate this question with the help of the arrangement shown in fig. 1.

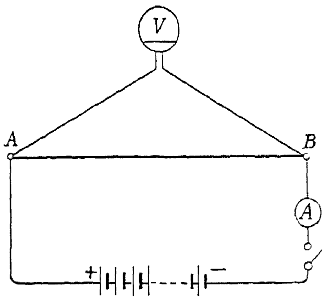

Fig. 1.—Determination of the resistance of a metallic conductor

The two binding screws A and B are connected with a source of electricity (e.g. with the poles of a battery of accumulators) and are thus kept at a certain difference of potential, which may be measured by means of a suitable voltmeter V (see § 6, p. 33) connected as shown in the figure. We now connect the two points A and B with a wire. The strength of the current flowing through the wire we measure by means of a coulombmeter A (§ 3, p. 152), observing the quantity of electricity which passes in a known time. This we do for a number of different potential differences between A and B, obtained by varying the number of accumulators in the battery. In this way we find that *the current strength I is proportional to the potential difference U.*

This may be expressed by means of the equation $I = GU$. In order to get an idea of the significance of the factor $G$, we keep the potential difference $U$ constant and vary the material and dimensions of the conducting wire, e.g. substitute an iron wire for a copper one or use wires of different lengths and thicknesses. We find that the above law holds for each of the wires, but that the proportionality factor $G$ has a different value in each case. Thus $G$ is a quantity characteristic of the particular conductor under consideration, in the sense that the greater $G$ is, the greater also the quantity of electricity passing through the wire per second for a given potential difference. We see that $G$ measures the capacity of the conductor to allow the electric charges to pass through it. For this reason it is known as the **electrical conductivity** of the conductor.

The above equation may also be written in the form

$$\mathbf{U = RI},$$

in which it expresses the fact that the potential difference between the ends of a conductor is proportional to the strength $I$ of the current flowing through it. The constant ratio $U/I = R = 1/G$ is called the **electrical resistance** of the conductor; for the greater $R$ is, the greater also must be the potential difference $U$ in order to drive the same quantity of electricity per unit time through the conductor. We may express this relationship as follows:

**For metallic and also for electrolytic conductors the product of the current strength and the resistance is equal to the potential difference producing the current. This is called Ohm's* law.**

*Note.*—Ohm's law is not the only relationship which may hold between current strength and potential difference in the process of electrical conduction. Other cases may arise, for example the current (then called the saturation current) may be independent of the potential difference, or the potential difference may fall off as the current strength increases. These cases will be treated later; the limits of validity of Ohm's law for metallic and electrolytic conductors will also be considered. It should be noted that according to Ohm's law the resistance of a conductor is independent of the potential difference and therefore, since field strength and potential difference are proportional, also independent of the field strength. Thus Ohm's law can be expressed in the following form:

*The resistance of a metallic or electrolytic conductor is independent of the field strength.*

**Unit of Resistance.**—As is seen from the equation given above, the unit of electrical resistance $R$ is fixed by the already defined units of potential difference and current strength. Measuring U in volts and I in amperes, the unit of resistance is the **ohm.** It is denoted by the symbol $\omega$. The unit 1,000,000 ohms is called 1 *megohm* and is denoted by $\Omega$.

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* GEORG SIMON OHM, born in 1787 at Erlangen, was teacher at the Military College at Berlin from 1826 to 1833; he then became Director of the Polytechnic School at Nuremberg; in 1849 he was appointed Extraordinary Professor and in 1852 Ordinary Professor at Munich University. He died at Munich in 1854. The law named after him appeared in a treatise *Die galvanische Kette, mathematisch bearbeitet,* which he published in 1827.

**The resistance of a conductor is 1 ohm when a potential difference of 1 volt applied at its ends causes a current of 1 ampere to flow through it.**

The unit of resistance thus defined is known as the *absolute ohm.* Ohm's law holds perfectly for metals within the range of conditions corresponding to ordinary practice.* Since further the resistance of a conductor remains very constant so long as the physical conditions are not changed, and since the construction of a metal conductor of any desired form presents no serious difficulties, it follows that standard resistances can be made with relative ease and exactitude. What is also very important, standard resistances can be copied accurately and easily. Attempts have therefore been made to determine the value of the ohm with very great precision. It has been defined internationally (1893) as follows:

**The legal (international) ohm is the resistance at 0° C. of a column of mercury 1 mm.$^2$ in cross-section and 106·3 cm. in length.†**

The chief reasons for choosing mercury as standard metal are as follows. It can easily be brought into a chemically pure state (e.g. by distillation); it is liquid and therefore free from mechanical strains; and its temperature can be kept constant more easily than is the case with other metals.

Since the value of the international ohm as defined appears to be too great by about 1 part in 2000, a factor $f$ must be introduced into the equation of Ohm's law when the resistance is expressed in international ohms instead of absolute ohms, viz.

$$U = fRI.$$

The value of $f$ differs very little from unity, however, and hence it need only be taken into account in very precise work.

## 2. Dependence of the Resistance of a Conductor upon its Material and Dimensions

Making use of the arrangement shown in fig. 1, p. 219, we investigate the current strength produced in various wires by the same potential difference. In this way we arrive at the following general relationships:

1. *For wires of the same material and the same cross-sectional area* A, *the resistance is proportional to the length* l.

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* Indications of deviations from Ohm's law have been observed by Bridgman in the case of very thin metal films at current densities of about 10 million amperes per cm.$^2$.

† According to measurements made by the German *Physikalisch-technische Reichsanstalt* the international ohm defined in this way is not equal, within the limits of error of the measurements, to the "absolute" ohm defined previously, but is about 1 part in 2000 too large (p. 196).

2. *For wires of the same material and the same length* l, *the resistance is inversely proportional to the cross-sectional area* A.

3. *Wires of the same length* l *and the same cross-sectional area* A *but made of different materials have different resistances, but the ratio of the resistances of two wires of the same dimensions is independent of the magnitude of these dimensions.*

These results may be combined in the equation:

$$R = \rho\, l/A.$$

Here $\rho$ is a factor of proportionality, whose value depends upon the substance of which the conductor is made and also upon the choice of the unit of resistance.

This factor $\rho$ is called the **specific resistance.** It is numerically equal to the resistance of a cubical block of the substance with 1 cm. edge. Its dimensions are [ohm.cm.].

Multiplying $\rho$ by $10^4$ we obtain the value of the resistance of a wire 1 m. long and with 1 $mm^2$. cross-sectional area. The values of $\rho . 10^4$ for a number of substances are given in Table V.

The reciprocal value $1/\rho = K$ is called the **specific electrical conductivity.** Its dimensions are [$ohm^{-1}$ . $cm.^{-1}$]. The value of K is numerically equal to the length of a conductor measured in centimetres which has a resistance of 1 ohm when its cross-sectional area is 1 $cm.^2$.

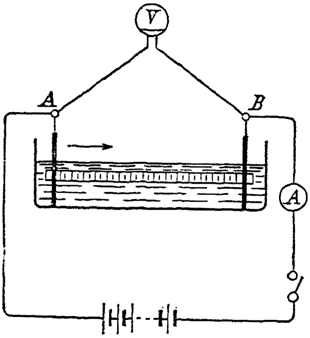

Fig. 2.—Resistance of an electrolyte

Multiplying this number by $10^{-4}$ (see Table V) we obtain the length in metres of a wire of 1 $mm.^2$ cross-sectional area which has a resistance of 1 ohm.

The relationships discussed here are not restricted to metallic conductors, but hold also for electrolytes. This can be shown easily with the help of the arrangement shown in fig. 2.

The circuit is essentially the same as that of fig. 1, p. 219, but the metal wire is replaced by a rectangular trough filled with copper sulphate solution, into which dip two copper electrodes. These are just large enough to form complete partitions across the trough and may be coated with paraffin wax upon their backs. The length $l$ of electrolyte between the electrodes can be varied by sliding the electrodes along, and the cross-sectional area A may be varied by filling up the liquid to different depths. The current strength and potential difference are measured as in fig. 1, p. 219. In order to avoid variations in the composition of the electrolyte, only weak currents must be used and care must be taken to keep the solution stirred. In this connexion see p. 285.

### TABLE V. SPECIFIC RESISTANCES AND CONDUCTIVITIES OF SOME SOLIDS AT 18° C.

$\rho \,.\, 10^4$ gives the resistance in ohms for a length of 1 m. and a cross-sectional area of 1 mm.$^2$. The last column ($\kappa \,.\, 10^{-4}$) gives the length in metres for a resistance of 1 ohm when the cross-sectional area is 1 mm.$^2$. Under $\beta$ (*temperature coefficient*) is the fractional alteration of the resistance for a rise of temperature of 1° C. between 0° C. and 30° C. A + sign indicates an increase, a — sign a decrease of resistance with rise of temperature.

| Substance | $\rho \,.\, 10^4$ | $\beta$ | $\kappa \,.\, 10^{-4}$ |
|---|---|---|---|
| Aluminium | 0·032 | +0·0036 | 31 |
| Antimony | 0·45 | +0·0041 | 2·2 |
| Lead | 0·21 | +0·0042 | 4·8 |
| Iron | 0·09–0·15 | +0·0045 to +0·006 | approx. 10 |
| Gold | 0·023 | +0·0040 | 43 |
| Copper | 0·017 | +0·0040 | 57 |
| Nickel | 0·08–0·11 | +0·0037 to +0·006 | approx. 10 |
| Osmium | 0·10 | +0·004 | 10 |
| Platinum (pure) | 0·108 | +0·0039 | 9·3 |
| Platinum (commercial) | 0·14 | +0·002 to +0·003 | approx. 7 |
| Mercury | 0·958 | +0·00092 | 1·0 |
| Silver | 0·016 | +0·0040 | 62 |
| Steel | 0·15–0·50 | +0·0052 to +0·006 | 6·5–2 |
| Tantalum | 0·15 | +0·0033 | 6·5 |
| Bismuth | 1·2 | +0·0042 | 0·8 |
| Zinc | 0·061 | +0·0037 | 16 |
| Carbon | 50–100 | −0·0002 to −0·0008 | approx. 0·02 |
| Graphite (from Siberia) | 11 | — | — |
| ALLOYS | | | |
| Brass | 0·07–0·09 | +0·0015 | approx. 12 |
| German silver (60% Cu, 21% Ni, 19% Zn) | 0·15–0·40 | +0·00022 to +0·0007 | 6·5–2·5 |
| Nickelin (58% Cu, 41% Ni, 1% Mn) | 0·42 | +0·00023 | 2·4 |
| Constantan (35–55% Ni, 65–45% Cu, 0–20% Zn) | 0·49–0·52 | 0 | approx. 2 |
| Manganin (70% Cu, 30% Mn) | 0·42 | 0 to +0·00003 | 2·4 |

**Practical Forms of Resistances.***—For practical measuring purposes use is made of **standard resistances**, whose external appearance and internal arrangement are shown in figs. 3 and 4 respectively. They consist of a wire made of a suitable alloy (e.g. manganin, constantan, nickelin, Table V, p. 223, and § 8, p. 243), mounted in a casing and kept at a definite temperature by means of an oil bath. The leads are two hook-shaped copper rods, whose resistance is negligibly small; they are hung in mercury cups during comparison measurements, thus establishing good metallic contact.

Use is also made of **resistance boxes** or **plug-in rheostats** (fig. 5), which consist of a set of wires of suitable resistances. The internal arrangement is shown in fig. 5 and also in fig. 30, p. 399. The two ends of each of the wires are connected to two thick blocks of metal of practically zero resistance attached to the insulating top plate of the box. The pairs of blocks can be put into conducting connexion by means of well-ground metal plugs. When the plugs are correctly inserted (they should be lightly pressed and slightly turned), the current entering and leaving by the respective binding screws of the resistance box flows directly through the plugs, which offer practically no resistance. But when a plug is removed, the current has to flow through the wire beneath it. By taking out one or more plugs any desired resistance may be inserted into the circuit.

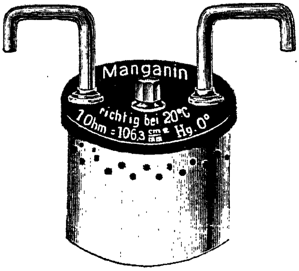

Fig. 3.—Standard resistance (the type adopted by the German Reichsanstalt).

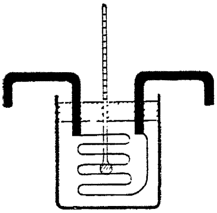

Fig. 4.—Internal construction of a standard resistance (diagrammatic).

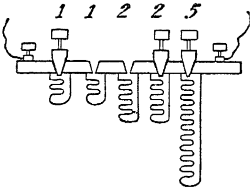

Fig. 5.—Diagram of a plug-in rheostat or resistance box.

Fig. 6 shows a **sliding resistance** and fig. 7 a **rotary resistance.** Both these types are especially employed for reducing current strength by the insertion of increased resistance or for regulating current strength to some desired value.

In the *sliding resistance* a nickelin wire (p. 244) is wound upon a cylindrical piece of slate or an enamelled iron tube so that the turns are close together but not in contact. The one end of the wire is attached to one binding screw of the instrument; the other is fixed to the slate cylinder. The other binding screw is connected to the metal rod mounted above the coil. This rod serves as a guide for the metal slider, which makes contact with the turns of wire. When the slider is at the extreme right-hand end (see figure), the whole of the resistance is included between the two binding screws; when the slider is at the extreme left-hand end (as in the figure), the current entering at one binding screw passes straight through the slider and the thick metal guide to the other.

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* The word "resistance" is unfortunately used with two different meanings. On the one hand it may stand for a certain *property* of the conductor with regard to electrical influences; on the other hand it may stand for the conductor itself, in so far as it is used to modify the current strength or potential difference by virtue of this property. The alternative word "rheostat" (Gr., *rheo*, I flow; rheostat = current regulator, i.e. a device for adjusting the strength of a current) may also be used in the latter sense, when the resistance in question is a variable one.

In the *rotary rheostat* any desired number of wire spirals may be inserted into the circuit by turning a switch-arm over a ring of contact studs. The upper ends of the spirals, which are carried upon a suitable frame, are connected together in pairs as shown in fig. 7; the lower ends are connected by thick copper wires with the contact studs of the switch.

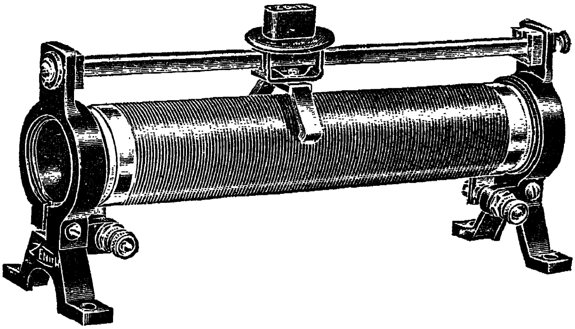


Fig. 6.—Sliding resistance

[The Zenith Electric Co. Ltd., London]

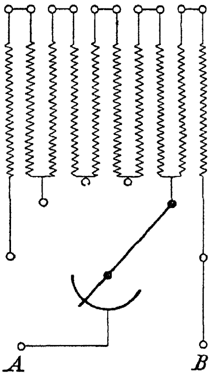


Fig. 7.—Rotary resistance

In this book the symbols given in fig. 8 will be used in diagrams of electric circuits.

Very high resistances are made of graphite, silicon carbide (carborundum), or compositions containing graphite or silicon. Very thin layers of metal, generally deposited upon mica and then sealed in glass tubes, are also employed.

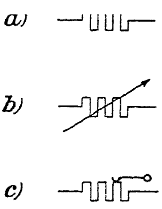


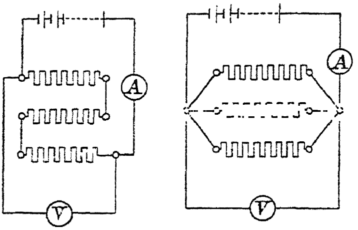
Fig. 8.—Diagram symbols for resistances: (*a*) general; (*b*) continuously variable; (*c*) variable in steps.

A line drawn with a lead pencil upon a ground-glass plate has a resistance of about a megohm, and is often useful. Contacts are made to the ends of the line by means of pieces of tinfoil held in place with clips or binding screws. For many purposes a resistance of the order of 100 megohms is required. A toluene solution of cadmium iodide in a U-tube furnishes a resistance of about this magnitude. Two wires, dipping into the respective limbs of the tube, are used as electrodes. The resistance may be varied by sliding the wires in or out so as to alter the distance between them.

## 3. Combinations of Conductors. Kirchhoff's Laws

We have already (p. 222) proved experimentally the formula $R = \rho l/A$. The proportionality of resistance to length can be expressed clearly by saying that, when two similar conductors are connected in series, the total resistance is double that of either of them. Thus resistances add together when connected in series. When two similar conductors are connected together in parallel, we may regard this as equivalent to doubling the cross-sectional area of either of them. The formula then shows that the reciprocals of the resistances add together; for $R_1 = R_2 = \rho l/A$, and $R = \rho l/2A$, whence:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$



Figs. 9 and 10.—Resistance of systems of conductors connected in series and in parallel.

Employing the circuits shown in figs. 9 and 10 (resistances in series and in parallel respectively), the resultant resistance of a combination of known resistances can easily be found by measuring current and pressure. In this way the following general relationships can be shown to hold (see p. 228):

1. *When several conductors are connected in series, the total resistance is equal to the sum of the individual resistances*, i.e.

$$R = \sum_{1}^{n} R_n.$$

2. *When several conductors are connected in parallel, the reciprocal value of the total resistance is equal to the sum of the reciprocal values of the individual resistances*, i.e.

$$\frac{1}{R} = \sum_{1}^{n} \frac{1}{R_n}.$$

Since $1/R$ is equal to G the conductivity, the second of the above cases can also be formulated as follows:

*When several conductors are connected in parallel, the total conductivity is the sum of the individual conductivities.*

The general case of any number of conductors connected together in any manner can be treated in the light of two fundamental rules first put forward in 1847 by KIRCHHOFF *.

---

* KIRCHHOFF (1824–87), was Professor of Physics at Heidelberg from 1854; from 1875 onwards at Berlin. The rules are not only valid for the electrical case, but hold also for the conditions of flow and pressure of an incompressible liquid in a branching system of pipes.

1. *The currents flowing to any point of a conductor being taken as positive and those flowing away from the point as negative, the sum of the current strengths is everywhere equal to zero,* i.e

$$\Sigma \mathbf{I} = \mathbf{0}.$$

This equation expresses the condition for a steady current (p. 611) at the point under consideration. As an example we may take the point in fig. 11 at which five wires join. If the two currents of strengths

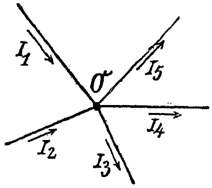


Fig. 11.—Branching-point of several conductors

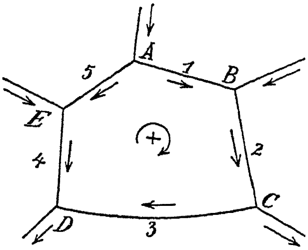


Fig. 12.—Loop of conductors without E.M.F.

$I_1$ and $I_2$ flow towards this point, and the currents of strengths $I_3$, $I_4$, $I_5$ away from the point, then the equation states that

$$I_1 + I_2 = I_3 + I_4 + I_5.$$

2. *In a closed electric circuit the sum of the electromotive forces* E *is equal to the sum of the products of the respective current strengths* I *and resistances* R *for the individual parts of the circuit,* i.e.

$$\Sigma \mathbf{E} = \Sigma (\mathbf{IR}).$$

The currents flowing in the clockwise sense are to be taken as positive, those in the anticlockwise sense as negative. Also the electromotive forces (p. 137) are to be reckoned as positive when they tend to cause the current to flow in the clockwise sense, and negative when they tend to cause it to flow in the opposite sense.

Let fig. 12 represent a portion of an arbitrarily branching electric circuit. The currents may be taken as flowing in any way to and from the branching points A, B, C, D, and E. Let ABCDEA be a closed conducting loop in the network of conductors. The arrows indicate the senses of the currents. We will assume that there is no source of electromotive force in the loop considered. Then we have:

$$I_1 R_1 + I_2 R_2 + I_3 R_3 - I_4 R_4 - I_5 R_5 = 0.$$

The significance of this equation can be seen as follows. The quantities $I_1R_1$, $I_2R_2$, &c., are the potential differences between the ends of the conductors AB, BC, &c. (see p. 220). Now by assumption there is no electromotive force in the loop ABCDEA. Hence the sum of the partial potential differences $I_1R_1$, $I_2R_2$, &c., must vanish, provided that there is no damming up of electricity at any point (incompressibility condition, see p. 148).

Now consider the case (fig. 13) in which sources of electromotive force have been inserted in the conductors BC, CD, and DE of the same system. The sense of the electromotive forces is represented by the dotted arrows. Then we have the equation

$$I_1R_1 + I_2R_2 + I_3R_3 - I_4R_4 - I_5R_5 = E_2 + E_3 - E_4.$$

The left-hand side of this equation is the sum of the potential differences for the conductors making up the closed loop; the right-hand side is the sum of the electromotive forces. The equation therefore expresses the obvious fact that, passing right round the loop and returning to the initial point, the total difference of potential is zero.

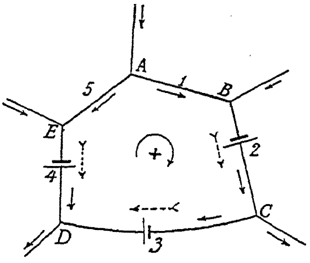


Fig. 13.—Loop of conductors with E.M.F.

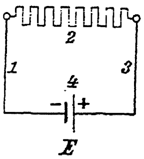


Fig. 14.—Resistanc in series

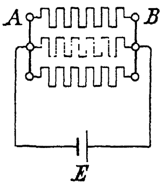


Fig. 15.—Resistances in parallel

**Applications.**—*Several Conductors connected in Series.* Let the electromotive force (or E.M.F.) E (fig. 14) be connected in series with the three resistances $R_1$, $R_2$, and $R_3$. Let the resistance of the cell producing the E.M.F. be $R_4$ (see p. 236). According to Kirchhoff's first law the current strength I must be the same throughout the whole circuit. By the second law we have

$$\begin{aligned} E &= IR_4 + IR_1 + IR_2 + IR_3 \\ &= I(R_4 + R_1 + R_2 + R_3). \end{aligned}$$

The resistance of the whole circuit being R, we have also $E = IR$. From the two equations it follows then that

$$\mathbf{R} = \mathbf{R}_4 + \mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_3.$$

Thus we obtain the result already given above, namely:

*The total resistance of a number of conductors connected in series is equal to the sum of their individual resistances.*

*Several Conductors connected in Parallel.*—The circuit in this case consists (fig. 15) of the cell with E.M.F. equal to E connected by leads to two points A and B, between which three resistances $R_1$, $R_2$, and $R_3$ are connected in parallel. Let the resistance of the cell together with its leads to A and B be $R'$. Let the total resistance of the three resistances connected in parallel be R. Then according to Kirchhoff's first law,

$$I = I_1 + I_2 + I_3,$$

where I is the total current strength through the cell and leads and $I_1$, $I_2$, $I_3$ are the partial current strengths through the resistances $R_1$, $R_2$, and $R_3$ respectively. Applying Kirchhoff's second law to the circuits formed by the cell, leads and the three resistances in order we obtain respectively:

$$E = IR' + I_1R_1 = IR' + I_2R_2 = IR' + I_3R_3.$$

Now consider the three resistances to be replaced by one equivalent resistance R and apply Kirchhoff's second law again. In this case we have

$$E = I(R' + R).$$

From these equations it follows that

$$I_1 = IR/R_1, \quad I_2 = IR/R_2, \quad \text{and} \quad I_3 = IR/R_3.$$

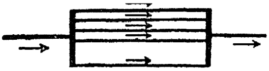


Fig. 16.—Addition of reciprocal resistances

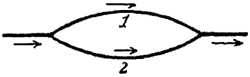


Fig. 17.—Current strengths in two conductors connected in parallel.

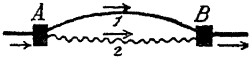


Fig. 18.—Adjustment of resistance by means of a shunt.

Substituting these values in the equation of the first law we have

$$I = IR/R_1 + IR/R_2 + IR/R_3,$$

whence

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

This is the result already given above, namely:

*The reciprocal value of the total resistance of a number of conductors connected in parallel is equal to the sum of the reciprocal values of their individual resistances.*

In the particular case of *n* equal resistances each of R′ connected in parallel, the total resistance R is given by

$$\frac{1}{R} = \frac{n}{R'} \quad \text{or} \quad R = \frac{R'}{n}.$$

We can obtain a mental picture of the meaning of this equation by imagining the *n* similar resistances to consist of *n* wires (see fig. 16). The combination can then obviously be replaced by a single wire of the same length but with *n* times the cross-sectional area. In this way the equation is referred back to the result already given on p. 222.

*The partial currents in two conductors connected in parallel* (*fig.* 17) *are in* [illegible] *respective resistances,* i.e.

$$\mathbf{I_1 : I_2 = R_2 : R_1}.$$

This can easily be proved by applying Kirchhoff's second law to the closed loop consisting of the conductors 1 and 2 (fig. 18). Conversely the experimental verification of the validity of this relationship can be taken as a proof of the law.

*Note.*—By connecting a large resistance (a so-called *shunt*) in parallel with a smaller resistance, the total resistance can be decreased by a small amount. Use is made of this when it is desired to adjust the resistance to a certain value, as in the construction of resistance boxes (fig. 5, p. 224). For example, in making a resistance accurately equal to 1 ohm a thick wire of rather more than 1 ohm resistance is soldered to two massive metal blocks A and B (fig. 18) which serve as leads. The two blocks are then joined by another long thin wire, the shunt, which is so chosen that the total resistance is still slightly greater than 1 ohm. The length of the shunt is then diminished until the desired resistance is attained.

## 4. Fall of Potential along a System of Conductors

The potential difference U between the end points of a system of conductors containing no sources of E.M.F. is given by Ohm's law. We will now consider the question of the magnitude of the potential difference between any two arbitrary points of the system.

If the current is constant, the answer to this question is obviously obtainable by applying Ohm's law. The potential difference between the points under consideration is equal to the product IR, where R is the corresponding resistance and I the current strength. On account of their importance we will consider more closely the problems arising in this connexion.

**Potential Distribution along a Homogeneous Conductor of Constant Cross-section.**—In accordance with Ohm's law, it is to be expected that the potential (measured relative to one end of the conductor) will increase linearly with the distance from this end; for the resistance (and therefore also the potential difference, since I is constant by the incompressibility condition) is in this case proportional to the length.

*Experiment.*—To investigate this problem experimentally we may use the arrangement shown in fig. 1, p. 219; but instead of connecting the voltmeter with the ends of the tightly stretched wire AB, we connect it to two points C and D (see fig. 19). The wire may be stretched over a scale. Then by measuring the distance between the points C and D and the corresponding pressure, we obtain the following result:

*The potential difference between two parts of a uniform conductor is directly proportional to the distance between them.*

A pictorial idea of the potential fall along the conductor may be obtained by plotting the distances $l$ from one end of the conductor as abscissæ and the corresponding potentials, relative to this end, as ordinates. The curve so obtained represents the nature of the potential fall along the conductor. In the case of a uniform conductor (fig. 20) a straight line is obtained. If the point A of the conductor be earthed, i.e. brought to zero potential, the ordinates give the actual potentials of the corresponding points.

The more complicated case of the potential fall throughout a number of different conductors connected in series will be treated in § 6, p. 236.

**Potential Distribution in Branching Systems of Conductors.**—In this case the conditions are more complicated, since the current strengths in the different portions of the system are different. The problem is of especial practical importance in dealing with the loading of an extensive network of electric mains.

The electrical devices used by the consumers of current (e.g. motors, incandescent lamps, cooking apparatus, &c.) are constructed so as to operate most efficiently at a certain applied *pressure*.* Hence it is necessary to furnish a constant pressure at the points of the mains where these devices are to be inserted; this pressure must be independent of the loading of the rest of the mains network. The sources of electricity are such that they maintain an (approximately) constant pressure at the starting-points of the mains system. Hence when a current-consuming apparatus is switched on at some point, the resistance of the whole network is reduced (all the consuming devices being connected in parallel with the mains), and the current strength is increased. In order, therefore, to keep up the initial pressure, the sources of electricity must furnish more current. They are generally so constructed that they can cope with any actually occurring current load. Cases do arise, however, in which they are unable to supply sufficient electricity per unit of time. The combination of the opposite charges then occurs more rapidly than their production, more lines of force disappear than are produced, and the field strength and the pressure both drop. If the resistance of the mains network remains constant, i.e. if the consuming devices remain switched on, the current strength drops off in accordance with Ohm's law until it becomes equal to that supplied from the sources. These relationships will be considered in greater detail in § 6, p. 236.

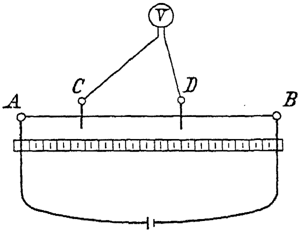


Fig. 19.—Potential difference between any two points of a conductor

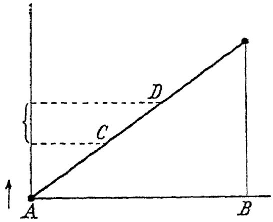


Fig. 20.—Fall of potential along a homogeneous conductor. (Lengths, horizontal; potentials relative to A, vertical.)

Quite similar conditions involving the insufficiency of the supply of electric charges in comparison with their reunion also occur when the current has to pass through leads of considerable resistance before it arrives at the actual consumer. The case is analogous to that of a system of water mains in which the source does not supply the water rapidly enough or in which the passage of the water is restricted by the pipes being too narrow or too long.

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* The terms *pressure*, *tension*, *potential difference* (P.D.), *electromotive force* (E.M.F.), as used by electrical engineers, are practically synonymous. When the pressure is measured in volts, the word *voltage* is frequently used.

*Experiment.*—The facts can be investigated experimentally with the arrangement shown in fig. 21. The two binding screws 1 and 1′ are connected by leads of negligibly small resistance to the poles of a large battery of accumulators. To each of the binding screws is also connected a thin wire (e.g. florists' wire) made of iron, which thus has a considerable resistance. These two wires (1234 and 1′2′3′4′ in the figure) are arranged parallel to one another, and are connected to one another at various points through current-consuming devices (in the figure three incandescent lamps $L_1$, $L_2$, $L_3$, which burn brightly at normal pressure, and an arc lamp A. The leads to these current consumers are of thick copper and have practically no resistance.

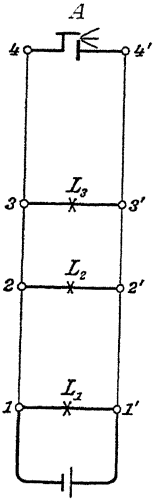

Fig. 21.—Potential fall under heavy load and with inadequate supply.

The experiment is carried out as follows. We first switch on the incandescent lamp $L_2$. It burns fairly brightly. We now switch on $L_3$ and notice that $L_2$ no longer burns so brightly as before. We now light the arc lamp A. At the moment of starting of the arc $L_2$ hardly glows. When $L_3$ and the arc lamp A are switched off, however, $L_2$ burns once more with its original intensity. When $L_2$ and $L_3$ are identical lamps and are switched on together, $L_2$ is brighter than $L_3$. The lamp $L_1$, on the other hand, which is connected directly through thick copper leads to the battery, burns with normal brightness under all circumstances, provided that the battery itself can provide a great enough current.

These phenomena can be understood at once in the light of Ohm's law. In the first place, it follows at once from Kirchhoff's first rule (p. 227), applied to the junctions 1 and 1′, that the current in the section 21 is equal to that in 1′2′, say $I_2$. Let the potentials at 1, 2, 2′, 1′ be $u_1$, $u_2$, $u_2'$, $u_1'$; and let $r_2$, $r_2'$ be the resistances of 21, 1′2′. Then, by Ohm's law,

$$u_2 - u_1 = r_2 I_2,$$

and

$$u_1' - u_2' = r_2' I_2.$$

Therefore, by addition,

$$(u_1' - u_1) - (u_2' - u_2) = (r_2 + r_2') I_2.$$

If then we write $u_1' - u_1 = U_1$, $u_2' - u_2 = U_2$, $r_2 + r_2' = R_2$, and similarly for the other sections, so that $U_1$ is the E.M.F. of the battery (considered constant), we have:

$$R_2 I_2 = U_1 - U_2;\; R_3 I_3 = U_2 - U_3;\; R_4 I_4 = U_3 - U_4.$$

Hence

$$U_2 = U_1 - R_2 I_2;\; U_3 = U_1 - R_2 I_2 - R_3 I_3;$$
$$U_4 = U_1 - R_2 I_2 - R_3 I_3 - R_4 I_4.$$

Thus the effective potential difference becomes smaller as we get farther along the leads from the battery. Its value also decreases as the strength of the current flowing in the circuit is increased.

*The loss of potential difference in any section of a circuit is equal to the product of the resistance of the section and the strength of the current flowing through it.*

**Potentiometers.**—The fall of potential along an electric circuit is very often used for the purpose of tapping off any desired smaller potential difference from a larger one. The arrangement used is shown in fig. 22. Let R be the resistance of the uniform conductor joining A and B and $R/n$ that of the portion between C and B. Then the potential difference between C and B is also $1/n$ of that between A and B. This potential difference between C and B may be employed as E.M.F. for some arrangement D, provided that the resistance of D is very great compared with that of CB. Thus, for example, suppose that it is desired to obtain a potential difference of 0·001 volt with the aid of an accumulator whose E.M.F. is 2 volts. In this case the resistance of AB might be chosen to be 2000 ohms and a tapping taken off at a point C such that the resistance of CB is 1 ohm. If the resistance of the arrangement D, for which the small voltage is desired, is small, i.e. of the order of magnitude of 1 ohm, then in order to fulfil the necessary conditions the resistance of CB must be made correspondingly smaller (say 0·01 ohm) and the resistance of AB reduced in the same proportion (i.e. to 20 ohms). In using a potentiometer care must be taken that the battery is short-circuited through the main resistance AB and that the current through it is great in comparison with that required through the apparatus D.

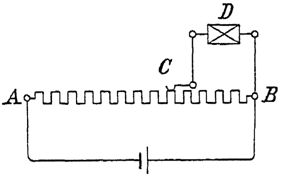

Fig. 22.—Potentiometer circuit

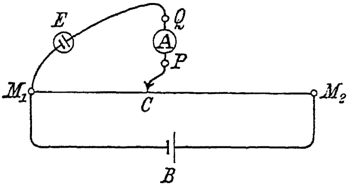

Fig. 23.—Compensation method for measuring E.M.F.

**Measurement of Electromotive Forces by Compensation.**—The possibility of tapping off any desired potential difference from the potential fall along a conductor carrying a current is made use of in Poggendorff's compensation method for the measurement of E.M.F. The method is as follows (see fig. 23).

$M_1M_2$ is a uniform metal wire, through which passes a steady current from the battery B. One pole of the electromotive force E to be measured is connected to the end $M_1$ of the wire; the other pole in connected through the current indicator A to a sliding contact C. The indicator A shows no current when there is no potential difference between its terminals P and Q. In order to attain this, the sliding contact C must be moved along the wire until the potential difference between $M_1$ and C (i.e. between $M_1$ and P) is just equal to the E.M.F.

to be measured between $M_1$ and Q. It is obvious that the senses of both potential differences must be the same as seen from $M_1$; that is to say, $M_1$ must be connected to similar poles of E and B. The potential difference between $M_1$ and C may be read off directly upon a scale placed beneath the wire $M_1M_2$; for the potential fall is linear, i.e. if $M_1C:M_1M_2 = a:l$, we have $E:E_B = a:l$ or $E = E_B a/l$, where $E_B$ is the E.M.F. of the battery B.

When actually carrying out measurements of this type, it is usual to employ accumulators, whose E.M.F. is practically constant and equal to 2 volts. If the plates of the accumulators are sufficiently large and if the measuring wire is sufficiently thin, the decrease in the E.M.F. of the battery caused by the current can be neglected. In this case each centimetre of a measuring wire 1 metre long corresponds to a drop of potential of 0·02 volt.

## 5. Application of Ohm's Law for the Measurement of Potential Difference and Current Strength

Ohm's law enables us to use a current-measuring instrument to measure potential difference and vice versa.

**Use of a Current-measuring Instrument to measure Potential Difference.**—The principle is as follows. We measure the strength of the current flowing through a conductor; then, knowing its resistance, we calculate the potential difference between its ends by means of Ohm's law. Suppose we desire to measure the potential difference between the two points A and B of an electric circuit (see fig. 24). We connect the points by a wire of known resistance, inserting also a current measurer. In many cases the resistance of the instrument itself will suffice. The total resistance is so chosen that the pointer indicates directly upon a scale the pressure applied to the meter (including the resistance, if any).

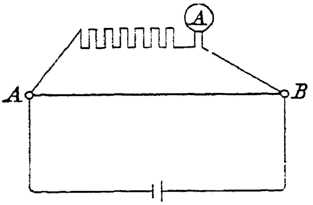

Fig. 24.—Arrangement of a current-measuring instrument to measure the potential difference between the points A and B.

*Example.*—Suppose that a galvanometer with an internal resistance of 100 ohms gives a deflection of one scale division when a current of 1/1000 milliampere ($= 10^{-6}$ amp.) is flowing through it. Then this deflection corresponds to an applied pressure of $10^{-4}$ volt; for by Ohm's law $U = IR$. If it is desired to make this deflection of one scale division correspond to a potential difference of $10^{-2}$ volt, a resistance 99 times that of the galvanometer (i.e. 9900 ohms) must be connected in series with it; for then, if the total potential difference applied to resistance and galvanometer is $10^{-2}$ volts, that actually applied to the galvanometer alone is $10^{-4}$ volt, which produces the desired deflection. If one scale division of the galvanometer is to correspond to 1 volt, the resistance inserted in the series must be 9999 times greater than that of the galvanometer, i.e. 999,900 ohms.

**Extension of the Measuring Range of Ammeters by means of Shunts.**—By means of *shunts* (i.e. resistances connected in parallel) every current-measuring instrument can be adapted for the measurement of current strengths greater than those for which it is otherwise suitable. Only a certain known fraction of the larger current is allowed to pass through the ammeter (fig. 25). Thus the galvanometer mentioned in the above example may be made to give a deflection of one scale division for a total current of $10^{-3}$ amp. instead of $10^{-6}$ amp. For this purpose the terminals of the instrument are connected by a shunt whose resistance is 1/999 of that of the galvanometer itself. Then according to p. 229 the current strengths through the shunt and the galvanometer are in the inverse ratio of the respective resistances, i.e. 999 : 1. Thus of the total current 999/1000 goes through the shunt and 1/1000 through the instrument.

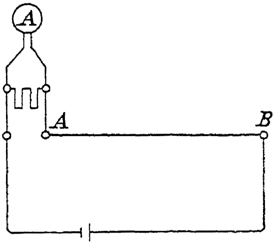

Fig. 25.—Extension of the range of measurement of a current-measuring instrument.

**Electrometer as Measurer of Current.**—This arrangement is seldom employed, since the difficulties of insulation make the use of ordinary current-measuring instruments much more convenient. The method is sometimes of advantage, however, for very weak currents in circuits of high resistance. The principle is to measure the potential difference between two points of the circuit, the corresponding resistance being known. Then application of Ohm's law gives the current strength between the two points and, since the electrometer takes no current, the current strength in the whole circuit.

**General Rules.**—The following points must be borne in mind. The introduction of the measuring instruments must have the least possible influence upon the process to be measured. In measuring *current strengths* the increase of resistance $\Delta R$ occasioned by the introduction of the measuring instrument must be small in comparison with the resistance R of the remainder of the circuit; otherwise the current flowing through the instrument, i.e. $I = U/(R + \Delta R)$, will differ considerably from the actual value $U/R$ which it is desired to measure. This condition can easily be fulfilled by employing a suitable shunt. In measuring the *potential difference* between two points A and B, however, the resistance of the instrument introduced in parallel must be *very great* in comparison with the resistance already between A and B; otherwise the potential difference will be considerably reduced. Care must be taken to see that this condition is fulfilled in the case of voltmeters, whose resistance may not always be sufficiently great for the proposed purpose. Mistakes may be avoided by leaving the voltmeter connected throughout the other measurements or observations which have to be made upon the process under investigation, and then reading off the pressure last of all; the fall of pressure due to the presence of the voltmeter is then present throughout the whole investigation, and the pressure read off is that at which the observed process actually took place.

Thus we have the following general rules: *Current measurers have small resis-*

*tance and are inserted in the main circuit. Measurers of potential difference have great resistance and are connected in parallel to the section of the circuit under investigation.* (Compare figs. 24 and 25, above.)

## 6. Methods of Connecting Up Sources of Current

The pressure losses in a system of conductors can be regarded as due to an insufficient passage of electricity on account of the resistance of the conductors. Analogous conditions occur in systems of water pipes not involving mechanisms producing differences of level. Insufficient passage of electricity may also be due to the incapacity of the sources to supply the necessary quantities of electricity with sufficient rapidity. This incapacity may be occasioned by the existence of too great an *internal* resistance in the sources themselves, or by some other process which limits the supply of electricity. We shall now go on to consider these factors. The problem is essentially the same as the following: What are the factors limiting the supply of a pump and how should pumps be arranged so as best to supply the desired flow of liquid or to maintain the desired pressure in a system of water mains? Before going farther into this problem, we will first consider in more detail the behaviour of sources of electric current when actually in operation.

**E.M.F. on Open and Closed Circuit.** — The potential difference between the poles of a source of current, in particular a galvanic cell, is called its *electromotive force* or E.M.F. When the poles are connected by a conductor the cell is said to be on *closed circuit*; when there is no conducting connexion, the cell is said to be on *open circuit* (p. 41).

*Experiment.*—Having verified that the E.M.F. of a chromic acid cell on open circuit is 1·8 volt, we connect the two poles by means of an iron wire 1 m. long and 0·2 mm. thick. The E.M.F. is now found to be only 1·5 volt. We then shorten the iron wire to $\frac{1}{2}$ m., and find that the E.M.F. falls to about 0·8 volt. As the wire is made still shorter, the E.M.F. decreases still more, until, when the two poles are put into direct contact, it vanishes altogether. Thus the smaller the resistance between the poles, the smaller also the E.M.F. When the circuit is broken, the E.M.F. (now on open circuit) reassumes its original value of 1·8 volt, provided that the short-circuiting was not of such long duration as to cause serious chemical changes of the electrodes and the electrolyte.

**Internal Resistance of Cells and Accumulators.**—We make use of an arrangement corresponding exactly to that (fig. 2, p. 222) previously employed in investigating the resistance of an electrolyte. In this case, however, we use chromic acid as the liquid and plates of zinc and carbon (with their backs paraffin waxed) as electrodes. The potential difference between these plates is measured with an instrument (e.g. a string electrometer) which takes no current. No other source of current is required in this case; in its place (fig. 26, compare fig. 2, p. 222) we insert a variable resistance in the external part of the circuit.

Keeping constant the resistance R of the external part of the circuit, we find that the E.M.F. between the poles of the cell decreases as the electrodes are moved farther apart. As the distance between the electrodes is decreased, the E.M.F. increases and becomes more and more nearly equal to the E.M.F. on open circuit.

We now vary the experiment, this time keeping constant the external resistance R and the distance between the electrodes, but varying the effective area of the latter by filling the containing vessel to different levels with the electrolyte. In this way we find that the E.M.F. of the cell on *closed* circuit (i.e. when delivering a current) increases as the cross-sectional area of the liquid column between the electrodes is made greater.

Thus, as must be expected, the electrolyte of the cell behaves like a resistance connected between the electrodes. Since this resistance lies within the source of current, it is called the *internal resistance.*

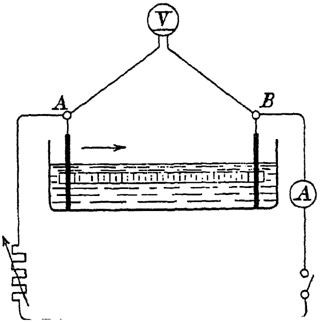

Fig. 26.—Internal resistance of a cell

We can get a pictorial representation of the potential fall by plotting distances (measured from electrode to electrode through the electrolyte and then along the conductor forming the external resistance) as abscissæ and corresponding E.M.F.s (measured between the carbon electrode and the zinc electrode) as ordinates (see fig. 27). The E.M.F. of the cell on open circuit is represented by the dotted ordinate E. Then $C_0A_0$ represents the E.M.F. $E_0$ of the cell when the electrodes are very close together and connected by a large external resistance. When the length of the liquid column between the electrodes is increased to an extent corresponding to the point C and the length of the external wire is made equal to CO = R, the fall of potential is represented by the broken line $A_0BO$. The E.M.F. of the cell (now on closed circuit) is given by CB = *e*. The potential fall within the electrolyte is represented by $A_0B$, the fall in the external circuit by BO.

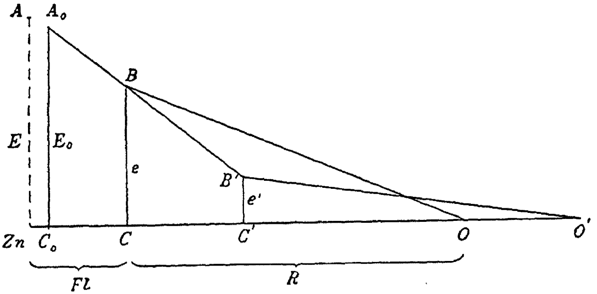

Fig. 27.—Fall of potential inside and outside of a cell

If now the carbon electrode is separated still farther from the zinc electrode to a position corresponding to the point C′, the external resistance being kept the same as before, we get the broken line $A_0B'O'$. The E.M.F. between the electrodes is now only $C'B' = e'$.

The influence of the internal resistance upon the current strength can be compensated, of course, by suitable adjustment of the external resistance. Thus we may arrange that the current strength, as indicated by the measuring instrument of the apparatus shown in fig. 27, is always constant. In order to attain this, we find that a variation of the distance between the electrodes must be compensated by a corresponding displacement of the slider of the rheostat. As the electrodes are moved farther apart, the resistance of the external part of the circuit must be reduced, the *increase* of the distance between the electrodes being approximately proportional to the *decrease* of the external resistance. Thus a decrease of the distance between the electrodes has the same effect upon the current strength as a decrease of the external resistance.

We must not deduce from this that the internal resistance of a cell is accurately proportional to the distance between its electrode plates. As a matter of fact this is not the case. The internal resistance of a cell is actually made up of two parts, the one being proportional to the distance between the electrodes but the other (at the same current strength) being constant. The latter part of the internal resistance is associated with the passage of the current out of the electrodes into the electrolyte. Thus the internal resistance $R_i$ of a cell may be written as the sum of two resistances $R_g$ and $R_u$. The former varies with the area A of the electrode plates and the distance $l$ between them, being given by the equation

$$R_g = \alpha l/A,$$

where $\alpha$ is a constant. The latter term $R_u$ is dependent only upon the strength of the current through the cell, decreasing as the current strength decreases. When the current strength is very small, $R_u$ may be assumed to be zero.

**Strength of the Current furnished by a Galvanic Cell.**—We may apply Ohm's law to a closed electric circuit including a source of electromotive force, provided that we take account of the internal resistance of that source (p. 228). The simplest case is that of a cell of E.M.F. E and internal resistance $R_i$ with its poles connected by a conductor of resistance $R_e$. The current strength I is then given by

$$I = \frac{E}{R_i + R_e}.$$

In this equation E is the E.M.F. of the cell on *open* circuit.

When several similar cells are connected **in parallel** (fig. 50, p. 45), the E.M.F of the battery is equal to that of a single cell. The battery acts like one cell with plates equal in area to the sum of the areas of the plates of the individual cells. Thus in the case of $n$ cells connected in parallel the internal resistance of the whole battery is only $1/n$ of that of a single cell. Hence the current strength $I_p$ when the external resistance is $R_e$ is given by

$$I_p = \frac{E}{\frac{1}{n}R_i + R_e},$$

where E and $R_i$ are respectively the E.M.F. and internal resistance of each of the cells.

In the case where $R_e$ is large in comparison with $R_i/n$, the first term of the denominator on the right-hand side of the equation may be neglected in comparison with the second, and the expression simplifies to

$$I_p = E/R_e.$$

From this we see that, when the external resistance is large, the current strength from a battery of cells connected in parallel is the same as that which would be given by a single cell.

On the other hand, when $R_e$ is negligibly small in comparison with $R_i/n$, the expression for $I_p$ becomes

$$I_p = nE/R_i.$$

Thus in the case where the external resistance is small the current strength is increased in proportion to the number of cells connected together in parallel to form the battery.

The E.M.F. of a battery of cells connected **in series** (fig. 49, p. 44) is equal to the sum of the E.M.F.s of the individual cells. The internal resistances of the cells also add together. Hence in the case of $n$ cells we may write

$$I_s = \frac{nE}{nR_i + R_e}.$$

When $R_e$ is large compared with $nR_i$, this simplifies to

$$I_s = nE/R_e.$$

Thus in this case the current strength is directly proportional to the number of cells in the battery.

When $R_e$ is small in comparison with $nR_i$, we obtain

$$I_s = \frac{nE}{nR_i} = \frac{E}{R_i}.$$

From this it is seen that, when the external resistance is small, no advantage is obtained by using a number of cells in series rather than a single cell.

In the case of **series-parallel arrangement,** in which $m$ groups each of $n$ cells in series are connected in parallel, the E.M.F. of the whole battery is $E' = nE$ and the internal resistance $R_i' = nR_i/m$. Hence the current strength is

$$I_{sp} = \frac{nE}{\frac{n}{m}R_i + R_e}.$$

The greatest possible current for any given case is obtained by suitable choice of $m$ and $n$. A simple calculation shows that for a given number of cells and a given external resistance the condition for maximum current is that the internal resistance of the whole series-parallel battery shall be as nearly as possible equal to the external resistance of the circuit.

## 7. Measurement of Resistance

**Resistance Measurements with the help of Ohm's Law.**—The unknown resistance R may be inserted into an electric circuit, and the potential difference U between its ends and the current strength I flowing through it may be measured. Then $R = U/I$.

*Method of Substitution.*—Alternatively the unknown resistance may first be inserted in a circuit fed by a battery of constant E.M.F. and then replaced by a variable resistance of known value. The latter is adjusted until the current strength in the circuit is the same as when the unknown resistance was present. The value in ohms read off upon the variable resistance is then the desired magnitude of the resistance to be measured.

These methods give only moderate accuracy. Numerous methods have been proposed for the more precise measurement of resistance. We cannot discuss them all in detail here, but will confine our attention to the following one, which is capable of wide application and which is sufficiently accurate for most purposes.

**Wheatstone's* Bridge.**—The two metal blocks A and B (fig. 28) are connected to the poles of a battery of electromotive force E.

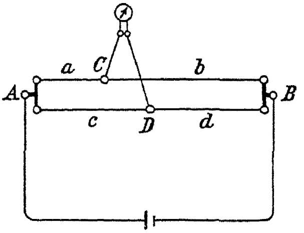

Fig. 28.—Fall of potential along two wires connected in parallel. Principle of Wheatstone's bridge.

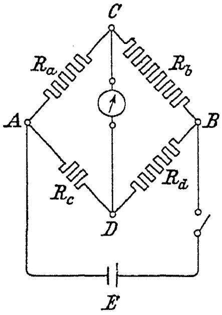

Fig. 29.—Arrangement of resistances in Wheatstone's bridge

In addition A and B are joined by the two wires ACB and ADB, along which there is therefore a certain uniform potential drop U dependent upon the magnitude of E, the internal resistance of the battery and the resistance of the wires and leads. If the point C is such as to divide the one wire into the parts $a$ and $b$, the potential difference $U_1$ between A and C is given by $U_1 : U = a : (a + b)$. Similarly, if the point D divides the other wire into the parts $c$ and $d$, the potential difference $U_2$ between A and D is given by $U_2 : U = c : (c + d)$. In general, therefore, $U_1$ and $U_2$ are different. Hence if they are touched with two wires leading to the poles of a current indicator, this instrument will in general show a deflection; the magnitude and sense of this deflection indicate the nature of the potential difference between C and D. By sliding the contact points C and D along the wires it is possible to find pairs of points such that the current-indicating instrument shows no deflection. There is no potential difference between such a pair of points, i.e. $U_1 = U_2$, or

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* Charles Wheatstone (1802–75), English physicist and the practical founder of modern telegraphy. He devised the bridge circuit in 1843.

$$U . \frac{a}{a+b} = U . \frac{c}{c+d}.$$

We see that U vanishes from this equation, so that the position of the pair of points is quite independent of the magnitude of the total potential difference between A and B. Written in another form, the equation states that

$$a : b = c : d,$$

a result which can easily be verified by direct measurement.

The whole arrangement (and more particularly also the connecting wire, with current indicator, joining C and D) is known as *Wheatstone's bridge.*

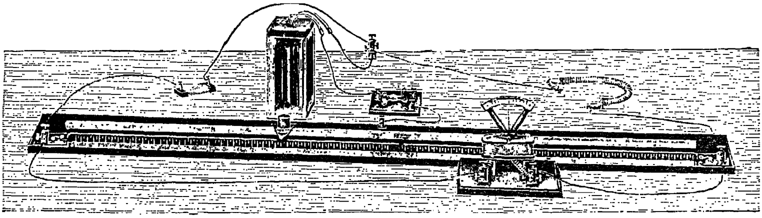

Fig. 30.—Apparatus for measuring resistance by Wheatstone's bridge method

We may now replace the four portions of wire $a$, $b$, $c$ and $d$ by resistances $R_a$, $R_b$, $R_c$ and $R_d$ (fig. 29). The condition for no current through the bridge-wire remains the same as before. Hence we can write in general:

*The condition for no current through the bridge-wire is*

$$R_a : R_b = R_c : R_d.$$

Wheatstone's bridge is especially useful for the measurement of unknown resistances. For if three of the four quantities be known in the relation $R_a : R_b = R_c : R_d$, the fourth can at once be calculated. Indeed, it is only necessary to know one of the resistances, say $R_a$, and the ratio of the two resistances on the other side of the bridge. A convenient arrangement is therefore to use a uniform wire stretched over a measuring scale as the conductor ADB. The ratio $R_c : R_d$ is then equal to $c : d$, which can be read off at once from the scale.

The bridge circuit may be modified by interchanging the positions of the battery and the current-indicating instrument. An actual apparatus connected up in this way is depicted in fig. 30, the corresponding circuit being shown diagrammatically in fig. 31. The unknown resistance is X, the known resistance R. The sliding contact is pushed along the measuring wire, which is stretched over a scale, until the galvanometer shows no deflection. Then we have $X = Rc/d$.

The measuring wire is often wound upon a cylinder made of insulating material (fig. 32). In this way a long wire can be accommodated in a relatively small space.

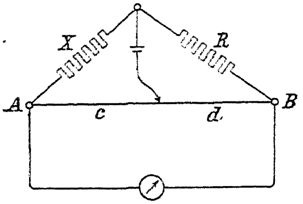

Fig. 31.—Interchange of battery and current-indicating instrument in Wheatstone's bridge.

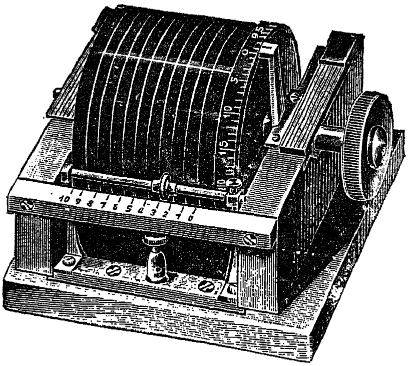

Fig. 32.—Drum measuring wire for Wheatstone's bridge method

The Wheatstone's bridge method is also used for the **determination of the resistance of electrolytes.** A complication is introduced here, however, by the fact that a direct current through the electrolyte gives rise to chemical changes which affect the results (see § 3, p. 286, polarization of electrodes). This difficulty can be overcome by using a current of rapidly alternating sense, so that the changes produced are continually reversed. Such currents can be obtained from an induction coil (p. 516), or better from a valve oscillator (p. 599), which therefore takes the place of the battery shown in figs. 29 and 31 above. As null-instrument it is necessary, of course, to use a device which indicates alternating currents, e.g. an electrodynamometer (p. 215) or, simpler and very sensitive, a telephone. This gives an arrangement of the kind shown in fig. 33. The vessel or

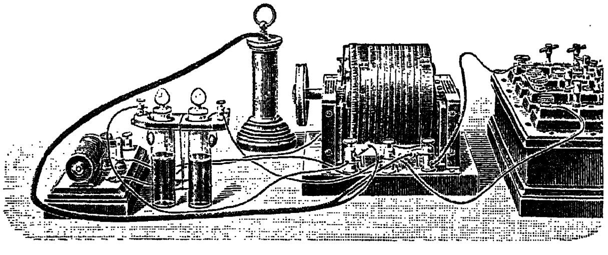

Fig. 33.—Bridge apparatus for measuring the resistance of electrolytes

cell containing the electrolyte usually has one of the shapes shown in fig. 34. The electrodes are made of platinized platinum (see p. 286).

In the case of electrolytes it is the specific and not the absolute resistance that is of interest (p. 222). The cell must therefore be calibrated. If the cell were cylindrical and the electrodes just large enough to fit inside it and form complete partitions across it, the specific resistance of the contents could be calculated from the observed absolute resistance and from the dimensions of the cell. In practice, however, the cell (see fig. 34) is filled with an electrolyte of known specific resistance and the corresponding absolute resistance is measured. From this the absolute resistance is calculated for the case of an electrolyte with unit specific resistance when contained in the same cell. This is called the *resistance capacity of the cell* or the *cell constant*. If the cell constant is C, the absolute resistance R when the cell is filled with an electrolyte of specific resistance $\omega$ is

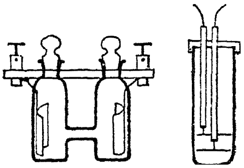

Fig. 34.—Forms of cell for conductivity measurements upon electrolytes.

$$R = C\omega.$$

## 8. Electrical Conductivity

The specific resistance $\rho$ (i.e. the resistance of a cube of the substance of 1 cm. edge) and its reciprocal, the specific electrical conductivity $\kappa$, are characteristic constants of the substance or solution under consideration. Numerical values of these very important quantities have already been given for a selection of substances in Table V, p. 223.

The conductivity of metals is about $10^5$ times greater than that of the best conducting electrolytes (see Table VI, p. 248). The conductivity of metals is affected to an extraordinary degree by the presence of impurities; it is usually diminished and to an extent far greater than would be expected from the concentration of the impurity. The conductivity of metals also varies up to several per cent with the previous chemical or mechanical treatment of the sample. It should be noted also that in the case of single crystals not belonging to the regular system very marked differences of conductivity can be observed in different directions. Thus the specific resistance of tin (tetragonal) is 0·131 ohm parallel to the axis, but only 0·09 ohm perpendicular to the axis.

**Dependence of Conductivity upon Temperature.**—The specific resistance varies with the temperature. In the case of metals it increases with rise of temperature; but in the case of carbon, silicon, tellurium, and a few other substances the variation is in the opposite sense (see fig. 35). Very pure graphite, however, behaves like metals. The change of resistance per 1° C. of a conductor whose initial resistance is 1 ohm is called the temperature coefficient of resistance. Hence if $d\mathrm{R}$ be the change of resistance for a change of temperature $d\mathrm{T}$ of a conductor of initial resistance R, the temperature coefficient $\beta$ is given by

$$\beta = \frac{1}{\mathrm{R}} \frac{d\mathrm{R}}{d\mathrm{T}}.$$

It follows that if the resistance of a conductor at a certain temperature is R, its resistance at a temperature $t°$ higher will be $\mathrm{R}(1 + \beta t)$. The curves in fig. 35 are not straight lines, from which it follows that $\beta$ is not a constant. The value usually given is a mean value between 0° C. and 30° C. For the common metals $\beta = \frac{1}{200}$ to $\frac{1}{300}$. It is remarkable that this value is not very different from the coefficient of expansion or the pressure coefficient of gases (Vol. II, p. 11). In order to represent more accurately the variation of resistance with temperature, it is necessary to employ a quadratic formula of the form $\mathrm{R} = \mathrm{R}_0(1 + \beta t + \gamma t^2)$.

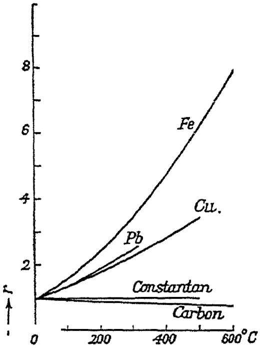

Fig. 35.—Variation of the resistance of solids with change of temperature ($r = \rho/\rho_0$)

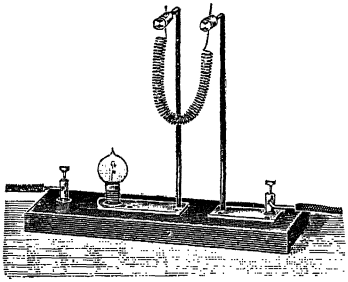

Fig. 36.—Iron wire resistance

Certain alloys similar to German silver, especially those containing nickel (constantan) or manganese (manganin) in certain proportions, have the peculiar property that their resistance varies only very slightly with the temperature, i.e. they have a very small temperature coefficient (Table V, p. 223). They are chiefly used for making standard resistances.

Fig. 36 shows a suitable apparatus for demonstrating the change of resistance with temperature. Upon a base-board are mounted a small incandescent lamp and two vertical rods carrying binding screws. A small spiral of iron wire is clamped between the upper ends of the rods. The lamp and spiral are connected in series with a battery of such an electromotive force that the lamp lights up brightly. The iron wire spiral is then heated with a Bunsen burner. This causes the lamp to burn less brightly and almost to go out, because the resistance of the iron wire is increased to four or five times its value when cold. As the spiral is allowed to cool down again, the lamp gradually lights up brightly once more.

The experiment may be repeated with a spiral of wire made of nickelin or constantan. In this case the intensity of the lamp is undiminished even when the wire is made red hot.

The high temperature coefficient of *iron* has found practical application for the automatic regulation of current strength. The inclusion of an iron wire in a circuit along with other resistances makes the whole sensitive to changes of temperature. If now the pressure goes up, the current strength tends to increase, but this in turn increases the resistance of the iron wire on account of the rise of temperature. By suitable choice of the length and cross-section of the iron wire it is possible to arrange that, in spite of the increase of pressure, the current remains constant or at least increases only very slightly. In actual practice the iron wire is usually sealed in a small airtight glass tube filled with hydrogen.

A simple relationship holds for the specific resistance of impure metals, which (as mentioned above) is greater than for the pure metal. The product of the specific resistance $\rho$ and the temperature coefficient $\beta$ has the same value at any temperature both for the pure and also for the impure metal (*Mathiessen's rule*). This means that the increase of the resistance of the metal by the presence of the impurity is accompanied by a corresponding decrease in the change of resistance with temperature. Thus the impurity adds to the original resistance of the metal an additional resistance which is independent of the temperature.

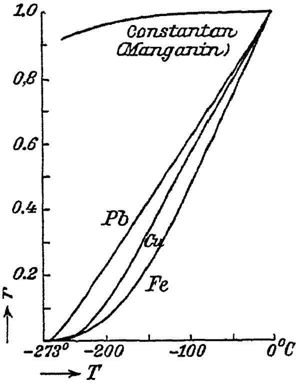

Fig. 37.—Change of resistance ratio $r = \rho/\rho_0$ of metallic conductors at low temperatures.

The variations of resistance at low temperatures are of especial interest. As is seen from fig. 37 the resistance decreases below about 0° C. more rapidly than would correspond to direct proportionality to the temperature. But at very low temperatures (about —250° C., i.e. 23° abs.) it still has a measurable value amounting to between 1/1000 and 1/10000 of the value at room temperature.

**Super-conductivity.**—In 1911 KAMERLINGH ONNES discovered a very surprising phenomenon, the causes of which still remain unexplained. He found that the resistance of mercury disappears at about 4·19° abs. (i.e. about —269° C.). Just above this temperature the specific resistance $\rho$ of mercury is about 1/500 of the extrapolated value for 0° C. (mercury melts at —38·9° C.). But a decrease of temperature of only about 0·02° then causes $\rho$ to become practically zero (compare the corresponding curve for tin shown in fig. 38).

This state, in which the metal offers no measurable resistance to the passage of an electric current, is called the *super-conducting state.* Up to the present it has been possible to bring the following pure elements (in addition to mercury) into the super-conducting state: thorium (1·4° abs.), thallium (2·47°), indium (3·41°), tin (3·75°), tantalum (4·5°), and lead (7·2°). The temperatures given in brackets are those at which the sudden fall occurs in the respective resistance-temperature curves. All the numerous other pure metals have a measurable, though very small, resistance even at the lowest temperatures. Recently, however, Meissner (1928) has succeeded in bringing into the super-conducting state an alloy of metals each of which alone is incapable of showing the effect (Au–Bi, 2·15°). Super-conductivity has also been observed in the case of compounds, e.g. carbides and nitrides, amongst which NbC possesses the highest upper temperature limit (namely 10° abs.) of super-conductivity yet known.

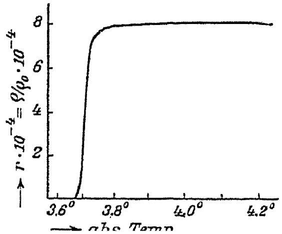

Fig. 38.—Variation of resistance of tin at very low temperatures, showing the sudden decrease to the super-conducting state.

The order of magnitude of the specific resistance in the super-conducting state is probably about $10^{-12}$ of that at 0° C. Thus in the super-conducting state a wire conducts electricity a billion times better than at ordinary temperatures.

This explains the fact, at first very surprising, that a current in a super-conducting circuit, when once it has been started (e.g. by induction, see p. 370), can continue to flow for a very long time with undiminished strength and without any E.M.F. It has been found possible to produce persistent currents of 350 amperes in a circle of quite thin lead wire. Investigation of these currents showed that, when once they were set flowing, their strength decreased at most by 1/40000 in an hour. (For further details of these experiments see Vol. V.) Practical importance attaches to the fact that the upper temperature limit of the super-conducting state is depressed by *external magnetic fields*, and that at magnetic field strengths above a certain value (generally a few hundred gauss, but in the case of Pb–Bi alloys at 1·9° abs. as high as 20,000 gauss) the super-conducting state cannot be attained. Hence it is unfortunately impossible to construct very strong electromagnets with the help of super-conductors. Even the magnetic field of the current within the super-conducting circuit has a disturbing effect, so that there is an upper limit to the current strength with which a super-conductor can be loaded at a given temperature (e.g. for lead 407 amperes per mm.$^2$ at 4·21° abs.). As is to be expected from the vanishingly small resistance, there is no observable development of heat in super-conductors, even at the highest possible current strengths.

**Resistance Thermometers.**—The alteration of the electrical resistance of a wire with change of temperature can be used to measure temperatures. Thermometers of this type are the most accurate up to 600° C.; for low temperatures they are indispensable. The metal usually employed is platinum, and for lower temperatures also lead and gold. The wire is wound loosely upon a piece of mica and contained within a protecting tube. The thermometer is calibrated by means of the usual fixed points (Vol. II, p. 2). For platinum the changes of resistance with temperature can be represented by a quadratic formula (p. 244). The measurement of the resistance may be performed with a Wheatstone's bridge arrangement. In order to measure temperatures correct to 0·001°, the apparatus must be made sensitive enough to measure changes of resistance of about 4 parts in a million.

*Bolometers.*—Resistance thermometers which are heated by radiation instead of by conduction are called bolometers.* They are chiefly used for measuring the intensity of the incident radiation. They consist of very thin wires or strips of platinum, iron, or nickel. Platinum sheet brazed to silver sheet of ten times its thickness can be rolled out to a thickness of 0·5 $\mu$; it is then stretched upon a slate frame and the silver dissolved off. In this way it is possible to obtain platinum foils only 0·00005 mm. thick. The foil, which is often cut into a zig-zag strip, is coated with lamp black or platinum black. When radiation (e.g. light) falls upon it, the strip absorbs the energy and transforms it into heat. As a result the temperature of the strip is raised and therewith also its electrical resistance, which is measured by a bridge method.

**Influence of other Factors upon the Electrical Conductivity.**—The specific resistance of metals is affected by *mechanical influences.* In general the resistance is diminished by increase of *pressure.* When the material is *extended,* an increase of resistance can usually be observed in the direction of the extension. The effects of the previous mechanical treatment of a metal can usually be removed by annealing at certain temperatures.

*Magnetic fields* also have an influence upon conductivity (W. THOMSON, 1858). The effect is very small for most metals; but in the case of **bismuth** the resistance may be doubled by the application of a strong magnetic field at right angles to the direction of the current. Hence bismuth can be used to measure magnetic field strength. For this purpose use is made of a small bifilar spiral of thin bismuth wire (fig. 39) mounted between two thin discs of mica (LENARD). This is placed in the magnetic field whose strength is to be measured, and the electrical resistance of the spiral is determined by the Wheatstone's bridge method.

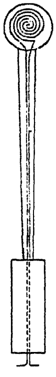

Fig. 39.—Lenard bismuth spiral

*Light or X-rays* can also affect the conductivity of solid crystals (not of metals). These phenomena will be treated later, see p. 367.

**Relation between Electrical Conductivity and Thermal Conductivity. Wiedemann-Franz Law.**—Experiment has proved that the following simple relationship holds between the thermal conductivity $\lambda_h$ (specific conductivity, Vol. II, p. 176) and the electrical conductivity $\kappa$: the ratio of $\lambda_h$ to $\kappa$ is approximately a constant for all pure metals at a given temperature (WIEDEMANN, FRANZ, 1853).

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* Gr., *bolē,* ray, from *ballein,* to throw.

Thus the greater the electrical conductivity of a material, the better it conducts heat. We are all familiar with this fact in a qualitative way. All poor electrical conductors such as glass, wood, and the like are also poor conductors of heat, while metals conduct heat much better. Also among the metals themselves the best conductors of heat are the good electrical conductors like copper and silver; whereas iron, which is a relatively poor electrical conductor, is also a relatively poor conductor of heat (compare Vol. II, p. 176).

It is found further that the ratio $\lambda_h/\kappa$ is proportional to the absolute temperature T, i.e.:

$$l = \frac{\lambda_h}{\kappa T} = \frac{\lambda_h \rho}{T},$$

where $l$ is a constant independent of the temperature and of the nature of the material. The above law ceases to hold at low temperatures, the thermal conductivity no longer increasing as rapidly as the electrical conductivity. At the transition into the super-conducting state the thermal conductivity does *not* change in the same way as the electrical conductivity.

**Conductivity of Electrolytes.** — The conductivity of electrolytes is very much smaller than that of metals. For aqueous solutions it amounts to only about 1/1000000 of that of the best conducting metals. Pure liquids have a much smaller conductivity even than aqueous solutions (Table VI). For the purest water $\kappa = 1$ to $2 \,.\, 10^{-6}$. The best distillate in air has $0{\cdot}7 \,.\, 10^{-6}$ ohm$^{-1}$ cm.$^{-1}$. On the average 1 mgm. of a salt dissolved in a litre of water raises the conductivity by about $10^{-6}$. Thus conductivity offers a very good means of testing the purity of water.

TABLE VI.—CONDUCTIVITY OF PURE LIQUIDS IN OHM$^{-1}$ CM.$^{-1}$

| | | | |
|---|---|---|---|
| Sulphuric acid, | $1 \,.\, 10^{-2}$ | Sulphur dioxide, | $< 1 \,.\, 10^{-7}$ |
| Ethyl alcohol, | $1\text{–}2 \,.\, 10^{-7}$ | Chloroform, | $< 1 \,.\, 10^{-8}$ |
| Ammonia, | approx. $1 \,.\, 10^{-7}$ | Benzene, | $< 1 \,.\, 10^{-8}$ |

*Influence of Concentration.*—In very dilute solutions the conductivity of an electrolyte increases approximately proportionally to the concentration.

In fig. 40 the conductivity of dilute sulphuric acid is plotted as a function of the concentration. The abscissæ are the concentrations expressed as percentages; the ordinates are the corresponding values of the conductivity. It is seen that in dilute solution the conductivity increases approximately in proportion to the concentration, reaches a maximum at about 30 per cent, and then decreases again. At about 84 per cent there is a minimum. At this concentration the composition of the solution corresponds to $H_2SO_4 \,.\, H_2O$. As the concentration is further increased the curve rises slightly, but then falls off rapidly towards the axis of abscissæ as the concentration 100 per cent is approached. Pure sulphuric acid is therefore a "non-conductor" for electric currents (see Table VI above). Similar curves are obtained for other electrolytes (see fig. 41).

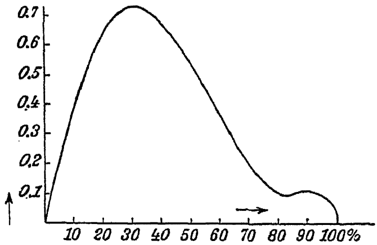

Fig. 40.—Variation of the conductivity of sulphuric acid with concentration. (Concentrations, horizontal; conductivities, vertical.)

In most cases there is a maximum electrical conductivity at a certain concentration.

The relationships involved were clarified by the introduction by F. Kohlrausch * (from 1876 onwards) of the concept of "equivalent conductivity".

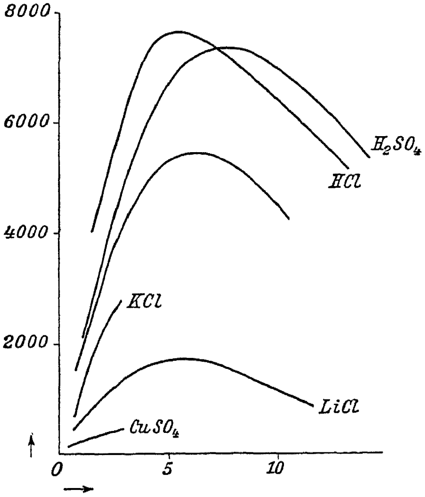

Fig. 41.—Variation of the conductivity of aqueous electrolytes with concentration. (Concentrations in equivalents per litre, horizontal; conductivities × $10^4$, vertical.)

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* Friedrich Kohlrausch, born in 1840 at Rinteln, in 1875 became Professor of Physics at Würzburg, in 1888 at Strasburg, from 1895 to 1905 President of the *Physikalisch-technische Reichsanstalt* at Charlottenburg, died at Marburg in 1910.

**The equivalent conductivity $\lambda$ is the quotient of the conductivity divided by the number of gramme-equivalents of the solute contained in 1 cubic centimetre of the solution.**

Kohlrausch found that the equivalent conductivity increases with increasing dilution, approaching a limiting value for infinite dilution.

The significance of this can be understood clearly by imagining the following experiment to be carried out. Think of a deep prism-shaped vessel of square cross-section and cross-sectional area 1 cm.$^2$. Two opposite sides must be imagined to be made of platinum, the other pair of sides and the base being of glass.

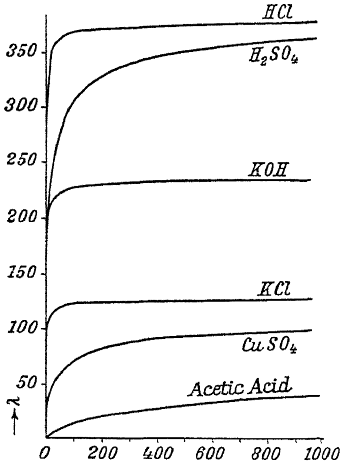

Fig. 42.—Variation of the equivalent conductivity of aqueous solutions with the dilution. (Litres per equivalent, horizontal; equivalent conductivities, vertical.)

The two platinum sides are connected by suitable leads in an electric circuit. Into this vessel are poured 1000 cm.$^3$ of an aqueous electrolyte containing exactly one gramme-equivalent of the solute: the solution is thus normal. The metal sides are then connected up and the conductivity determined. The value so obtained is the equivalent conductivity at the concentration 1 normal or 1/1000 equivalent per cm.$^3$. The electrolyte is now diluted with water. Suppose that the conductivity is again measured when the total volume is 10 litres. The value obtained is the equivalent conductivity at the concentration $\frac{1}{10}$ normal = 1/10000 equivalent per cm.$^3$. No matter to what extent the solution is diluted, there is always 1 gramme-equivalent of the solute between the platinum electrodes. At every dilution the equivalent conductivity $\lambda$ is given by 1000 $\varkappa/n$, where $n$ is the number of gramme-equivalents contained in a litre.

In the above experiment the observed conductivity at any dilution is the equivalent conductivity at that dilution. Such an experiment would prove that the equivalent conductivity increases with increasing dilution. When the measurements are not carried out with the kind of vessel imagined above, the equivalent conductivity for a given concentration can be calculated from observed results with the help of Ohm's law.

Fig. 42 shows the variation of equivalent conductivity with dilution for a number of solutes. As is seen from the curves, *the equivalent conductivity $\lambda$ always increases with increasing dilution and approaches a limiting maximum value $\lambda_\infty$ for infinite dilution.*

Concerning the conclusions as to the mechanism of electrolytic conduction which may be drawn from this behaviour see Chap. VIII, p. 266 *et seq.*

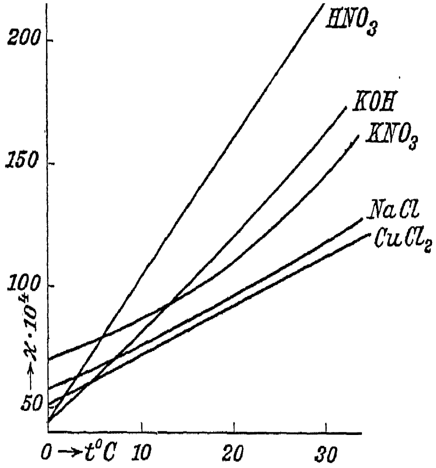

Fig. 43.—Effect of temperature upon the conductivity of aqueous electrolyte solutions, $\frac{1}{10}$ normal. (The ordinates represent conductivities × $10^4$ and are to be increased by 200 for $HNO_3$ and by 100 for KOH.)

*Influence of Temperature.*—In contrast to the case of metals, the conductivity of electrolytes *increases* with rise of temperature. The value usually given is the equivalent conductivity at 18° C. ($\lambda_{18}$). The temperature variation of $\kappa$ is shown for a number of substances in fig. 43. The curves illustrate the fact that the increase is only in a few cases proportional to the rise of temperature. Around room temperature the change of conductivity can generally be taken as about 2·5 per cent per degree.

## CHAPTER VII

# Electric Current and Heat. Mechanical Equivalent of a Current

### 1. Mechanical Equivalent of a Current at given Potential Difference

We start once more with the basic experiment upon the discharge of a charged condenser. Work is required in order to charge up the condenser to the potential difference U. According to §§ 12 (p. 69) and 16 (p. 96) the magnitude W of this work is $\frac{1}{2}QU$, where Q is the charge. We now put the two condenser plates in conducting connexion and allow the opposite charges to neutralize each other. After this has taken place the condenser is once more uncharged. According to the law of the conservation of energy (Vol. II, p. 32) the energy expended in the charging process must have been changed into some other form. Now in the above experiment it is easy to observe that the temperature of the conductor joining the plates has risen, maybe to such an extent as to make it red hot or white hot. The electrical energy has thus been transformed into thermal energy and possibly also into light energy. Our purpose in this section is to consider the quantitative measure of this transformation. As has been previously mentioned (p. 136), it is possible by supplying charges from a suitable source to maintain the potential difference of the condenser at a constant value in spite of the conducting connexion between the plates. The current flowing between the plates is then maintained by a constant electromotive force and itself remains of constant strength. The quantity of heat developed per unit of time is also constant under these conditions. The *quantity of work* W *necessary to maintain the potential difference* U *for a certain time* is given by the equation:

$$\mathbf{W = QU,}$$

where Q is the quantity of charge supplied during that time; for according to p. 29 we must perform the work QU to supply the charge Q to the condenser plates at the constant potential difference U. Thus we arrive at the general result:

*The mechanical equivalent* W *of an electric current flowing in a con-*

*ductor is equal to* QU, *where* Q *is the quantity of electricity involved and* U *is the potential difference between the ends of the conductor.*

When an influence machine is used, for example, the energy required to maintain the current between the condenser plates is supplied as mechanical work in turning the machine. The energy necessary to maintain the current through an ordinary incandescent lamp is derived from the turbines and steam-engines of the power station. The energy required to keep up the lighting current of a pocket flash-lamp is supplied at the expense of the chemical energy of the dry cells.

## 2. Joule's Law. The Thermal Equivalent of an Electric Current

If the whole of the energy W of discharge of the condenser is transformed into heat, the quantity H of heat obtained is given by $H = QU = W$. When the current is of constant strength I we may write $Q = It$, where $t$ is the duration of the current. A constant K must be introduced when the heat is measured in calories, the potential difference in volts and the current strength in amperes. We thus obtain:

$$\textbf{H cal.} = \textbf{K . U volts . I amperes . } t \textbf{ seconds.}$$

This is known as **Joule's* law.** The constant K gives the number of calories of heat corresponding to a current of 1 ampere flowing for 1 second through a potential fall of 1 volt. This quantity of energy is given the name **1 international joule.**

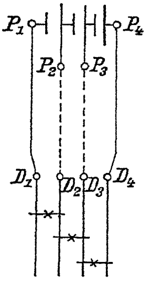

Fig. 1.—Variation of Joule heat with the potential difference.

For the experimental investigation of the development of heat by an electric current we may make use of several small incandescent lamps burning normally at 2 volts and as source of current a battery of low internal resistance (e.g. large accumulators), so that the pressure drop when the lamps are switched on may be neglected (see p. 230). Fig. 1 shows an arrangement in which three accumulators are combined to form a battery and fitted with the four terminal screws $P_1$, $P_2$, $P_3$, and $P_4$, between each successive pair of which there is thus a potential difference of 2 volts. Each of these terminal screws is connected to a corresponding piece of bare copper wire, $D_1$, $D_2$, $D_3$, or $D_4$. The four wires are stretched out parallel to each other upon a base-board. The small lamps are held in suitable devices by means of which they can be put into circuit by simply placing them as a bridge across two neighbouring wires.

Using only one lamp, the brightness with which it lights up is independent of which pair of wires it is placed upon; for the potential difference is the same between each pair of neighbouring wires. A certain current strength flows through the lamp (about $\frac{3}{4}$ ampere) and the transformation of the electrical energy into heat takes place at a corresponding rate. This rate can be judged by the brightness with which the lamp burns. Thus in the above case we obtain the result that the rate of evolution of heat is the same for all positions.

---

* First established experimentally in 1841 by JOULE (see Vol. II, p. 29).

When now two or three lamps are bridged across the same pair of wires, they have no effect on each other, provided that the accumulators are so large that there is no pressure drop. The strength of the current through each of the lamps is the same as when only one lamp is present. Through the *n* lamps, therefore, the total current strength is *n* times as great as before. Observation shows that all the lamps burn with normal brightness, whence we deduce that the rate of evolution of heat is also *n* times as great as before. Therefore:

*At constant potential difference the quantity of heat developed in a given time is directly proportional to the current strength.*

We now place three lamps as in fig. 1, one across each of the pairs of neighbouring wires. The lamps light up with the same brightness as in the preceding experiments. We then break the connexions (shown dotted in fig. 1) of the terminal screw $P_2$ with the wire $D_2$ and the terminal screw $P_3$ with the wire $D_3$. The lamps burn with unchanged brightness. In the new circuit the current strength is the same as that in the first experiment with one lamp alone; but the pressure is now three times as great, since the cells are connected in series. The rate of evolution of heat is now three times as great as in the experiment with one lamp. Hence:

*At constant current strength the quantity of heat developed in a given time is directly proportional to the potential difference.*

It is at once obvious that the quantity of heat developed is directly proportional to the time, for under given conditions the same quantity is developed every unit of time.

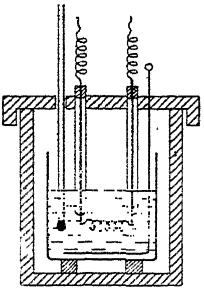

Fig. 2.—Determination of the thermal equivalent of an electric current.

Summarizing the experimental results we obtain the formula already given above:

$$\mathbf{H = KUI}t.$$

In this equation we still have to determine the constant factor K. For this purpose we may suspend a spiral of thin nickel or constantan wire in a calorimeter, e.g. in a glass beaker containing water and standing upon pieces of cork within a second beaker (see fig. 2). The spiral is held by copper leads which pass up through glass tubes and are held in place with sealing-wax.

We pass a current of known strength I and known potential difference U for a time *t* through the spiral. We observe the resultant rise in temperature $\theta$ of the calorimeter system (water, vessel, stirrer, thermometer), whose water equivalent is *m*. The quantity of heat evolved is then $H = m\theta$. We now know all the quantities in the equation $H = KUIt$ except the constant K.

In order to prevent errors due to radiation from the calorimeter system, it is best to start with water at a temperature somewhat *below* room temperature, and to let the current go on passing through the spiral until the temperature has risen as much above room temperature as it was originally below (Rumford's method). The results of an actual experiment were as follows: $U = 76$ volts; $I = 1{\cdot}2$ amperes; $t = 523$ sec.; $m = 1079$ gm. of water; $\theta = 10{\cdot}6°$ C. The value of K calculated from these figures is $(1079 \,.\, 10{\cdot}6)/(1{\cdot}2 \,.\, 76 \,.\, 523) = 0{\cdot}24$ cal. per volt-ampere-second.

An incandescent lamp may be used in place of the wire spiral, but then the fraction of the energy which appears as light and which may amount to a few hundredths must also be absorbed in the calorimeter. For this purpose the water must be made opaque or a metal pot must be used as calorimeter; otherwise, if light escapes through the transparent water and glass walls, the value obtained for K will be too small.

In measuring a physical quantity care must always be taken to make sure that the observed quantity really represents the whole of what is to be measured. For example, the current flowing in a circuit may perform mechanical work as well as produce heat. In this case the observed quantity of heat evolved would fall short of the total energy by the amount which is used up in performing mechanical work. Conversely in determining the amount of mechanical work done by the current, allowance must be made for the fraction of the total energy which is transformed into heat (the so-called **Joule heat**).

Accurate measurements have given the following result:

**The value of K (i.e. 1 international joule) is 0·2390 cal. per volt-ampere-second.**

This is the quantity of heat evolved during the passage of 1 coulomb through the potential difference 1 volt. By the first law of thermodynamics we can express this quantity of heat in mechanical units (Vol. II, p. 32). We then obtain the result that 1 volt-ampere-second (1 international joule) is equivalent to approximately $10^7$ ergs.* It follows also that 1 cal. = 4·184 volt-ampere-sec.

**Power of a Current.**—The work performed per unit of time is called the *power*. The power of a current is therefore given by the product *volts* × *amperes*. From above we have:

$$\begin{aligned} 1 \text{ volt-ampere} &= 0{\cdot}2390 \text{ cal./sec.} = 1 \text{ international joule/sec.} \\ &= 0{\cdot}00134 \text{ horse-power} = 1 \text{ international watt.}\dagger \end{aligned}$$

The international watt is *defined* as the power of 1 volt-ampere ‡. Hence:

**The power of an electric current is measured in watts, 1 watt being equal to 1 volt-ampere.**

From the above equation it follows that 1 horse-power (h.p.) is equal to 746 watts.§ A filament lamp of medium power requires about 60 watts, i.e. nearly $\frac{1}{10}$ h.p. Each such lamp therefore requires the supply of work at a rate of nearly $\frac{1}{10}$ h.p., which is about as fast as a man can work continuously. Remembering that this power is very small compared with that required in industrial processes, we realize the tremendous multiplication of human power by modern engineering.

*Note.*—The attempt to obtain simple numerical correspondence between the units of electrical energy and power and the mechanical units is the historical reason for the remarkable powers of ten which occur in the practical definitions of the volt (p. 43) and the ampere (p. 155). As is clear from the footnote on p. 255, this attempt has not been quite successful. The numerical correspondence which has been sought after is actually not at all necessary. In the science of heat, for example, the calorie has been adopted as unit of energy and a factor is used in transforming calories into mechanical units.

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* The electrically defined international joule (= 0·2390 cal.) is not exactly equal to $10^7$ ergs (see Vol. II, p. 32); for $10^7$ ergs are equivalent to 0·23865 cal. (probable better value 0·2389).

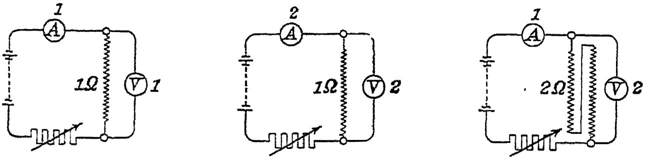
† Vol. I, p. 90.

‡ Thus 1 international watt is not exactly equal to $10^7$ ergs per sec., which is called 1 *absolute* watt.

§ The British horse-power differs somewhat from the continental, which is equal to 736 watts (see Vol. I, p. 91).

The following alternatives therefore present themselves: either, as in the case of the international agreement, the correspondence between electrical and mechanical units may be disregarded and the former fixed once and for all by legal definition; or the units may be altered in accordance with each new and more accurate determination of their values. There is no doubt that the first course, which was also adopted in fixing the metre as unit of length and the kilogram weight as unit of weight, is the more practical. In this book, therefore, we shall deal exclusively with the legal units in which all measuring instruments are calibrated (see § 14, p. 80, and § 7, p. 193).

It follows from the above that the *work* performed by a current can also be measured in watt-seconds. As larger units use is made of the watt-hour = 3600 watt-seconds and the kilowatt-hour = 1000 watt-hours. The latter is also known as the Board of Trade (B.O.T.) unit.



$H_1 = 1$ $H_2 = 4$ $H_3 = 2$

Figs. 3, 4, and 5.—Dependence of Joule heat upon current and pressure

**Alternative Forms of Joule's Law.**—With the help of Ohm's law $I = U/R$ we can express Joule's law in the forms

$$\mathbf{H} = \mathbf{I}^2\mathbf{R}t \quad \text{and} \quad \mathbf{H} = \mathbf{U}^2 t/\mathbf{R}.$$

Here H is expressed in watt-seconds. From the first of these equations it follows that:

*The quantity of heat developed by a current flowing through a conductor of given resistance is directly proportional to the square of the current strength.*

*For a given current strength the quantity of heat evolved is directly proportional to the resistance.*

From the second equation it follows that:

*The quantity of heat developed by a current flowing through a conductor of given resistance is directly proportional to the square of the potential difference between its ends.*

*For a given potential difference the quantity of heat evolved is inversely proportional to the resistance of the conductor.*

The circuits shown in figs. 3, 4, and 5 each consist of a battery, a variable resistance, an ammeter A, and the wire (represented by a wavy line) in which the development of heat is to take place. The potential difference between the ends of the wire can be read off upon a voltmeter V. By adjusting the variable resistance, the current strength as indicated by the ammeter A is made to assume the value given in each figure just above A. The magnitudes of the resistances of the wires are also shown in the figure, and the potential difference between the ends of the wire is given in each case just to the right of V. Let the quantities of heat developed in the three cases be $H_1$, $H_2$, and $H_3$ respectively.

It is found that $H_2 = 4H_1$, because the resistance is the same in both cases but the current in the first is only half that in the second. The potential difference in the first case is also only half that in the second. Further $H_3 = 2H_1$, because the current strength is the same in both cases but the resistance in fig. 5 is twice that in fig. 3. Also $H_3 = \frac{1}{2}H_2$, since the potential difference is the same in both cases but the resistances are in the ratio 2 : 1.

Since the quantity of heat developed in a conductor is proportional to the *square* of the current strength or the *square* of the potential difference, it is independent of the *sense* of the current.

## 3. Applications of Joule Heat

**Hot-wire Instruments.**—The expansion of a wire produced by the heat generated in it by a current may be used to measure the current strength. The wire must be made of a material whose electrical resistance varies only slightly with the temperature and whose increase of length is proportional to the rise of temperature. This rise of temperature also depends upon the temperature of the surroundings. If this is sufficiently constant and if the above conditions are fulfilled, then according to p. 256 the expansion is directly proportional to the square of the current strength. The expansion can be transmitted in various ways to a pointer: see, for example, figs. 6 and 7. The measuring range of an instrument of this type can be extended to any desired degree by the use of suitable shunts (p. 235). The accuracy obtainable is not very high, but these instruments possess the advantage that their indications are independent of the *direction* in which

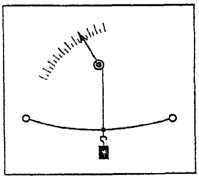


Fig. 6.—Principle of a hot-wire current-measuring instrument

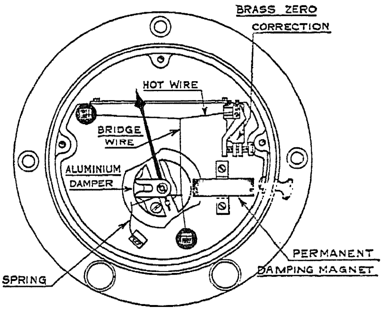


Fig. 7.—Hot-wire ammeter
[Johnson and Phillips, Ltd., London]

the current passes through them. They are therefore suitable for measuring the strength of alternating currents. For reasons which cannot be discussed until later (Chapter XIV, p. 488), they are chiefly used nowadays in high-frequency work (fig. 31, p. 489). The sensitivity can be greatly increased by enclosing the wire in a high vacuum.

The change of resistance of a wire with the rise of temperature produced by a current may also be used. A Wheatstone's bridge method may be used and with suitable arrangement a very high sensitivity may be attained.

**Production of High Temperatures.**—The transformation of electrical energy into heat offers a convenient method of producing high temperatures. This is the principle underlying the action of electric furnaces and incandescent lamps.

**Electric laboratory furnaces** consist of a porcelain tube around which is wound a spiral of thin platinum or iridium wire; for lower temperatures (up to 800° C.) wires made of nickel alloys may be used. As a protection against mechanical damage and more particularly against loss of heat by radiation, the tube is surrounded by an outer tube and the space between them is filled with magnesium oxide. The body which is to be heated is introduced into the porcelain tube in a crucible *and the two ends of the wire spiral are connected to a source of current. Fig.* 8 shows a furnace of this type.

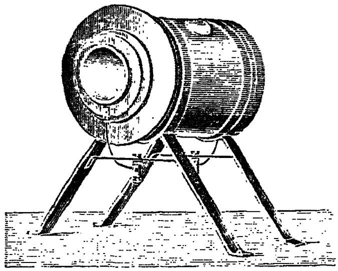


Fig. 8.—Laboratory electric furnace

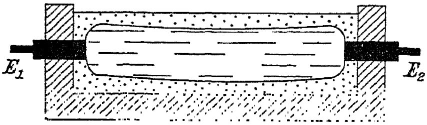


Fig. 9.—Heating of a melt by resistance

**Electric kettles** work on the same principle. The actual kettle is surrounded by an insulated conductor, through which the current flows, and the whole is covered on the outside with a protecting casing. The conductor consists either of thin iron wires fused into an insulating and protective layer of enamel, or else of a film of platinum about 0·00025 mm. thick deposited upon mica by a special process. Thin strips of nickel alloys are also used.

In **electric resistance furnaces** a coarse mixture of carborundum, graphite, and clay is placed between the metal electrodes, which are connected to the source of current. Because of the high resistance it offers, the mixture becomes very hot. Temperatures as high as 2000° C. can be produced in furnaces of this type.

In large-scale operations very wide use is also made of the Joule heat in molten electrolytes. Fig. 9 shows a diagram of such an arrangement (see further p. 357).

**Incandescent Lamps.**—Davy was the first to observe (1801) that a platinum wire may become so hot as to glow and send out light when an electric current is passed through it. The practical application of this discovery has required an extraordinary amount of work and ingenuity. The economic problem of the incandescent lamp, namely the problem of the rational production of light from glowing bodies, cannot be treated quantitatively until Vol. V. At present it is sufficient to remark that the amount of light, i.e. the quantity of energy appearing as light, which can be obtained from 1 watt of electrical energy increases very rapidly with rise of temperature of the filament. It is necessary, therefore, to employ substances which can be raised to a very high temperature and which will stand this treatment for a long time without deterioration. The first practical filaments were made of carbon and were prepared by raising threads of cellulose to incandescence in absence of air. These filaments were enclosed in evacuated bulbs so as to prevent them from burning away.* In spite of the high melting-point of carbon (about 4200° C.), however, filaments of this substance can only be heated to a relatively low temperature (running temperature 1865° C.) owing to the fact that they disperse, blackening the walls of the containing bulb, and finally break. Metals behave in this way to a very much smaller extent, and the further steps in the development of incandescent lamps have involved the use of metal filaments of ever higher and higher melting-points (osmium †, melting-point 2740° C., running temperature 1925° C.; tantalum, melting-point 2930° C.; and finally, tungsten, melting-point 3390° C., running temperature 2080° C.). The preparation of the fine filaments and their mounting in the small lamp bulbs presented technical problems of great difficulty. Two further advances (Langmuir, 1913) were the winding of the filament into narrow spirals whose length is only a tenth of that of the filament itself, and the filling of the lamps with a chemically inert gas (nitrogen or argon) at about $\frac{2}{3}$ atmospheric pressure. The pressure of this gas, which is approximately at atmospheric pressure when the lamp is alight owing to the rise of temperature, represses the disintegration of the filament to a very great extent, so that these lamps have a useful life of 1000 hours or more. The winding of the filament into a spiral diminishes the conduction of heat away through the gas filling the bulb, a factor which reduced the efficiency of the earlier lamps to such an extent that they had to be very thoroughly evacuated. When a spiral filament is used the fractional loss by conduction becomes so small (actually one-seventh when the length of the spiral is one-tenth of that of the actual filament) as to be of no special practical significance. Gas-filled lamps give a very white light whose spectral composition is similar to that of daylight. For a given size they far exceed in brilliance all earlier types of incandescent lamps. They are made for E.M.F.s from a few volts up to 260 volts and for intensities (see Vol. IV) from a few candle-power up to 60,000 candle-power.

The temperature of the tungsten spiral in a gas-filled lamp is about 2625° C. The larger lamps require about 0·5 watt per candle-power (see Vol. IV); for this reason they are often referred to as half-watt lamps.

**Nernst Filament.**—The Nernst incandescent lamp, nowadays only occasionally used for scientific purposes, employs as carrier of the current a small rod made of a mixture of zirconium oxide with oxides of rare earth metals (NERNST, 1900). This material conducts electrolytically and is made to glow in the air. Since the resistance at room temperature is too great, the rod must first be warmed up. On account of the large negative temperature coefficient (a common property of all electrolytes) a resistance must be connected in series with the lamp, otherwise the rod, having once been made sufficiently conducting, would become hotter and hotter, would continue to conduct better and better, and would finally burn through. Concerning other light sources see Vol. V.

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* The inventor of the carbon filament lamp is the Hanoverian teacher Heinrich Goebel, who emigrated to New York; he made carbon filament lamps as early as 1855, and used them to light his watch-maker's workshop. The first lamps suitable for practical use were made by Edison (born 1844) in 1879. He was the first to prepare the carbon filaments by carbonization of bamboo threads. To him is due also the Edison cap, by means of which the lamp can be inserted conveniently and with certainty into the electric circuit by being screwed into a holder. Edison was the first to connect up a number of lamps in parallel in the same circuit, so that each could be lit independently of the others.

† The osmium lamp was invented in 1898 by Auer von Welsbach (born 1858 at Vienna, died 1929), the inventor of the incandescent gas mantle and of the *cereisen* lighter. The first osmium lamps were only designed for low E.M.F.s. Then by using osmium mixed with other substances, especially with tungsten, metal filaments, suitable for use at higher E.M.F.s, were subsequently obtained. Auer chose osmium because according to the contemporary data it appeared to be the metal with the highest melting-point.

**Safety Fuses.**—In deciding the cross-section for a conducting wire a certain normal current load is taken as a basis. It may subsequently happen, however, that for some unforeseen reason (e.g. **short-circuit,** i.e. direct contact of the wires from the opposite poles of the battery) the current strength may rise far above the normal. It is necessary, therefore, to have some automatic device to prevent the disadvantageous consequences (e.g. heating of the wires) of such a rise of current. For this purpose safety fuses are often used. At some convenient point of the circuit a thinner piece of wire is included; rise of current strength then causes this to melt. Formerly a wire of lead or a strip of tinfoil was generally used. Nowadays a thin wire of copper or silver is employed. This is totally enclosed in a porcelain casing, so as to prevent danger of causing fire by the molten pieces of metal and the metal vapour which are formed when the fuse burns through. Fig. 10 shows a practical form of enclosed, screw-in fuse unit of the type known as the "Zed" safety fuse. The two fusible wires are completely enclosed within the porcelain casing. In parallel with them (in the middle of the casing) is connected a very thin, short wire carrying on a spiral spring a small coloured disc which is visible from outside in a depression of the casing. This *indicating wire* melts without any danger directly after the actual fuse wires. Nowadays electromagnetic cut-out switches are frequently used in place of fuses. In these devices an increase of current strength automatically opens a switch by means of an electromagnet, thus breaking the circuit.

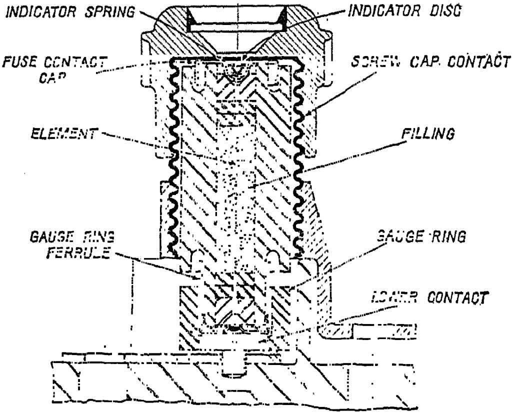

Fig. 10.—"Zed" safety fuse
[Siemens Electric Lamps and Supplies, Ltd., London]

## 4. Thermoelectricity and Peltier Effect

**Thermoelectricity.** — If a conducting circuit be made of two different metals and if one of the junctions be heated, an electric current flows in the circuit. The presence of the current may be demonstrated, for example, by the deflection of a magnetic needle. Fig. 11 shows a suitable form of apparatus. In the circuit depicted in the figure the *negative* current (i.e. the current of electrons) flows from the antimony to the copper across the heated junction. Thus in consequence of the difference of temperature between the junctions there is an electromotive force, the **thermoelectric E.M.F.** The magnitude of this E.M.F. is not affected by the presence of other conductors in the circuit, provided that all their junctions are kept at the same temperature. Hence the current flowing in the circuit may be measured by an ammeter. A pair of metals which give a current when one of their junctions is heated is called a **thermo-couple.** If both of the junctions are heated to the same temperature, no current is produced. Within a wide temperature range the current strength and hence also the thermoelectric E.M.F. increase as the temperature *difference* between the junctions is increased.

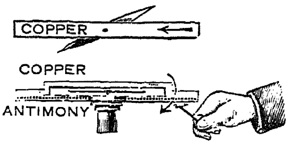

Fig. 11.—Demonstration of thermoelectric effect

For small temperature differences and when the temperature of the one junction is kept constant, the E.M.F. produced in a thermocouple is directly proportional to the temperature difference between its junctions. For more extended temperature ranges the thermoelectric E.M.F. is given with sufficient accuracy by a quadratic equation of the form $E = a + b\theta + c\theta^2$, where $\theta$ is the temperature difference and $a$, $b$, and $c$ are constants.

On the basis of their thermoelectric behaviour the metals may be arranged in a so-called **thermoelectric series,** viz.:

+ Selenium (850), antimony (100), iron (83), brass (76), tin (73), copper (72), silver (72), platinum with 10% rhodium (71), gold (71), zinc (71), lead (69), mercury (65), platinum (65), nickel (51), constantan (30), bismuth (0) —.

The order of the metals in the thermoelectric series is not quite certain: small quantities of foreign substances often considerably alter the position of a metal in the series.

The thermoelectric behaviour of an alloy depends very largely upon the proportions of its constituents. The alloy constantan, which lies between nickel and bismuth in the thermoelectric series, is often used for thermo-couples.

In the case of a thermo-couple made of two of the metals of the thermoelectric series the metal coming first in the series becomes positive at the heated junction. The magnitude of the thermoelectric

E.M.F. is greater, the farther the two metals are apart in the series. For a temperature difference of 1° C. the E.M.F. expressed in microvolts (1 microvolt $= 10^{-6}$ volt) is approximately equal to the difference of the respective numbers given in brackets above after the names of the metals. In comparison with the electromotive forces of galvanic cells, therefore, thermoelectric E.M.F.s are very small.

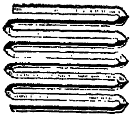

Fig. 12.—Arrangement of couples in a thermopile.

In order to obtain a total thermoelectric E.M.F. of 1 volt by connecting thermo-couples of bismuth and antimony in series, we should have to use 100 elements for a temperature difference of 100° C. between the junctions.

The E.M.F. of a thermo-couple is only proportional to the temperature difference within narrow limits. It depends not only upon the temperature difference but also upon the actual temperature. Thus when the one junction is at 0° C. and the other at 10° C., the thermoelectric E.M.F. is in most cases not the same as when the one junction is at 100° C. and the other at 110° C. Consider, for example, the behaviour of a thermo-couple made of a piece of iron wire and a piece of copper wire soldered together with hard solder at one end, the other ends being connected to the terminals of a galvanometer. When the soldered junction is heated, the galvanometer indicates a current, the direction of which is from copper to iron across the heated junction. At first the current strength increases in proportion to the rise of temperature; but as the soldered junction is heated further, the current strength falls off again, reaching the value zero when the temperature of the hot junction is 276° C. (The other junctions are supposed to be kept at 0° C.) If the temperature of the hot junction be raised still more, the current is reversed. Most thermo-couples show a similar behaviour. The point at which the reversal of the current direction occurs is called the **neutral point** of the couple.

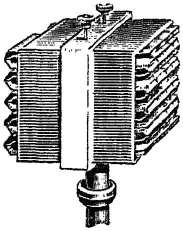

Fig. 13.—Thermopile

*Thermopiles.*—The connexion of a number of thermo-couples in series to form a thermopile is shown diagrammatically in fig. 12. The junctions of the small metal rods lie alternately to the one side and to the other, so that all the odd junctions lie to the right and all the even junctions to the left of the group. The small rods are held together in a common casing (fig. 13), from the opposite sides of which project the alternate junctions. The casing carries two binding screws connected to the free ends of the outermost rods.

Although the thermoelectric E.M.F.s are very small, very strong currents can be produced provided that the resistance of the whole circuit is kept small. In a circuit consisting of a copper rod 10 cm. long and a nickel rod 4 cm. long, each of the rods having a cross-sectional area of 1 cm.$^2$, a temperature difference of only 200° C. will produce a current of about 10 amperes.

Concerning the views as to the fundamental processes involved in the thermoelectric effect, see p. 366.

**Application of Thermo-couples.**—The chief application of thermo-couples is for the measurement of temperatures *. Long wires are soldered or welded together at one end and this junction is placed at the point where the temperature is to be measured. The wires lead to a region of constant temperature (either room temperature or for more accurate purposes 0° C. obtained by the use of melting ice) and from there to the measuring instrument (fig. 14). With this arrangement the circuit almost always consists of three conductors, say M, N, and K. If the temperature of the one junction MN (i.e. the temperature to be measured) is $\theta_m$ and that of the other two NK and KM is $\theta_z$ (e.g. room temperature), then the thermoelectric E.M.F. of the circuit is the same as if there were only one junction NM at the temperature $\theta_z$ instead of the two junctions NK and KM.

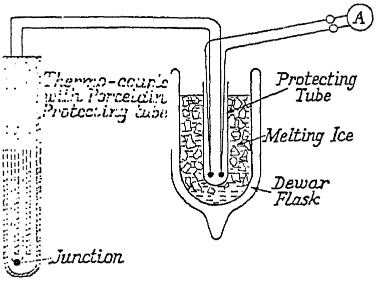

Fig. 14.—Measurement of temperature with a thermo-couple

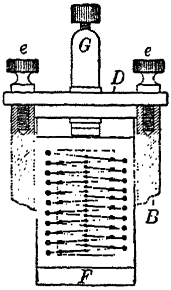

Fig. 15.—Rubens thermopile

For temperatures below 0° C. and up to +500° C. thermo-couples of copper-constantan or iron-constantan are generally used. For higher temperatures the noble metals are employed. A couple of platinum and platinum-rhodium (alloy with 10 per cent rhodium) is suitable for temperatures up to 1600° C.; up to 2000° C. iridium and iridium-rhodium may be used. The one wire is electrically insulated from the other by means of a tube of unglazed porcelain or quartz slipped over it, and the whole couple is protected from flame-gases, &c., by a surrounding jacket of glazed porcelain. The current corresponding to the temperature difference between the junctions is read off upon a sensitive galvanometer, which for practical purposes is usually fitted with a scale upon which the temperature can be read off directly.

*Linear Thermopiles.*—In order to measure the energy of spectrum lines in different regions of the spectrum (see Vol. IV), the alternate junctions are arranged one above the other in a straight line (*linear* thermopile).

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* Thermometers for high temperatures are sometimes called *pyrometers*. From Gr. *pyr*, fire.

Fig. 15 gives a diagram of such a thermopile, in which thin (1 mm. diameter) iron and constantan wires are so arranged that 20 alternate junctions lie directly above one another. The diagram is $\frac{2}{3}$ natural size. The thermopile is shielded from stray radiation by means of a metal casing, which covers it on all sides except that from which the rays under investigation are to be received. On this side is placed a metal funnel, opening outwards. This type of thermopile is particularly suitable for radiation measurements on account of its very small heat capacity. Occasionally a thermopile is used as a source of current.

The sensitivity can be very much increased by making the mass of the thermo-couples as small as possible and enclosing them in a high vacuum to prevent losses of heat by convection and conduction in the air.

Fig. 16 shows a vacuum thermo-couple consisting of strips of constantan and manganin 0·001 mm. thick, 0·1 mm. wide and 2 mm. long soldered on to thick nickel leads within a highly evacuated glass bulb. The time taken to assume constancy is about $\frac{1}{5}$ sec. The resistance is about 10 ohms. A supply of heat at the rate of $5 \, . \, 10^{-8}$ cal. per second produces a thermoelectric E.M.F. of $1 \, . \, 10^{-6}$ volt. On account of their smallness, thermo-couples of this kind are chiefly used for the measurement of the absolute intensities of spectrum lines.

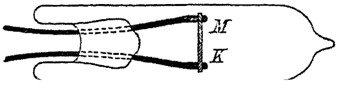

Fig. 16.—Moll vacuum thermo-couple

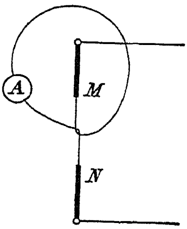

Fig. 17.—Thermo-galvanometer

Their very small mass and their consequent very small heat capacity make thermo-couples particularly suitable for measuring the changes of temperature of small bodies; for these changes are not seriously affected by the measurement. Thermo-couples are therefore used, for example, for the investigation of the temperature variations in small insects, living cells, &c.

*Thermo-Galvanometer.*—It is possible to construct a hot-wire current-measuring instrument in which the thermal expansion is not used (p. 257) to actuate the pointer, but in which instead the rise of temperature of the wire is measured directly by means of a thermo-couple. To the leads M and N (fig. 17) are soldered two wires of different materials (e.g. iron and constantan). These wires are then crossed and soldered or welded together. The free ends are connected to the terminals of a sensitive galvanometer to indicate the thermoelectric current. This arrangement is chiefly used for measuring very weak alternating currents.

**Peltier Effect** *.—This is the reverse of the thermoelectric effect considered above. When an electric current is passed through a thermo-couple consisting of rods of bismuth and antimony soldered together, the temperature of the junction rises when the negative current flows across it from bismuth to antimony, and falls when the negative current flows across it from antimony to bismuth. The change of temperature is thus always in the sense which tends to produce a thermoelectric current opposite to the current which is sent through the junction.

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* Named after PELTIER (1785–1845), who first observed it in 1834.

The Peltier effect can be demonstrated with an air thermoscope. The arrangement is shown in fig. 18. Each of the two bulbs of the air thermoscope contains a thermo-couple. The two thermo-couples are connected together in such a way that the current from the battery flows in one of them from the antimony to the bismuth and in the other from the bismuth to the antimony across the junction. With this arrangement the fall of temperature in the one thermo-couple and the rise in temperature in the other both act in the same sense upon the thermoscope, whereas the Joule heat is the same on both sides and therefore has no effect.

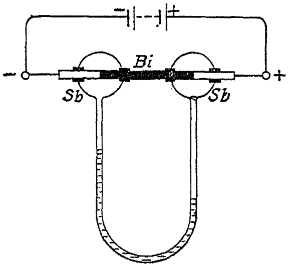

Fig. 18.—Demonstration of the Peltier effect

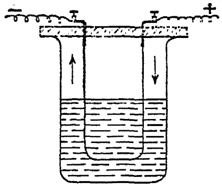

Fig. 19.—Thomson effect

**Thomson Effect.**—As was shown by W. Thomson (Lord Kelvin) there is a thermoelectric E.M.F. not only between different metals but also between pieces of the same metal at different temperatures.

When an electric current is passed through a wire in which there is a temperature gradient, a kind of Peltier effect occurs between the regions of different temperature. The total quantity of heat evolved by the current is therefore greater when the flow is in one sense than when it is in the reverse sense.

This can be shown very well with the help of the apparatus shown in fig. 19. A U-shaped platinum wire dips partially into cold water or mercury. A current is passed through it so that the parts not under the liquid become incandescent. Above the liquid surface there is then a rapid fall of temperature. It is observed that the limb of the wire in which the current flows from the region of higher to the region of lower temperature glows considerably brighter just above the liquid surface than does the other limb. Reversal of the current sense reverses the brightnesses of the two limbs.

In the case of lead the Thomson effect is vanishingly small. This metal is therefore often chosen as comparison substance in tables. (For further discussion of the Peltier and Thomson effects, see Appendix.)

## CHAPTER VIII

# The Mechanism of Electrical Conduction

### 1. The Dissociation of Electrolytes in Solution.

As has already been explained in § 1, p. 144, the passage of a current through a solution of an electrolyte is accompanied by chemical effects. The experiment described on p. 140 shows that it is the solute which is principally effective in transporting the current. The relations between quantity of substance liberated and quantity of charge transported (i.e. Faraday's laws of electrolysis) make it very probable that not only the *liberation* of the charges at the electrodes but also the *transport* of the electricity through the liquid is in some way associated with a transport of matter, i.e. of the electrically charged atoms or groups of atoms (the *ions*, p. 158). It can be shown that the ions of the dissolved electrolyte do indeed migrate when a current is passed through the solution.

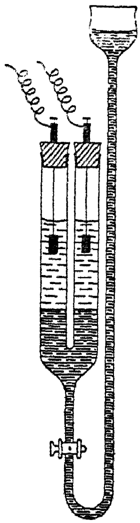

Fig. 1.—Demonstration of the migration of ions (after Nernst).

*Experiment.*—This migration of the material constituents of the liquid during the passage of an electric current can be demonstrated very elegantly as follows (experiment due to Nernst). The funnel tube of the apparatus shown in fig. 1 is filled with an aqueous solution of potassium permanganate ($KMnO_4$) (about ½ gm. per litre) whose specific gravity has been somewhat increased by the addition of 5 gm. of urea per 100 cm.³ of the solution. The tap is then opened cautiously and the solution allowed to rise in the left-hand side of the apparatus until it just reaches the lower part of the U-tube. The U-tube is then partially filled with an aqueous solution of potassium nitrate (about 0·3 gm. per litre) and the permanganate solution is allowed to rise up the U-tube so as to form a sharp liquid junction in both limbs (see figure). A current of about 0·3 to 0·4 ampere is now passed between the two platinum electrodes dipping in the two parts of the potassium nitrate solution. The level of the permanganate solution is seen to rise in the anode limb and fall in the cathode limb. Thus the violet-coloured constituent of the solution migrates in the direction of the negative current. More detailed investigation proves this constituent to be the negative $MnO_4'$ ions.

In order to demonstrate the migration of ions of *both* signs, the lower part of the U-tube is filled with a mixture of equal volumes of ½-normal solutions of copper sulphate ($CuSO_4$) and potassium dichromate ($K_2Cr_2O_7$), the specific gravity of which has been somewhat increased by the addition of urea. Above this is placed in the two limbs of the U-tube a dilute solution of sulphuric acid or of potassium sulphate. When a current is passed between platinum electrodes dipping in the upper layers, the blue cupric ions are presently seen to rise in the cathode limb and the deep yellow dichromate ions in the anode limb.

We will now consider the questions of the formation of charged ions in solution and of the detailed nature of the transport of electricity through electrolytes.

**Historical.**—It was originally thought that the electric current caused the separation of the electrolyte into its ions, the molecules being as it were torn into pieces by the electric field.*

The experimental results of GMELIN (1838) and POUILLET (1845), according to which the decoloration of a solution of an electrolyte like copper sulphate takes place principally at the cathode and not uniformly throughout the whole liquid (p. 272), presented the first difficulties in the explanation of the phenomena of electrolysis by means of Grotthus's theory (see footnote).

Moreover this theory leads to the conclusion that the greatest resistance to the passage of a current would be offered by those electrolytes whose molecules are composed of atoms held together with the greatest chemical forces. As a matter of fact, however, this is not the case. Experiment shows, for example, that an aqueous solution of potassium chloride, in which compound the chemical linkage between the atoms is a very strong one, has a much higher conductivity than a solution of mercuric chloride, although in the latter compound the atoms are held together by relatively weak forces. Particularly significant also is the fact (BUFF, 1853) that under suitable experimental conditions very small applied potential differences, i.e. very small field strengths, suffice to bring about electrolysis with complete obedience to Faraday's laws. This indicates that no special force is required to decompose the molecules into ions.

It must therefore be assumed that *free ions* are present in every solution of an electrolyte. This conclusion was first drawn by CLAUSIUS (1857).†

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* GROTTHUS (1805) was the first to put forward a complete theory of electrolysis. He assumed the different parts of the electrolyte molecule to be associated with equal quantities of opposite charges. In the case of the KCl molecule, for example, he supposed the K-atom to be charged with $+Q$ and the Cl-atom with $-Q$. According to the theory, when an electric field is applied between two platinum electrodes dipping in a KCl solution, the molecules all become orientated with their positively charged K-side towards the negative electrode and the negatively charged Cl-side towards the positive electrode. All the molecules are then pulled into halves. At each of the electrodes an atom of the opposite sign is deposited and the remaining charged atoms in the space between combine again in neighbouring pairs to form molecules. These newly formed molecules are then turned through 180° by the electric field, and once more torn into halves. This process is repeated continually, one atom being deposited at each electrode for every repetition. The characteristic feature of Grotthus's theory is the continual alternate decomposition and rebuilding of the molecules. The theory was generally accepted until the middle of last century. It was adopted and extended by various workers, amongst them being BERZELIUS and FARADAY.

† CLAUSIUS made the assumption that the molecules of the solute split up during the process of solution. Thereafter he supposed them to be in a state of continual dissociation and recombination. The electric current acts only upon the dissociated molecules, the oppositely charged dissociation products (the ions) being driven in opposite directions by the applied electric field.

**Theory of Arrhenius.**—It still remains to be determined what fraction of the electrolyte is **dissociated** into ions, i.e. **ionized** in solution. The solution of this problem was obtained by consideration of the anomalous behaviour of electrolytically conducting solutions with regard to the depression of the freezing-point and the elevation of the boiling-point of the solvent (ARRHENIUS, 1887).*

It had been shown (Vol. II, p. 63) that the gas laws can be applied to calculate the osmotic pressure of dilute solutions (VAN'T HOFF, 1887), the osmotic pressure $\pi$ taking the place of the gas pressure. If $c$ be the number of gramme-molecules of the solute per litre (i.e. $1/c$ the gramme-molecular volume), then in analogy to the gas equation $pv = \mathrm{RT}$ we have $\pi/c = \mathrm{RT}$ (see Vol. II, pp. 14 and 63), where R is the gas constant and T the absolute temperature. Accordingly it is to be expected that dilute aqueous solutions of different substances but of the same molecular concentration will all have the same osmotic pressure and also (Vol. II, pp. 70 and 79) the same depression of the freezing-point and elevation of the boiling-point.

This is indeed found to be true for many solutes, namely for all those whose solutions do *not* conduct an electric current (see the example given in Vol. II, p. 63). The aqueous solutions of electrolytes, on the contrary, give considerably *higher* values of the osmotic pressure, depression of the freezing-point and elevation of the boiling-point. Electrolyte solutions thus behave as if they contained *more* molecules than would correspond to the molecular concentration reckoned in accordance with the chemical formula of the solute. ARRHENIUS interpreted these anomalies as due to a dissociation of the dissolved electrolyte into ions, and made the assumption that these ions behave in solution like independent molecules. In the case of an electrolyte which is completely dissociated, the molecular concentration will appear to be multiplied by the number of ions into which each molecule dissociates. This is actually found to be the case for a large number of substances in dilute solution. The values for potassium chloride, for example, are twice as great as would otherwise be expected; and the same is true for many other salts of the same type. *We must conclude, therefore, that salts are completely dissociated into anions and cations in dilute solution.*

Great importance attaches to the effects of *variation of concentration.* As we pass to more concentrated solutions we find deviations which may be interpreted as being due to the fact that only a fraction of the molecules are dissociated. The ratio of the number of dissociated molecules in a solution to the total number if none were dissociated is called the **degree of dissociation** or **degree of ionization** of the electrolyte. From what has been said it follows, therefore, that the degree of dissociation is dependent upon the concentration of the solution: it increases as the concentration is decreased. At very low concentrations the degree of dissociation may be put equal to unity; the whole of the solute is then present in the form of ions.

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* SVANTE AUGUST ARRHENIUS, born in 1859 at Wijk near Upsala, died in 1927. He received the Nobel prize in 1903.

According to ARRHENIUS there is an equilibrium in the solution between ions and undissociated molecules, e.g. $K^{\cdot} + Cl' \rightleftharpoons KCl$. With increasing dilution the equilibrium is displaced towards the left, i.e. in favour of the ions, until at infinite dilution the dissociation becomes complete.

Hence if $\alpha$ is the degree of dissociation and $i$ the ratio, for example, of the observed to the normal depression of the freezing-point, we have:

$$\alpha = \frac{i-1}{n-1},$$

where $n$ is the number of ions into which each molecule dissociates. (For KCl, for example, $n = 2$, namely $K^{\cdot}$ and $Cl'$.) In the case in which $n = 2$ (i.e. the solute molecule may split up into 2 ions) and $i = 2$ (i.e. the observed freezing-point depression is twice that which would normally be expected), we obtain $\alpha = 1$. The dissociation is therefore complete. If, on the other hand, $i$ were equal to 1 (i.e. if the freezing-point depression were normal), we should have $\alpha = 0$. This would mean that the solution contained no ions, the solute being completely undissociated. In this case there would be practically no electrical conductivity on account of the absence of material carriers of charge.

One of the strongest supports for the views of ARRHENIUS is the *variation of the equivalent conductivity* (p. 250) with change of concentration. It is obvious that, other things being equal, the greater the number of ions, the greater also the conductivity. Thus the greater the degree of dissociation, the greater the conductivity for a given quantity of solute (§ 8, p. 248). This is exactly what had been shown by KOHLRAUSCH to occur with increasing dilution (see fig. 43, p. 251, and the imagined experiment of p. 250). The limiting value $\lambda_\infty$ (p. 251) is reached when all the molecules are dissociated, i.e. when the degree of dissociation $\alpha$ becomes equal to unity. When $\alpha < 1$, the observed equivalent conductivity $\lambda$ is only the fraction $\alpha$ of the limiting value, i.e.

$$\lambda = \alpha\lambda_\infty.$$

This formula is important in that it offers a way of calculating the degree of dissociation from the observed values of $\lambda$ and $\lambda_\infty$. The value of $\lambda_\infty$ can be obtained, for example, by extrapolation of the curve (fig. 43, p. 251) representing $\lambda$ as a function of concentration, or from other data (e.g. see p. 275).

Thorough experimental investigation has shown that the theory of ARRHENIUS cannot be maintained in the form given above. The main difficulties in the theory are as follows. The variation of the degree of electrolytic dissociation with concentration is calculated in analogy with the chemical decomposition of neutral molecules (Law of Mass Action). The results so obtained are in agreement with experiment in the case of weakly dissociated electrolytes (so-called weak electrolytes), but completely at variance with experiment in the case of strongly dissociated (so-called strong) electrolytes. Secondly, it is found that the degree of dissociation calculated from the depression of the freezing-point or the elevation of the boiling-point is by no means always the same as that obtained from conductivity measurements. Moreover, the increase of solubility of slightly soluble salts by the addition of salts with a common ion cannot be explained by Arrhenius's theory. The conclusion is, therefore, that the theory of Arrhenius is in harmony with observation in the case of weak electrolytes, but that it fails to account for the behaviour of strong electrolytes.

The observed facts can be explained by assuming strong electrolytes to be *completely dissociated at all concentrations* (SUTHERLAND and BJERRUM). On account of the electrostatic forces between them the oppositely charged ions are not completely free to move. It is this mutual influence (**interionic forces**) which causes the variation of the equivalent conductivity or of the freezing-point depression, &c., with change of concentration. The quantitative treatment of the problem, which cannot be discussed further here, has been supplied chiefly by DEBYE and HÜCKEL. It gives an explanation of the differences between the "degrees of dissociation" obtained from conductivity measurements and freezing-point depression determinations, and allows of the calculation of values which are in agreement with observation over a considerable range (see further p. 277).

## 2. Migration of Ions. Transport Numbers

**Migration of Ions.**—In accordance with what has been said above, we have to assume that there is an approximately uniform distribution of positive and negative ions throughout a solution of an electrolyte. The application of a potential difference between two electrodes, the anode A and the cathode K, dipping into the solution will cause the ions to be set into motion.

The current may consist either of the motion of positive electricity from A to K, or of the motion of negative electricity from K to A, or of both of these processes simultaneously. Suppose, for example, that the quantity of positive electricity $+Q_1$ flows from A to K in unit time and that the quantity of negative electricity $-Q_2$ flows in the same time from K to A. The total current strength, reckoned as positive from A to K, is then equal to $Q_1 + Q_2$. In solutions of electrolytes both the positively charged and also the negatively charged ions take part simultaneously in the transport of the electricity.

Let us consider the processes in detail with the help of fig. 2 (after Hittorf, 1853). Let the vessel containing the electrolyte solution be bounded by the electrodes K and A and also divided as shown into three compartments by means of permeable partitions. The cations of the electrolyte (e.g. in the case of HCl the hydrogen ions H˙) are represented (within a circle) by +, the anions (e.g. Cl′) by —, and the deposited atoms or discharged ions by a plain circle. At the top of the figure (*a*) is shown the state of affairs before the potential difference is applied. Next below this (*b*) is shown the case in which both the anions and the cations migrate with the same velocity. After four ions have given up their charges at each electrode, the number of anions which has passed from left to right through the partitions is the same as the number of cations which has passed through in the opposite direction. Hence, as is seen from the figure at (*b*), the concentration has remained constant in the central compartment and has decreased by the same amount (namely to 0·6 of its original value) in each of the electrode compartments. At (*c*) is shown the case in which only the anion migrates. Consider again the moment at which four anions have become discharged at the anode and have either been liberated there in solid or gaseous form or have entered into secondary reactions liberating other products. This discharge of negative ions at the anode tends to make the liquid correspondingly positive; but since the liquid is in conducting connexion with the cathode, this tendency is at once suppressed by the supply of four negative charges (electrons) from the cathode. These negative charges neutralize four cations in the immediate neighbourhood of the cathode. (It is assumed that the liquid in the electrode compartments is sufficiently mixed by stirring or diffusion, so that there are always enough ions in the immediate neighbourhood of the electrodes.) The neutralized cations are either deposited in the solid or gaseous form or undergo secondary reaction (e.g. Na). Since now by assumption the cations have not migrated, there must be a fall of concentration of the electrolyte in the cathode compartment. We see that as the electrolysis is continued the electrolyte eventually migrates completely out of the left-hand part of the vessel. This causes such a dearth of ions near the cathode that the electrolysis is automatically brought to a standstill. The lowest section (*d*) of fig. 2 represents the case in which the velocity of migration of the anion is twice that of the cation. Here the concentration of the electrolyte is decreased near both electrodes but remains constant in the central compartment.

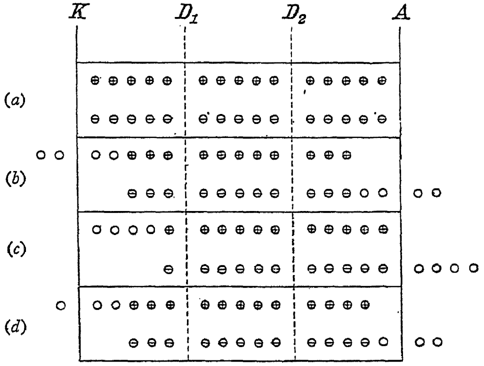

Fig. 2.—Change of concentration near the electrodes owing to the migration of the ions

These considerations show that the concentration changes near the electrodes are connected with the migration velocities of the ions. If the fall of concentration at the anode is $D_a$ and that at the cathode $D_k$, then obviously $D_a : D_k = u : v$, where $u$ and $v$ are the migration velocities of the cations and anions respectively. In the three cases (*b*), (*c*) and (*d*) of fig. 2 we have respectively $D_a/D_k = u/v = 1$, 0, and $\frac{1}{2}$.

Hittorf * was the first to recognize the important part played by ionic migration in electrolysis. His classical researches (1853–59) were carried out with the apparatus shown in fig. 3. The vessel containing the electrolyte solution was made in several separate parts A, B, C, D, and E, each of which was closed below with parchment paper and fitted on to the next as shown. In agreement with the result of the above considerations Hittorf found that the concentration of the electrolyte remained constant in the central layers, but that there were changes of concentration in the electrode compartments A and E.

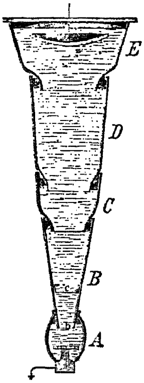

Fig. 3.—Hittorf's apparatus for the investigation of the changes of concentration due to electrolysis.

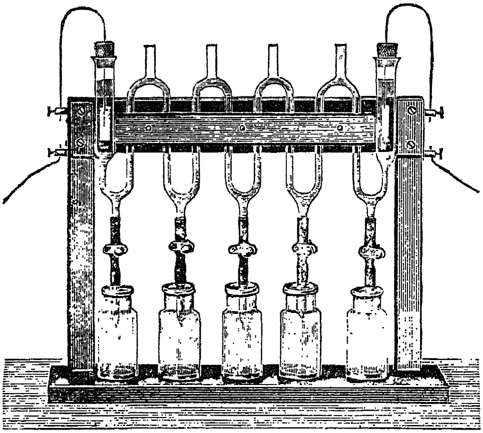

Fig. 4.—Determination of changes of concentration due to electrolysis

Fig. 4 depicts a simplified apparatus consisting of a tube bent several times as shown and provided with glass side tubes at the upper and lower bends. The upper side tubes are open; the lower ones carry pieces of rubber tubing closed with clips. The ends of the tube are somewhat wider, so as to admit platinum electrodes. An ammeter to register the current strength is included in the circuit. The whole of the glass apparatus is filled with normal sulphuric acid solution. On account of the high resistance a potential difference of say 110 volts should be used. When the current is switched on, hydrogen is evolved at the cathode and oxygen at the anode, i.e. the hydrogen ions migrate to the cathode and are discharged and liberated there, while $SO_4''$ (or $HSO_4'$) migrates to the anode and is discharged there, liberating an equivalent quantity of oxygen from the water in accordance with the equation:

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* Joh. Wilh. Hittorf, born in 1824 at Bonn, from 1852 onwards Professor of Physics at the Academy (subsequently University) of Münster, died in 1914.

$$SO_4 + H_2O \rightarrow 2H^{\cdot} + SO_4'' + O \text{ or } H^{\cdot} + HSO_4' + O.$$

The result of this chemical reaction is an increase in the sulphuric acid concentration at the anode. This increase can be measured by titration.

The results of an actual experiment were as follows: A current of 0·05 ampere was passed through the apparatus for 10 hours. The central U-section of the tube was then emptied into the corresponding vessel (see figure). In this way the anode and cathode liquids were completely separated. The other sections of the tube were then emptied into the respective vessels and an exactly equal volume of normal caustic soda solution added to each. Addition of a small amount of litmus showed that the middle three portions of the liquid were neutral, but that the anode liquid was acid and the cathode liquid alkaline. The anode liquid actually required 4·14 cm.$^3$ of normal caustic soda, the cathode liquid 4·14 cm.$^3$ of normal sulphuric acid, to effect neutralization.

A current of 0·05 ampere flowing for 10 hours, i.e. a quantity of electricity equal to 0·5 ampere-hours, liberates 0·5 . 37·29 = 18·65 milligramme-equivalents (p. 157). If the whole current had been due to the migration of $SO_4''$ anions to the cathode, an addition of 18·65 cm.$^3$ of normal solution would have been required for neutralization. The quantity actually required, viz. 4·14 cm.$^3$ of normal solution, is only $\frac{2}{9}$ of this. Hence only $\frac{2}{9}$ of the current can have been due to the migration of $SO_4''$ ions. The remaining $\frac{7}{9}$ must have been borne by the migration of the $H^{\cdot}$ ions in the opposite sense.

**Transport Numbers.**—The above result may be expressed as follows: During the deposition of one equivalent at the cathode the quantity of cations in the cathode compartment does not also decrease by one equivalent, but by $\nu$ equivalents (where $\nu < 1$), because cations simultaneously migrate into this compartment. The number of equivalents of cations entering the cathode compartment during the time that $\nu$ equivalents of anions are leaving this compartment is $1-\nu$. Hence of the electricity passing through the central portion of the vessel (e.g. through the partition $D_1$ in fig. 2, p. 271), the fraction $\nu$ is borne by the anions towards the anode and the fraction $1-\nu$ by the cations towards the cathode. But these fractions are proportional to the velocities of migration of the respective ions, i.e.

$$u \quad 1 - \nu$$

The number $\nu$, i.e. the fraction of the current borne by the anions, was named by HITTORF the *transport number* of the anion. The transport number of the cation is therefore $1 - \nu$.

Table VII (p. 274) contains a few transport numbers for $\frac{1}{10}$-normal solutions. The values for compounds giving rise to $OH'$ and $H^{\cdot}$ differ very markedly from 0·5. This indicates, for example, that $OH'$ migrates faster than $Na^{\cdot}$ and that the velocity of migration of $H^{\cdot}$ is about six times greater than that of $Cl'$.

The *transport numbers for solid electrolytes* are remarkable in that the value for one of the ions is in many cases equal to unity (TUBANDT). The conductivity is then due to the cations alone or the anions alone. This is a realization of the case (*c*) of fig. 2, p. 271. In AgCl, AgBr, NaF, and NaCl only the cations can move; in $BaCl_2$, $BaBr_2$, $PbF_2$, and $PbCl_2$ only the anions. In these cases, however, the transport numbers vary considerably with the temperature. Thus at lower temperatures $PbCl_2$ exhibits anion conductivity, but at higher temperatures the conductivity is chiefly due to the cations.

TABLE VII

TRANSPORT NUMBERS OF ANIONS IN $\frac{1}{10}$-NORMAL AQUEOUS SOLUTION

| | | | |
|---|---|---|---|
| KCl, | 0·506 | LiCl, | 0·687 |
| NaCl, | 0·607 | $KNO_3$, | 0·497 |
| KOH, | 0·735 | HCl, | 0·164 |
| NaOH, | 0·82 | $\frac{1}{2}H_2SO_4$, | 0·185 |

## 3. Conditions for the Validity of Ohm's Law

The considerations of the preceding section enable us to obtain a deeper insight into the relation between current strength and the potential difference and resistance in the case of an electrolyte.

Consider a liquid column $s$ cm. in length enclosed in a tube of uniform cross-sectional area A cm.². Suppose that there are $n$ ions of each sign per 1 cm.³ and that the charge carried by each ion is $e$. Then when a potential difference U is applied between the ends of the column, the ions will migrate. The quantity of electricity transported per second through any cross-section of the tube will be $+neAu$ towards the cathode and $-neAv$ towards the anode. The resultant current strength will therefore be $I = neA(u + v)$.

The resistance R is defined (p. 220) as the quotient U/I. Hence, introducing the field strength $E = U/s$, we have:

$$R = \frac{s}{A} \frac{1}{ne} \cdot \frac{E}{u + v}.$$

Now Ohm's law states that the resistance is independent of the field strength (p. 220). From the experimentally proved fact that Ohm's law holds for electrolytes we deduce that both $ne$ and also $E/(u + v)$ must be independent of the field strength.

The first condition, namely the constancy of $ne$, means that the total charge of the ions per unit volume must be independent of the field strength.

The second condition, namely the constancy of $E/(u + v)$, means that the sum of the migration velocities must be proportional to the field strength. In accordance with their physical significance we may also set each of the individual migration velocities proportional to the field strength, i.e. $u = l_k E$ and $v = l_a E$, where $l_k$ and $l_a$ are the respective velocities of migration when $E = 1$, i.e. under a field strength of 1 volt per cm. Substituting these values in our equation for R, we obtain:

$$R = \frac{s}{A} \cdot \frac{1}{ne} \cdot \frac{1}{l_k + l_a}.$$

The fact that the action of a constant electric force upon an ion does not impart to it an accelerated motion but only causes it to move with constant velocity is due to the *frictional resistance* to which the ion is subjected. In the case of a particle subjected to a frictional resistance which increases with the velocity, the effect of a constant force is merely to increase the velocity until the frictional force just balances the moving force, after which the velocity remains constant (Vol. I, p. 372). Thus the motion is only accelerated for the first instant after the application of the force (cf. p. 307).

The conditions for the validity of Ohm's law are therefore as follows: (1) The concentration of the carriers of electricity must be independent of the field strength (and therefore also independent of the current process). (2) The velocity of migration of the carriers must be proportional to the field strength. Only when these conditions are fulfilled can Ohm's law hold, and conversely the validity of Ohm's law for a particular process of conduction proves that the mechanism of the process is such as to fulfil the conditions. This is by no means always the case, as will be shown later (pp. 302 and 354).

## 4. The Variation of Equivalent Conductivity with Concentration. The Principle of the Independent Migration of Ions.

As was mentioned on p. 251, the equivalent conductivity approaches a limiting value $\lambda_\infty$ as the dilution is increased. Remarkable regularities have been found to exist amongst the $\lambda_\infty$-values of different salts. This is illustrated by the following data:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| KCl | 130·10 | $KNO_3$ | 126·50 | NaCl | 108·99 | LiCl | 98·88 |
| NaCl | 108·99 | $NaNO_3$ | 105·33 | $NaNO_3$ | 105·33 | $LiNO_3$ | 95·18 |
| | 21·11 | | 21·17 | | 3·66 | | 3·70 |

We see that the difference between the $\lambda_\infty$-values of two corresponding salts with a common ion is independent of the nature of that ion. This can only be explained by assuming that $\lambda_\infty$ is made up of two independent parts, one characteristic of the cation, and the other of the anion. Since the conductivity is due to the migration of the ions, it is obvious that these parts of $\lambda_\infty$ must be related to the migration velocities. We have $\lambda_\infty = C(l_a + l_k)$, where $l_a$ and $l_k$ are the velocities of migration in unit field and C is a constant whose value is yet to be determined. We conclude therefore that:

*At very great dilution the anions and cations move with their own characteristic velocities and transport their respective fractions of the current independently of each other.* (KOHLRAUSCH: **Principle of the Independent Migration of Ions.**)

**Calculation of the Equivalent Conductivity at Infinite Dilution.**—It follows that $\lambda_\infty$ can be calculated from the migration velocities of the ions, as obtained from the transport numbers. We will now carry out this calculation.

Putting $1/R = K$, the conductivity, we may write the equation for R on p. 275 in the form:

$$K = ne(u + v)\frac{A}{sE} = ne(l_k + l_a)\frac{A}{s}.$$

The value of K for a cube of 1 cm. edge being $\varkappa$, we have:

$$\varkappa = ne(l_k + l_a).$$

In order to obtain the relationship with the equivalent conductivity, we must introduce the connexion between the number of ions and the total number of dissolved molecules. If $m_k$ is the mass of a cation, $nm_k$ is the total mass of all the cations taking part in the process of conduction. The total mass of the cations which would be produced by complete dissociation of the solute being $M_k$, then $nm_k = \alpha M_k$, where $\alpha$ is the degree of dissociation.

Since $e/m_k = 96{,}494/W_k$, where $W_k$ is the equivalent weight of the cation, we may write:

$$\varkappa = n(l_k + l_a)m_k.\frac{96{,}494}{W_k} = \alpha(l_k + l_a)M_k.\frac{96{,}494}{W_k}.$$

Here $M_k/W_k$ is the number of gramme-equivalents per cm.$^3$. Putting this equal to $c$, we obtain:

$$\varkappa = \alpha c(l_k + l_a).96{,}494, \text{ or } \frac{\varkappa}{c} = \lambda = \alpha(l_k + l_a).96{,}494,$$

where $\lambda$ is the equivalent conductivity defined on p. 250.

The quantities $96{,}494\, l_a = U$ and $96{,}494\, l_k = V$ are called the **ionic mobilities**. Defined in this way they have the dimension [ohm$^{-1}$]. In order to get the *absolute ionic mobilities* $l_k$ and $l_a$, i.e. the velocities in cm. per sec. for a field strength of 1 volt per cm., we must divide the above mobilities, expressed in reciprocal ohms, by the factor 96,494 or multiply them by the factor $1/96{,}494 = 0{\cdot}000010363$. From above we have $\lambda = \alpha(U + V)$, and since $\lambda/\lambda_\infty = \alpha$ (p. 269):

$$\lambda_\infty = U + V = 96{,}494(l_a + l_k).$$

Thus the undetermined constant C of p. 275 has the value 96,494 when the $l$-values are expressed in terms of the cm., sec., and volt. Conversely, we can obtain the sum of the ionic mobilities when the value of the equivalent conductivity at infinite dilution in known. In this case we can also calculate the sum of the ionic velocities. Finally, since the ionic mobilities and ionic velocities are in the ratio of the respective transport numbers, we can also calculate the actual ionic mobilities and velocities from their respective sums.

*Example.*—The equivalent conductivity of sulphuric acid at infinite dilution is $\lambda_\infty = 388$, i.e.

$$U + V = 388.$$

Now the transport number of $SO_4''$ in very dilute sulphuric acid is $\nu = 0{\cdot}212$ (see p. 273). The transport number of the cation $H^{\cdot}$ in the same solution is therefore $1-\nu = 0{\cdot}788$. Hence for the mobilities of the ions we obtain:

$U = 388 \,.\, 0{\cdot}788 = 306$, and $V = 388 \,.\, 0{\cdot}212 = 82$. The absolute mobilities are found from these figures by multiplication by 0·00001036. The values obtained are:

$$u_{H^{\cdot}} = 0{\cdot}0032 \frac{\text{cm./sec.}}{\text{volts/cm.}} \text{ and } v_{SO_4''} = 0{\cdot}00085 \frac{\text{cm./sec.}}{\text{volts/cm.}}.$$

TABLE VIII

ABSOLUTE MOBILITIES OF IONS (I.E. VELOCITIES OF MIGRATION IN CM./SEC. IN A FIELD OF 1 VOLT/CM.)

| | | | |
|---|---|---|---|
| $Li^{\cdot}$ | 0·000346 | $Cl'$ | 0·000679 |
| $K^{\cdot}$ | 0·000669 | $NO_3'$ | 0·000639 |
| $H^{\cdot}$ | 0·003263 | $OH'$ | 0·001803 |

Table VIII gives the absolute mobilities of a number of ions. One striking fact is that the mobilities of the $H^{\cdot}$ and $OH'$ are relatively very large (see also p. 278). This is probably connected with the fact that they are also constituents of the solvent water, in consequence of which peculiar possibilities of migration (by exchange) are opened up. In the introductory experiment described on p. 266 it was the absolute mobility (multiplied by the field strength) which was observed. The progression of the red zone in the first experiment indicated the migration of the $MnO_4'$ ions. In the second experiment the blue $Cu^{\cdot\cdot}$ ions migrated up the cathode limb and the yellow $Cr_2O_7''$ ions up the anode limb. By this method the calculated migration velocities may be checked by direct experiment.

**Deviations from Ohm's Law in Electrolytes.**—In the above treatment it has been assumed that $l_a$ and $l_k$ are independent of the concentration; only when this is true is it permissible in calculating the equivalent conductivity at infinite dilution to use the same value of $(l_a + l_k)$ as in the calculation of $\lambda$ at higher concentrations. Now according to the recent theories already mentioned on p. 270, this assumption is not quite justified. Without going more deeply into these theories it is at once obvious that the electrostatic forces between the ions must have an effect upon their mobilities, and that this effect will vary with the concentration. In the neighbourhood of a cation, for example, there will (on the average) be a preponderance of anions over cations; for the anions will be attracted and the cations repelled. We must conclude, therefore, that each cation forms the centre of a cluster of anions (known sometimes as its ionic atmosphere). When an electric field is applied to the solution, the cation tends to migrate in the opposite direction from its ionic atmosphere; this atmosphere thus disappears behind the cation as it moves, and is built up anew in front of it. Since these processes take a certain time, the charge distribution around the central ion is not symmetrical, so that the ion is subjected to a resultant braking force. Hence when currents of very short duration are used in the conductivity measurement, i.e.

alternating currents with a period shorter than the time required to establish the lack of symmetry of the ionic atmospheres, a different value of the conductivity is obtained from that observed with direct current. Moreover *the conductivity must be dependent upon the applied potential difference,* since the formation of the asymmetry obviously depends upon the field strength (DEBYE). The experimental discovery of these deviations from Ohm's law (Wien effect, M. WIEN) provides strong support for the views outlined above.

**Hydration of Ions.**—Conductivity measurements have revealed the striking fact that the electrical resistance of electrolyte solutions varies in a parallel manner to the viscosity (Vol. I) of the *solvent.* This indicates that during the passage of the current the particles of the *solvent* are displaced relatively to one another, i.e. that the ions are surrounded by shells of solvent molecules. This is known as *solvation* or in the case of water *hydration.* The quantities of water transported by the ions can also be measured directly by enclosing the electrodes in suitable diaphragms. The calculation of the absolute ionic mobilities offers the possibility of determining the size of the moving complexes. The number of solvent molecules associated with each ion can only be obtained roughly in this way. For cations it varies between 4 and 15 (according to LENARD and others), while for $H^{\cdot}$ it is probably only 1. Thus the hydrogen ion probably exists in solution in the form $H_3O^{\cdot}$. The solubility of a substance in a liquid appears to be due to a large extent to the formation of solvated complexes.

CHAPTER IX

# Transformation of Chemical into Electrical Energy and the Reverse. Calculation of E.M.F.s of Galvanic Cells

## 1. Electrolytic Solution Pressure

In § 18, p. 105, we have already recognized as a general property of matter the transference of electricity between two different substances placed in contact with one another. This transference is eventually automatically stopped by the formation of an electrical double layer. This takes place, for example, when a metal is placed in a liquid (see fig. 129, p. 110). We will now consider more closely the case of a metal dipping into a solution of one of its own salts. The action of a Daniell cell (p. 42) depends upon the passage of zinc into the ionic state at the zinc electrode, i.e. $Zn \rightarrow Zn^{\cdot\cdot} + 2(-)$, and the passage of copper ions into the metallic state at the copper electrode, i.e. $Cu^{\cdot\cdot} + 2\,(-) \rightarrow Cu$. These two processes take place spontaneously until they are stopped by the opposing fields of the double layers to which they give rise. But by destroying the double layers, namely by allowing the excess electrons at the zinc electrode to pass through an electric conductor to the copper electrode, where there is a deficit of electrons, the processes may be kept in progress and made to yield energy continuously. Our problem is to calculate the quantity of energy derivable from such a reaction. The electrical energy is given by the product of quantity of electricity and potential difference. The quantity of electricity associated with one gramme-equivalent is always 96,494 coulombs. The problem therefore reduces to calculating the work which can be obtained by the passage of 1 gramme-equivalent of metallic zinc into the ionic state, i.e. to calculating the potential difference involved in this process. In order to be able to treat the problem quantitatively we will make use of a concept which was introduced by Nernst in 1889, and which has proved to be of great value.

For every metal we may assume a certain pressure, the **electrolytic solution pressure,** tending to drive it into solution in the form of ions when it is placed in a liquid.

This pressure is analogous to the vapour pressure of a liquid, which is a measure for the tendency of the liquid molecules to leave the liquid and pass over into the vapour state.

Similarly we may regard the process of solution of a substance like cane sugar in water as due to a solution pressure driving the molecules into the liquid phase. The molecules which have passed into solution also exhibit a pressure, the osmotic pressure (Vol. II, p. 61), acting in opposition to the solution pressure and tending to make the solution more dilute. The final equilibrium between these two opposing tendencies is a dynamic one in which there is a continual passage of molecules into and out of solution. In the case of solid solute in contact with unsaturated solution more molecules are driven into solution by the solution pressure than are driven out again by the osmotic pressure. When the solution is supersaturated the reverse is true. In the same way the *electrolytic* solution pressure acts in opposition to the osmotic pressure of the ions.

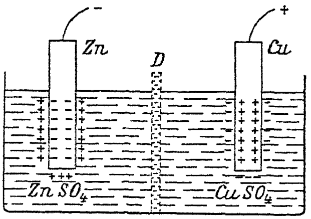

Fig. 1.—Double layers in the Daniell cell

The magnitude of the potential difference involved is therefore determined by the difference between the electrolytic solution pressure of the metal and the osmotic pressure of its ions in the solution. Since the latter varies with the concentration, it follows that the potential difference between the metal and the solution must also vary with the concentration of the dissolved salt. This is borne out by experiment (p. 283).

The potential difference we are considering is known as the **electrolytic potential** or **electrode potential** of the metal relative to the solution. In stating the value of such a potential it is necessary also to give the concentration of the salt solution. For many metals the electrolytic solution pressure is extremely small: a minute concentration of the metal ions in solution is sufficient to produce an osmotic pressure exceeding the solution pressure of the metal. This is the case, for example, for copper. When a piece of copper is dipped into the solution of a copper salt, the copper ions are deposited upon the metal until the process is stopped by the double layer to which it gives rise (fig. 1). Thus copper becomes positively charged when dipped into a solution containing $Cu^{\cdot\cdot}$ ions. The **electromotive force** (E.M.F., p. 137) of a cell is now clearly equal to the difference between the potentials of the two electrodes relative to the solutions in which they dip.

Thus if $-E_1$ (the sign is minus because zinc becomes negatively charged) is the potential of a zinc electrode relative to normal zinc sulphate solution and $+E_2$ the potential of a copper electrode relative to normal copper sulphate solution, then the E.M.F. of the resultant Daniell cell is equal to $E_2 - (-E_1) = E_2 + E_1$. *

## 2. Relation between E.M.F. and Heat of Reaction

The maximum amount of work which can be obtained from a chemical reaction is not determined alone by the quantity U of heat evolved when no external work is performed, but by the free energy H, which may be either greater or smaller than U (Vol. II, § 13, p. 148). We have the general equation (HELMHOLTZ),

$$H = U + T\frac{dH}{dT},$$

where T is the absolute temperature. Only when $\frac{dH}{dT}$, the differential coefficient of the free energy with respect to the temperature, is very small, can we put $H = U$.

In the case of a zinc rod dipped into copper sulphate solution, for example, the zinc passes into solution and copper is deposited. A certain amount of heat is liberated during the process and the temperature of the whole system rises. The quantity of heat per gramme-atom of zinc and gramme-molecule of copper sulphate is 50,110 cal. When the process is allowed to take place in a Daniell cell, there is no appreciable evolution of heat and the process can perform work, e.g. by driving a motor in the external circuit. The quantity of electrical energy obtainable is given by the product of 2 . 96,494 coulombs (factor 2 because Zn and Cu are divalent) and the E.M.F. of the cell in volts (E say). Transforming to calories we have (§ 2, p. 255) $H = 2\,.\,96{,}494\,.\,0{\cdot}239E$ cal. Putting this equal to U, i.e. equal to 50,110, we can calculate E. This involves neglecting the term $-T\frac{dH}{dT}$. The value obtained is $E = 1{\cdot}0863$ volt. This is very near the value actually found in experiment. In this case, therefore, we are justified in neglecting the term $-T\frac{dH}{dT}$. When the valency is $n$ the factor 2 in the above calculation must be replaced by $n$.

As a rule $\frac{dH}{dT}$ cannot be neglected. Since $dH = 96{,}494\,.\,0{\cdot}239n\,dE = 23063n\,dE$ we have in general:

$$E = \frac{U}{23{,}063n} + T\frac{dE}{dT}.$$

---

* To this must be added also the potential difference between the solutions. This may, however, be neglected for the time being (p. 283).

Hence the E.M.F. of a cell can be calculated exactly from the heat of reaction of the processes involved and the temperature coefficient of the E.M.F. The above equation has been satisfactorily verified by accurate measurements.

**Calculation and Determination of Individual Electrode Potentials.**—Nernst has suggested another method of quantitative treatment of the relation between electrode potential, electrolytic solution pressure and osmotic pressure. The maximum work which can be obtained by the expansion of *one* gramme-molecule (i.e. $\nu = 1$) of an ideal gas is $W = RT \log_e(p_1/p_2)$ (Vol. II, p. 102), where R is the gas constant, T the absolute temperature, and $p_1$ and $p_2$ the initial and final pressures of the gas respectively. Nernst's idea is to apply this equation to the electrolyte solution pressure P (e.g. of the zinc) and to the osmotic pressure $p$ of the ions in the solution. This gives $W = RT \log_e(P/p)$. But this amount of electrical work obtainable by the transference of one gramme-atom of the metal (e.g. zinc) into the ionic state is also equal to the product of $96{,}494n$ coulombs ($n =$ valency) and the potential E of the metal relative to the solution. Hence:

$$E = -\frac{RT}{96{,}494n} \log_e \frac{P}{p} = -\frac{0{\cdot}000198\,T}{n} \log \frac{P}{p} \text{ volts.}$$

Here R has been transformed to volt-coulombs with the help of the thermal equivalent (giving $R = 8{\cdot}316$ volt-coulombs) and the natural logarithm has been reduced to an ordinary logarithm by dividing by 0·4343. The negative sign corresponds to the fact that the solid metal becomes negatively charged relative to the solution as a result of the ionization.

At room temperature ($T = 273 + 18 = 291$) we have:

$$E = -\frac{0{\cdot}058}{n} \log \frac{P}{p} \text{ volts.}$$

When $p > P$, as in the case of copper (p. 283), the sign of E is reversed; the metal then assumes a *positive* potential relative to the solution of one of its salts.

*Individual electrode potentials cannot be determined by direct experiment.*—In all measurements we necessarily have to use a cell with two electrodes, so that it is only a difference of electrode potentials which is actually observed. It would be possible to determine single electrode potentials, if the second electrode of the cell were such that its potential relative to the solution were zero. So far, however, a satisfactory electrode of this type has not been made. For most purposes (e.g. for the calculation of the E.M.F.s of compound cells) it is sufficient to know relative values. Hence in practice electrode potentials are expressed relative to an abitrarily fixed standard, which is taken as zero. The standard adopted is the **normal hydrogen electrode**, first suggested by NERNST. This consists of an electrode of platinized platinum containing absorbed hydrogen and dipping in an acid with a hydrogen ion concentration of 1 gm. per litre. This concentration corresponds, for example, to approximately 2-normal sulphuric acid at room temperature. Since the osmotic pressure $p$ of the ions in the solution is proportional to the concentration, the numerical value of electrode potentials usually quoted refers to an ionic concentration of one gramme-equivalent per litre. Referred to the normal hydrogen electrode as standard, the electrode potentials are then known as *standard electrode potentials*. Table IX (after DRUCKER) gives the values for a number of different substances.

TABLE IX

STANDARD ELECTRODE POTENTIALS

| Process | Volt | Process | Volt |
|---|---|---|---|
| $Li \rightleftharpoons Li^{\cdot}$ | −3·02 | $4OH' \rightleftharpoons O_2 + H_2O$ | +0·41 |
| $K \rightleftharpoons K^{\cdot}$ | −2·92 | $2I' \rightleftharpoons I_2$ (solid) | +0·58 |
| $Zn \rightleftharpoons Zn^{\cdot\cdot}$ | −0·76 | $2Br' \rightleftharpoons Br_2$ (dissolved) | +1·087 |
| $Fe \rightleftharpoons Fe^{\cdot\cdot}$ | −0·44 | $2Cl' \rightleftharpoons Cl_2$ (gas) | +1·36 |
| $Ni \rightleftharpoons Ni^{\cdot\cdot}$ | −0·25 | | |
| $Pb \rightleftharpoons Pb^{\cdot\cdot}$ | −0·13 | $Cu^{\cdot} \rightleftharpoons Cu^{\cdot\cdot}$ | +0·167 |
| $H_2 \rightleftharpoons 2H^{\cdot}$ | 0·000 | $Sn^{\cdot\cdot} \rightleftharpoons Sn^{\cdot\cdot\cdot\cdot}$ | +0·2 |
| $Sb \rightleftharpoons Sb^{\cdot\cdot\cdot}$ | +0·2 | $Fe^{\cdot\cdot} \rightleftharpoons Fe^{\cdot\cdot\cdot}$ | +0·75 |
| $Cu \rightleftharpoons Cu^{\cdot\cdot}$ | +0·345 | $H_2 + 2OH' \rightleftharpoons 2H_2O$ | −0·82 |
| $Ag \rightleftharpoons Ag^{\cdot}$ | +0·81 | $Pb^{\cdot\cdot} + 2H_2O \rightleftharpoons PbO_2 + 4H^{\cdot}$ | +1·44 |
| $Au \rightleftharpoons Au^{\cdot}$ | +1·5 | | |

These are not only useful for the calculation of the E.M.F.s of voltaic cells, they are of fundamental significance for the whole chemical behaviour of the respective substances. This cannot, however, be discussed further here.

**Concentration Cells.**—A very striking example of the difference of the electrode potential of a metal in solutions of different ionic concentration is provided by the following experiment.

In a vertical glass tube is placed a layer of concentrated copper sulphate solution with a layer of very dilute solution of the same salt above it. Into each of these layers is dipped a copper rod. It is then found that there is a potential difference between the two rods. When they are joined by a conducting wire, a current flows in the direction from the concentrated to the dilute solution in the external part of the circuit. Within the liquid the current is from the dilute to the concentrated solution. It is accompanied by electrolysis, the $Cu^{\cdot\cdot}$ ions migrating to the cathode, where they give up their positive charges, and the $SO_4''$ ions migrating towards the anode, where they give up their negative charges. Consequently copper is deposited upon the rod dipping into the more concentrated solution, while at the other rod the sulphate radicals react at once with the copper, forming more copper sulphate. On the whole, therefore, there is an increase in the concentration of the more dilute solution and vice versa. The process comes to an end when the two solutions reach the same concentration.

Since the equalization of concentration may also occur by diffusion, it is best to use two tubes as shown in fig. 2. These are identical except that the one on the right has its electrodes connected by a conducting wire, whereas the electrodes of the one on the left (the comparison tube) are unconnected. The effects of diffusion are then the same for both tubes, and any difference in their behaviour must be due to the current alone. Actually it is observed that the equalization of concentration takes place considerably more rapidly in the tube with its electrodes connected than in the comparison tube. In the right-hand tube the dilute solution becomes intensely blue near the copper electrode after one day, whereas in the left-hand tube no such coloration can be observed.

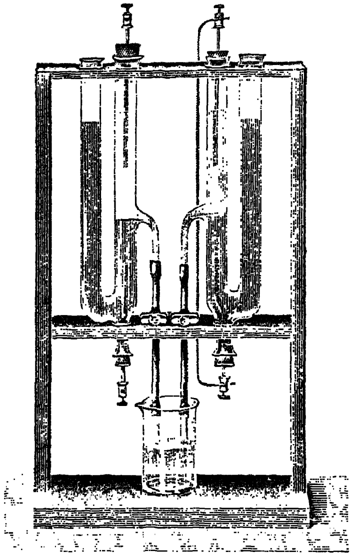

Fig. 2.—Equalization of concentration by simple diffusion and by electrical transport

Since the copper does not react directly with the copper sulphate, the cause of the potential difference and of the ionic migration accompanying the current can only lie in the difference of concentration of the two parts of the electrolyte. For this reason a cell of this kind is called a *concentration cell.*

It is interesting to modify this experiment by using electrodes of an indifferent material. The apparatus shown in fig. 2 may again be used and the solutions may be normal HCl and 1/1000-normal HCl. The electrodes are placed as near as possible to the liquid junction. Once again there is a potential difference, the electrode in the more concentrated solution becoming the cathode. The cause for this E.M.F. lies in the difference of the migration velocities of the $H^{\cdot}$ and $Cl'$ ions (see below).

**Kinetic Theory of the Processes in Voltaic Cells.**—The molecular mechanism of the phenomena at present under consideration can be most easily understood in the case of concentration cells.

Take for example the hydrochloric acid concentration cell described above. The ions of the electrolyte, viz. $H^{\cdot}$ and $Cl'$, diffuse from regions of higher concentration to regions of lower concentration. The two kinds of ions, however, have different mobilities and therefore move at different velocities. Hence they tend to become separated from one another and are consequently subjected to restraining electrical forces arising from their opposite electric charges. These forces establish equilibrium with the forces tending to produce the motion. The latter are the same forces which cause the displacements of suspended particles in the Brownian movement; their origin is the *thermal kinetic energy of the molecules;* with a different experimental arrangement they may be observed as osmotic pressure. The charge displacements arising in this way in the liquid are associated with potential differences between different points. When, therefore, a conducting wire is dipped into the liquid so as to connect these points, a current is produced. This removes the potential differences and the diffusion now proceeds more rapidly than before, being no longer restrained by electric forces.

These considerations show that a potential difference must always occur between two layers containing the same electrolyte at different concentrations. The E.M.F. of such a liquid junction can be calculated in a manner similar to that for ordinary electrodes. We cannot, however, go into this here. The essential factor is obviously the ratio of the migration velocities. No potential difference is to be expected when the anion and cation have the same velocities of migration. The actual E.M.F.s are not large: in the HCl case mentioned above, for example, the value is 0·114 volt.

In the same way double layers are formed at the boundary surface between solutions of *different* electrolytes, and a potential difference therefore occurs. In calculating the E.M.F.s of cells account must, of course, be taken of the potential differences at the liquid junctions.

Similar considerations also apply to the case of a metal dipping in the solution of one of its salts and hence to concentration cells. When, for example, a copper rod is placed in a solution of copper sulphate, the diffusion process drives $Cu^{\cdot\cdot}$ ions against the rod, where they give up their charge and are deposited and held fast as metallic atoms. At the same time, of course, $SO_4''$ ions are also driven against the rod; but more $Cu^{\cdot\cdot}$ ions than $SO_4''$ ions come into contact with the rod on account of the greater mobility of the former. In a given time, therefore, more $Cu^{\cdot\cdot}$ ions are captured by the rod than are dissolved off by the $SO_4''$ ions. On the whole we may regard the process as equivalent merely to the deposition of $Cu^{\cdot\cdot}$ ions at a certain rate. Other things being equal, the tendency causing this deposition is directly proportional to the ionic concentration. Opposing it is the electrolytic solution pressure, which tends to drive copper into the solution in the form of ions. The two tendencies come to equilibrium at a certain electrode potential, the value of which is dependent upon the concentration of the electrolyte. In the case of a concentration cell the observed E.M.F. arises from the difference of concentration of the two parts of the liquid. The positive copper cations enter the more dilute solution, causing its concentration to increase, and leave the more concentrated solution, causing its concentration to decrease. When the cell is closed by a wire of another substance (e.g. platinum), the conditions are more complicated.

In Nernst's theory the tendency opposing the electrolytic solution pressure, i.e. the tendency which gives rise to the Brownian movement, is measured by the osmotic pressure. In 1916, P. DEBYE * actually succeeded in deriving the Nernst formula for the E.M.F. of a concentration cell directly from the theory of the Brownian movement.

It may also be remarked that the processes of a concentration cell furnish a clear example of how natural processes take place in the direction corresponding to increase of entropy. In the case of a more concentrated and a less concentrated solution in contact with one another the distribution of the molecules is not one of maximum disorder. The probability and hence the entropy of the state (Vol. II, § 10, p. 130) are smaller than if the concentration were everywhere the same. The electric current of the concentration cell, like the diffusion process, takes place in the direction causing a more nearly random distribution and therefore also an increase of probability and of entropy.

The energy of the electric current furnished by a concentration cell is not of chemical origin, as is the energy of other galvanic cells. The equalization of concentration actually involves absorption of heat, like the process of solution of a substance which does not react with water, and like the expansion of a gas with performance of external work.

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* P. DEBYE, Dutch by birth (born 1884), was first Professor of Physics at Göttingen, then 1920–27 at the Technical College at Zürich, and subsequently at Leipzig.

## 3. Electric Polarization of Electrodes. Accumulators.

**Electrolytic Polarization Current.**—In the arrangement shown in fig. 3, B is a battery of about 4 volts, P is a vessel filled with dilute sulphuric acid into which dip two large platinum sheets or carbon plates A and K, the instrument A to the right is a galvanometer, and U is a change-over switch.

The switch being in the position represented in the figure by the continuous line, the current from the battery flows in the direction indicated by the corresponding arrows. Thus of the electrodes in P, A is the anode and K the cathode. Oxygen is therefore evolved at A and hydrogen at K. A certain amount of the gases remains clinging to the respective plates. The switch is now changed over into the other position (dotted in the figure). The galvanometer A then indicates an electric current flowing in the direction of the dotted arrows. This current is called the (*electrolytic*) *polarization current*. It is only of short duration.

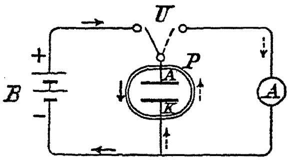

Fig. 3.—Polarization current

The explanation lies in the presence of gas upon the electrodes. The cathode K is covered with hydrogen molecules. When the current from B is stopped, the high electrolytic solution pressure of the hydrogen (p. 280) drives hydrogen cations back into the liquid, causing K to assume a negative potential relative to it. In the same way oxygen molecules enter the liquid as anions from the gas adhering to A, which thus assumes a positive potential. When now A and K are put into conducting connexion, a current is produced. This polarization current only continues to flow as long as there are still gas molecules adhering to the electrodes.

That it is really the gases adhering to the electrodes which cause the polarization current, follows from the fact that a galvanic cell can be made by dipping into sulphuric acid two pieces of platinum sheet, one of which has previously been hung for some time in a vessel full of hydrogen and the other in air. In order that the current may be of sufficient duration to allow of its observation, the platinum must be coated with platinum black (i.e. must be platinized) before being hung in the atmosphere of hydrogen or air. The quantity of gas taken up is then much greater than in the case of unplatinized electrodes.

In this connexion mention may be made of the Grove gas cell (1839) shown in fig. 4. Into the two side necks of a Woulfe bottle containing dilute sulphuric acid are inserted two tubes, closed at their upper ends, which carry long strips of platinized platinum. The two tubes are filled, through the central neck of the bottle, with hydrogen and oxygen respectively. When now the projecting ends of the platinum are joined through a galvanometer, the latter indicates a current in the direction from oxygen to hydrogen in the external part of the circuit (see arrows in the figure). A more convenient procedure is first to fill the electrode tubes quite full of dilute sulphuric acid, then generate the hydrogen and oxygen electrolytically by an auxiliary current. The gases are thus evolved in the respective electrode tubes in the correct proportions.

**Decomposition Potentials. Overvoltage.**—Very small applied potential differences are sufficient to cause the passage of a current between copper electrodes dipping in a solution of copper sulphate, the copper being dissolved off from the anode and deposited upon the cathode. If electrodes of platinum be used, the cathode soon becomes covered with copper, and oxygen is evolved at the anode. During the whole of the electrolysis we then have to overcome the polarization E.M.F. of the cell so formed. A polarization E.M.F. is always produced when salt solutions are electrolysed between inert electrodes. In order to produce a current in such cases, we must overcome both this polarization E.M.F. (P say) and also the drop of E.M.F. due to the resistance R of the solution (p. 237). The potential difference which must be applied between the electrodes to produce continuous electrolysis is thus

$$E = IR + P,$$

where I is the current strength.

When I is very small, we have E = P. This potential difference, the minimum necessary just to produce a current through the cell, is called the *decomposition potential* of the salt.

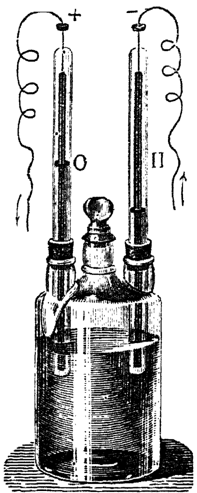

Fig. 4.—Grove hydrogen-oxygen cell

The potential of an electrolytically liberated gas or metal relative to the solution depends upon the nature of the electrode. Platinized platinum is chosen as standard. For almost all other electrode materials a greater E.M.F. is necessary to liberate hydrogen than for the standard platinized platinum. This phenomenon is known as *overvoltage.* It is of very great importance for the practical deposition of metals, as it makes this possible in cases where it could not occur according to the table of standard electrode potentials (see, for example, p. 283). Thus the overvoltage of hydrogen upon mercury is about 0·6 volts. In the deposition of metals there is also an increase of the electrolytic solution pressure, i.e. an enhanced deposition potential. Conversely, it is possible to reduce the electrolytic solution pressure by alloy formation at the electrode. This fact, together with the large overvoltage of hydrogen on mercury, makes possible, for example, the deposition of alkali metals from aqueous solution upon a mercury cathode.

**Accumulators.**—In the Grove gas cell we have already become acquainted with an example of how electrical energy can be stored up in the form of chemical energy. We use electrical energy to generate hydrogen and oxygen and can regain it by chemical combination of these two gases in the cell. Such storage devices for electricity are called *accumulators* (also *secondary* or *storage cells*). Two types have come to be of practical importance, namely the lead accumulator and the Edison accumulator.

*The Lead Accumulator* *.—This consists of two lead plates, usually originally coated with a layer of lead oxide, in a vessel containing dilute (20 per cent) sulphuric acid. The lead oxide is converted by the acid into lead sulphate. When a current is passed between the plates, the lead sulphate at the anode is oxidized to lead peroxide, while that at the cathode is reduced to metallic lead. When the current has passed a sufficient time, we have a plate of lead peroxide and a plate of metallic lead. The accumulator is then said to be *charged.*

When the poles of a charged accumulator are put into conducting connexion, a current flows in the opposite direction from the previous charging current, namely from the lead peroxide plate to the lead plate through the conducting wire. The result is to reproduce lead sulphate at both plates. This process is known as the *discharge* of the accumulator.

The chemical processes involved may be represented by the equation:

$$2PbSO_4 + 2H_2O \overset{\text{charge}}{\rightleftarrows} PbO_2 + Pb + 2H_2SO_4.$$

In the charged condition the anode consists of $PbO_2$, the oxide of quadrivalent lead, and the anode of metallic lead. The process of discharge consists essentially of the change of quadrivalent $Pb^{\cdot\cdot\cdot\cdot}$ ions into bivalent $Pb^{\cdot\cdot}$ ions at the anode and the change of metallic lead Pb into bivalent $Pb^{\cdot\cdot}$ ions at the cathode. The $Pb^{\cdot\cdot}$ ions so formed then combine with the $SO_4''$ ions at both plates to give insoluble lead sulphate $PbSO_4$. Hence the processes producing the E.M.F. are:

$$Pb^{\cdot\cdot\cdot\cdot} \rightarrow Pb^{\cdot\cdot} \quad \text{and} \quad Pb \rightarrow Pb^{\cdot\cdot}.$$

It follows that $SO_4''$ ions are used up during discharge, so that the sulphuric acid concentration of the accumulator sinks. Hence measurement of this concentration (e.g. by density determination with a hydrometer) is the best way to find out the degree of discharge of the accumulator. Various difficulties had to be surmounted before the process could be satisfactorily applied in a practical form of accumulator, in which the maximum fraction of the plates is effective for the production of current. In spite of the energetic chemical reactions at the plates, there must be no loss of rigidity; but on the other hand the weight of the plates must not be too great. It was therefore a great advance when Planté discovered that by repeated charging and discharging ("forming") the electrodes could be converted to an ever-increasing degree into lead oxide, without too serious an effect upon the rigidity of their texture. In this process of "forming" the surface of the plates becomes progressively loosened and hence the quantity of lead oxide entering into the chemical reactions of the accumulator is progressively increased. Only by "forming" could the efficiency of the lead accumulator be raised sufficiently to compete with the other known types of cells.

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* First described by the German physician Sinsteden in 1854. The later developments in lead accumulators have been based upon the investigations (1860) of R. Gaston Planté (1834–89), Professor of Physics at Paris.

The discovery did not assume commercial importance until the lead plates were replaced by cast lead grids, into the interstices of which a paste of lead oxide and concentrated sulphuric acid was mechanically pressed before "forming" (FAURE *). During charging this oxide is converted into brown lead peroxide at the anode, and into spongy grey metallic lead at the cathode.

The success of this type of accumulator depends upon the observation that lead oxide or lead powder mixed with sulphuric acid gives a paste which sets hard after a short while. The hardness can be increased by addition of glycerine and other substances. In order that the internal resistance may be as small as possible, it is usual to solder a number of similar plates on to one common lead rod. The anode and cathode are then placed in the containing vessel in such a way that the members of one set of plates are situated in the spaces between the members of the other set (fig. 5). The positive set usually has one plate less than the negative, and can be distinguished both by this fact and also by the brown colour of the lead peroxide when the accumulator is charged.

In ordinary portable accumulators the internal resistance usually amounts only to a few hundredths of an ohm.

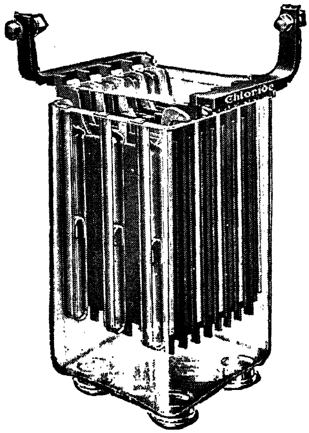

Fig. 5.—Lead accumulator
[Exide and Drydex Batteries, London]

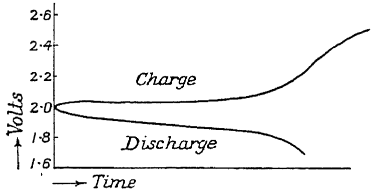

Fig. 6.—Charging and discharging pressures of a lead accumulator

The E.M.F. of a lead accumulator is almost exactly 2·0 volts and has a very small temperature coefficient, namely about 0·0003 volt per degree.

Although the processes producing the current in an accumulator are reversible, energy losses are unavoidable, particularly as the pressure required for charging is higher than that given during discharge. Fig. 6 represents the variations of pressure during charge and discharge. The difference of E.M.F. between charge and discharge is due to a concentration E.M.F. arising from differences of concentration set up by the chemical reactions on account of the narrowness of the pores of the plates. Towards the end of the charging process there is also an evolution of oxygen and hydrogen and consequently a high polarization E.M.F. The charging is regarded as complete when the pressure has risen to about 2·6. Discharge should not be continued after the pressure has fallen to 1·8. The average charging pressure is usually about 2·3, the average discharge pressure about 1·9. The efficiency of the accumulator is thus about 80 per cent as regards pressure. Since the efficiency as regards quantity of electricity is about 98 per cent, the efficiency as regards energy is also about 80 per cent. The capacity of a good lead accumulator is about 4 ampere-hours per square decimetre of the plates.

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* CAMILLE FAURE invented and named in 1881 the type of accumulator commonly used nowadays.

This capacity (the word "capacity" is used here in a general sense for the total effective amount of electricity which can be stored up) decreases as the strength of the discharging current is increased. As much as 32·4 watt-hours can be stored up in an accumulator weighing one kilogram.

*The Edison Accumulator.**—When charged the cathode consists of finely divided iron, the anode of nickel hydroxide $Ni(OH)_3$. The electrolyte is 20 per cent caustic potash solution containing a certain amount of lithium. During discharge the iron passes over into the ionic state, $Fe \rightarrow Fe^{\cdot\cdot}$, but is immediately precipitated as $Fe(OH)_2$ at the cathode. At the anode the trivalent nickel becomes bivalent $Ni^{\cdot\cdot\cdot} \rightarrow Ni^{\cdot\cdot}$, which also remains as insoluble hydroxide $Ni(OH)_2$ at the anode. These processes, to which the E.M.F. of the accumulator is due, may be represented by the equation:

$$Fe(OH)_2 + 2Ni(OH)_2 \underset{\text{discharge}}{\overset{\text{charge}}{\rightleftarrows}} Fe + 2Ni(OH)_3.$$

The discharging pressure falls off from an initial value of 1·4, the mean value being 1·2 (fig. 7). The charging pressure is considerably higher than this. Moreover, secondary reactions occur in the accumulator and reduce the efficiency as regards quantity of electricity to about 70 per cent. The efficiency as regards energy is therefore only about 50 per cent.

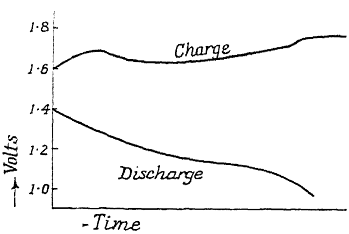

Fig. 7.—Charging and discharging pressures of an Edison accumulator

This accumulator has been considerably improved by the use of cadmium in place of iron (JUNGNER). In this way the efficiency as regards quantity of electricity could be increased to 75 per cent and the efficiency as regards energy to 57 per cent.

Especial mention must be made of the great insensitivity of the Edison accumulator to mechanical vibration and heavy discharge. These qualities make it particularly suitable for use in moving systems (vehicles, &c.), where its useful life is longer than that of the lead accumulator.

The capacity for one kilogram weight is about 30 watt-hours.

## 4. Practical Applications of Electrolysis

The first applications of electrolysis date back to DAVY, who decomposed the caustic alkalis in 1807 and prepared potassium and sodium in the metallic state, and to JACOBI,† who showed in 1838 how to produce plastic metallic reproductions of objects by electrolysis and how to plate ordinary metals with noble metals. But electrochemistry did not develop into an extensive branch of engineering and industry until the invention of the dynamo-electric machine for the production of any desired E.M.F. and current strength.

The following is a short account of the most important commercial applications.

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* THOMAS ALVA EDISON (1847–1931), once a newspaper-boy, became the most successful of American inventors.

† M. H. VON JACOBI (1801–74), German physicist in Petersburg, brother of the famous mathematician K. S. J. JACOBI (1804–51).

**Electrolysis of Metal Salts.**—This is one of the most extensive branches of applied electrolysis. It includes firstly the electrolytic preparation or purification of metals, and secondly, the metal plating of objects for practical or decorative purposes and the preparation of plastic reproductions in metal.

1. *Electrolytic Preparation of Metals.*—By employing an insoluble anode and a solution of a salt of the metal as electrolyte, it is possible under certain circumstances to deposit the pure metal upon the cathode.

Large quantities of copper (over 100,000 tons annually) are deposited in this way from electrolytes containing $CuSO_4$. The anodes used are made of an alloy of copper with silicon, iron, and other metals.

Zinc is also prepared in large quantities (in 1925 about 160,000 tons) by a similar process. The anode material is in this case manganese dioxide, $MnO_2$.

2. *Electrolytic Purification of Metals.*—In this process the impure metal is used as anode, the electrolyte being a salt of the same metal. The passage of an electric current then causes the anode to pass into solution and the pure metal to be deposited upon the cathode.

The most important application is for the *electrolytic refining of copper.* The anodes are cast of the unrefined metal which contains about 98–99 per cent Cu.

The impurities consist chiefly of silver, gold, platinum metals, bismuth, arsenic, and antimony, together with other metals. Those impurities which are less "noble" than copper (viz. nickel, iron, cobalt, zinc, antimony, arsenic, bismuth) pass into solution along with it. On the other hand the noble metals and also selenium and tellurium remain undissolved and sink to the bottom of the electrolysis vessel (anode sludge). The solutions employed contain 12–16 per cent of $CuSO_4 . 5H_2O$ and 5–10 per cent free $H_2SO_4$. The working temperature is 40–50° C. and the current density up to 300 amperes per $m.^2$. The electrolytic process is complicated by the capacity of copper to assume two valencies, i.e. to ionize both as $Cu^{\cdot}$ (cuprous) and $Cu^{\cdot\cdot}$ (cupric). One of the results of this is that a small fraction of the copper goes into the anode sludge. Another is that the quantities deposited may not correspond to Faraday's law, but under certain circumstances may show large deviations (e.g. about 50 per cent) from it. It is these same complications which make the copper-coulombmeter (p. 152) unsuitable for quantitative purposes. Of the impurities which pass into solution nickel, iron, cobalt, and zinc require a higher E.M.F. for deposition than does copper, and can therefore be concentrated to a considerable degree in the solution. Special methods must be employed to prevent the deposition of arsenic, bismuth, and antimony.

The metal deposited upon the thin sheets forming the cathode is very pure copper (up to 99·99 per cent pure). This degree of purity is absolutely necessary, however, for the purposes of modern electrical engineering, as the smallest traces of impurities may have a very great effect upon the conductivity of copper. Thus a few thousandths of 1 per cent of arsenic are said to render copper useless for electric mains.

The quantities of copper refined in this way are very large indeed. The annual production of the copper refining industry of the United States alone amounts at present to more than 1,200,000 tons. This means that in the U.S.A. alone a total average continuous current of 125 million amperes is used in this one process.

The anode sludge formed during copper refining contains such large amounts of noble metals that the value of these metals regained from it annually by electrolysis is estimated as more than £10,000,000.

The silver is cast into anodes containing 80–85 per cent Ag and is electrolytically deposited upon pure silver cathodes in a manner similar to that described above for copper, the electrolyte used in this case being 1 per cent $HNO_3$ with 1–2 per cent $AgNO_3$. The anode sludge from this silver refining process is treated chemically and then made into gold anodes. The gold is then refined electrolytically in a similar way, using gold chloride solution acidified with hydrochloric acid as electrolyte.

The electrolytic refining of other metals (e.g. iron, nickel, tin) is only undertaken on a relatively small scale.

3. *Electroplating and Galvanoplastics.*—Electroplating with noble metals (i.e. the deposition of a surface layer of these metals by electrolysis) is used to improve the appearance of the surfaces of objects or to render them more resistant to mechanical wear or chemical attack. The electrolytic deposition of a metal upon the conducting surface of a sunk impression or negative of an object can also be used for the preparation of plastic reproductions. It is estimated that a total average continuous current of 5 million amperes is used for electroplating and galvanoplastic processes.

The main problem of electroplating is to deposit a smooth (i.e. microcrystalline) compact layer of metal of uniform thickness and texture upon surfaces with very irregular depressions and elevations. In order to attain this, the objects to be plated must be scrupulously cleaned; in particular they must be freed from all traces of grease. Since the deposit only appears to adhere firmly when it can form an alloy with the metal of the base, it is necessary in certain cases to use an intermediate layer of a third metal. Thus silver will not adhere to iron, nor nickel to zinc; but a preliminary plating with copper removes the difficulty in both cases. In order that a smooth, microcrystalline surface layer may be obtained, it seems essential that the metal to be deposited shall be contained in the solution in the form of a complex salt. Solutions of complex cyanides are specially suitable, as they have the further advantage that the deposition is accompanied by high chemical polarization, so that non-uniformity at projections and depressions is avoided. The smoothness of the deposits is also enhanced by the presence of colloids (e.g. gelatine) in the plating solution.

The metals electrolytically deposited on the largest scale are zinc (galvanization of iron), nickel, copper, silver, and gold. Electroplating with chromium and cadmium is now being carried out to an increasing extent. Non-conducting objects can also be plated with metals. For this purpose they are first soaked in wax and painted over with gutta-percha lac, after which they are made conducting by being rubbed with powdered graphite and are used as cathode.

Plastic reproductions of objects can also be made electrolytically (**galvanoplastics**). A negative of the original is first made in wax, gutta-percha, or plaster of Paris, and its surface is rendered conducting with powdered graphite. Negatives for the preparation of electrotype can be made by pressing the original with great force into soft lead.

The metal usually deposited is copper, solutions containing 20–30 per cent $CuSO_4$ and 0·3–3 per cent $H_2SO_4$ being employed. The current density is about 0·015 ampere per cm.$^2$, or at higher temperature and with adequate stirring 0·08 ampere per cm.$^2$. The thickness of the deposit is generally from 0·15 to 0·25 mm.

The copper electrotype so obtained is often not sufficiently resistant for the purposes of printing. A preliminary deposit of nickel is then used, and upon this a layer of copper. With electrotype of this kind one million copies can easily be printed.

Another galvanoplastic operation carried out on a large scale is *the manufacture of seamless copper tubing.*

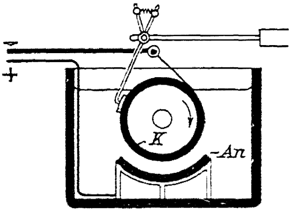

Fig. 8.—Electrolytic manufacture of seamless copper tubing (Diagram after Grube.)

The holders for the cathodes consist of rotating cylinders of cast-iron or brass (fig. 8). Beneath them are the anodes *An*, which have the form of arcs of coaxial cylinders. The deposited copper layer is continuously smoothed down by means of an agate shoe moving backwards and forwards along its length under pressure. The involved crystal disposition thus produced makes the deposit about three times as hard as ordinary electrolytic copper. When the desired thickness has been reached, the cylindrical deposit can be slipped off its holder and drawn into tubing of very small diameter.

4. *Application of Electrolysis for the Quantitative Estimation of Metals.*—The main problem of these methods is to precipitate the metal from its solution practically completely and in weighable form (**electroanalysis**).

The evolution of hydrogen at the cathode is a deciding factor. If this takes place more easily than the deposition of the metal (as is the case for the metals from manganese to the alkali metals, see the table on p. 283), i.e. if the metal requires a considerably higher potential than hydrogen for its liberation, then a quantitative deposition is impossible. Hence electroanalytic methods are employed chiefly for the estimation of silver, mercury, copper, bismuth and antimony.

By reducing the hydrogen ion concentration, e.g. by addition of excess of alkali, or by the formation of complex ions containing the metal, it is possible also to deposit other metals quantitatively. Thus zinc may be precipitated quantitatively by electrolysis of an alkaline solution of the hydroxide, and nickel by electrolysis of an ammoniacal solution containing the ion $[Ni(NH_3)_4]^{\cdot\cdot}$.

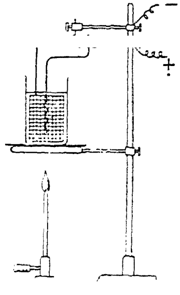

Fig. 9.—Electroanalysis apparatus

In the case where several metals are present simultaneously in the solution a quantitative separation and deposition is possible, provided that the respective deposition potentials are sufficiently different from one another. Thus silver can be separated from copper or lead by electrolysing the solution at about 1·23 volt. This deposits the silver quantitatively, but leaves the copper or lead in solution. The remaining metal can then be deposited subsequently by application of a higher E.M.F. The apparatus (fig. 9) usually consists of a cathode of platinum gauze and a spiral platinum anode. The rate of deposition of the metal to be estimated can be considerably increased by heating and agitation of the electrolyte. The time required for a copper estimation, for example, is about 10 minutes.

Less "noble" metals can sometimes be deposited upon the anode. Thus $Mn^{\cdot\cdot}$ is transformed into $Mn^{\cdot\cdot\cdot\cdot}$ at the anode, where it immediately gives a firmly adhering deposit of manganese dioxide $MnO_2$, which can be weighed. Similarly lead can be deposited as $PbO_2$ from solutions made strongly acid with nitric acid.

**Electrolysis of Sodium Chloride Solution.**—Great commercial importance attaches to *the electrolysis of common salt solution* (*sodium chloride solution*).

The ions $Na^{\cdot}$, $H^{\cdot}$, $Cl'$ and $OH'$ are present in the aqueous solution. Since the standard electrode potential (p. 283) of sodium is −2·71 volts and that of hydrogen in neutral solution only −0·415 volt, hydrogen alone is liberated at the cathode during the electrolysis of a neutral solution between platinum electrodes. The potential of oxygen relative to a neutral solution is +0·82 volt, that of chlorine +1·26 volt. Accordingly we should expect only oxygen to be liberated. But on account of the large overvoltage (p. 287) of oxygen upon platinum electrodes, it is chlorine which is actually evolved. Hence when a neutral sodium chloride solution is electrolysed between platinum electrodes, the substance liberated at the cathode is exclusively hydrogen, while at the anode the predominant product is chlorine. In consequence of the disappearance of $H^{\cdot}$ from the water near the cathode, an excess of $OH'$ is produced there, i.e. caustic soda is formed to some extent and the solution becomes alkaline.

1. *Preparation of Hypochlorite and Chlorate.*—When mixed, the electrolysis products NaOH and $Cl_2$ react to give hypochlorite according to the equation

$$Cl_2 + 2NaOH \rightarrow NaCl + NaClO + H_2O.$$

The hypochlorite solution is very widely used as a bleaching agent for textiles and cellulose articles.

The NaClO reacts partially with the water to give HClO according to the equation

$$NaClO + H_2O \rightleftharpoons HClO + NaOH.$$

The hypochlorous acid then reacts with more sodium hypochlorite, giving sodium chlorate, viz.:

$$2HClO + NaClO \rightarrow NaClO_3 + 2HCl.$$

By a suitable choice of working conditions this reaction may be made to predominate, so that in this way it is possible to prepare the valuable sodium chlorate from common salt.

2. *Preparation of Caustic Alkali and Chlorine.*—In order to obtain the chlorine and caustic alkali separately, great care must be taken to prevent their mixing. Various methods have been worked out for this purpose.

In one method a cathode of *mercury* is employed. The overvoltage of oxygen upon mercury is very large. Moreover, the formation of an amalgam, which is soluble in excess mercury, reduces the electrolytic solution pressure of the alkali metal to such an extent that a direct discharge of the alkali metal ions can take place at the cathode (fig. 4, p. 145). The alkali metal amalgam can then be removed and treated subsequently with water to give caustic alkali.

In the other methods the mixing of the electrolysis products is prevented by the use of diaphragms or by keeping the electrodes far apart. The various practical forms of plant which have been evolved cannot be discussed further here. Some of the processes require currents up to 16,500 amperes.

Formerly there was hardly an adequate demand for chlorine and hydrogen in the quantities in which they were produced by electrolysis. Nowadays, however, both gases are used on a large scale, the hydrogen being employed, for example, in oxy-hydrogen welding, for the hydrogenation of fats, in the synthesis of ammonia, &c.

**Oxidation and Reduction Processes.**—Electrolysis can be used with great advantage for the purpose of oxidation (at the anode) or reduction (at the cathode), there being no contamination of the solution as when chemical oxidizing or reducing agents are employed.

One electrolytic oxidation process which is carried out very extensively is the preparation of *potassium ferricyanide* from potassium ferrocyanide in accordance with the equation

$$[Fe(CN)_6]'''' \rightarrow [Fe(CN)_6]''' + (-).$$

In Germany potassium permanganate is made almost exclusively by electrolytic oxidation of potassium manganate solution:

$$[MnO_4]'' \rightarrow [MnO_4]' + (-).$$

Further applications are the preparation of persulphates, hydrogen peroxide, perchlorates, and also organic substances like iodoform.

**Electrolysis of Water.**—Since hydrogen and oxygen are produced in sufficient quantities from other sources, the electrolysis of water has not yet attained importance. In actual practice the water is either acidulated with sulphuric acid and electrolysed between lead electrodes in a lead vessel, or else it is made alkaline with chloride-free alkali and is electrolysed between iron electrodes, the anode sometimes being nickel-plated.

**Electrolysis of Fused Salts.**—This is chiefly used for the preparation of alkali and alkaline earth metals, magnesium, aluminium, cerium and beryllium.

*Metallic sodium** is mostly prepared by the electrolysis of fused sodium hydroxide at temperatures up to 330° C. in iron vessels with cathodes of iron and anodes of nickel. The yield is reduced mainly by the water formed at the anode, but recent plant design obviates this to a great extent. Attempts have also been made to prepare sodium by the direct electrolysis of fused sodium chloride. On account, however, of the fact that the melting-point of sodium chloride is not far from the boiling point of sodium, it is necessary to use a mixture of the fluorides and chlorides of sodium and potassium, by which means the melting-point of the electrolyte may be lowered from 800° C. to about 600° C. The world's output of metallic sodium in 1927 was about 25,000 tons.

A difficulty arises in the preparation of *metallic calcium* by the electrolysis of fused calcium chloride $CaCl_2$, in that the calcium combines with the $CaCl_2$ at high temperatures to form CaCl. To avoid this the cathode is only allowed to dip just beneath the surface of the melt and is gradually raised as its length increases by deposition of calcium. In this way a long stick of calcium is obtained. A layer of crystallized calcium chloride prevents surface oxidation.

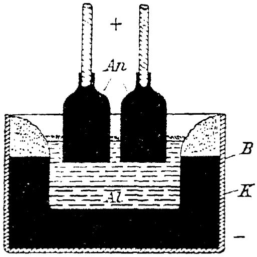

Fig. 10.—Diagram of the electrolytic preparation of aluminium.

*Magnesium*† is prepared by the electrolysis of fused carnallite ($KCl.MgCl_2.6H_2O$), which possesses the advantages of lower water content and lower melting-point as compared with pure magnesium chloride. In a recent process the $MgCl_2$ used up is replaced by allowing the chlorine liberated at the anode to react with magnesium oxide in presence of carbon monoxide, whereby magnesium chloride and carbon dioxide are formed. This offers the very desirable possibility of using magnesite (naturally occurring $MgCO_3$) as starting material. The annual world's production of magnesium is about 2000 tons.

The most important process of this type is the electrolytic preparation of *aluminium*.‡ The material used is a solution of $Al_2O_3$ in cryolite ($Na_3AlF_6$). Other substances (e.g. $CaF_2$) are added to lower the melting-point. The bottom of the electrolysis vessel forms the cathode. In order that the molten aluminium deposited there may not rise to the surface, the specific gravity of the melt must be smaller than that of the metal. The electrolysis vessel (fig. 10) is a wroughtiron tank B with a lining K of pressed and previously ignited carbon. The anodes A*n* are blocks of carbon. The electrolyte is first melted by placing thin carbon rods between the anodes and the cathode. These rods become hot owing to the passage of a current, and the electrolyte around them melts. When this process has gone on for a sufficient time, the rods are removed and the electrolysis is begun. Plants are constructed to take 16,000 amperes at 6 to 7 volts with an anodic current density of 1 to 2 amperes per $cm.^2$. The world output of aluminium in 1928 was about 250,000 tons.

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* Although first isolated electrolytically by Davy in 1808, potassium and sodium were formerly prepared commercially by reduction of the carbonates with carbon. Not until 1890 did Castner take out the first patent for the preparation of the alkali metals by electrolysis of the fused hydroxides.

† This metal was prepared by Faraday by the electrolysis of fused $MgCl_2$. Further experiments were later carried out by Bunsen, who obtained a yield corresponding to 60 per cent of the electricity used.

‡ First prepared by Bunsen and St. Claire-Deville in 1854 by the electrolysis of $Na_3AlCl_6$.

Concerning applications of the *electric furnace* (preparation of carbide, &c.) see p. 357.

## CHAPTER X

# The Conduction of Electricity in Gases and in High Vacua

### 1. Induced Conduction at Relatively High Gaseous Pressures.

**Fundamental Experiments.**—The experiments described in connexion with electrostatics have shown that under normal conditions of pressure and temperature air and other gases are excellent insulators, provided that the field strength is not too great. Only at very high field strengths, such as are produced by the crowding of the lines of force at angular projections and points or by the use of very great potential gradients, did we find that the insulation of the air broke down. We also observed conduction in gases at very high temperatures, e.g. in the flame gases of a Bunsen burner. Reduction of pressure likewise caused conduction in gases at relatively small field strengths, as was shown in the experiment of fig. 14, p. 140.

We will now consider what conditions must be fulfilled in order that there may be conduction through a gas. Let us first take a very simple example illustrating the nature of the processes involved.

Between the plates of a charged condenser we suspend a small, light, conducting ball upon an insulating thread. As a result of induction the ball is attracted to whichever of the plates is nearer to it. On contact with this plate it assumes a charge of the same sign, and is therefore attracted at once to the other plate. Here it gives up its charge and becomes electrified oppositely from before. It is then attracted back again to the first plate, and the process is repeated a large number of times. In this way a large fraction of the charges of the condenser plates may be neutralized. A similar process of neutralization also takes place when an electrified ebonite rod is held over a pile of paper snippings or pith balls.

The experiments of § 8, p. 47, upon suspended charged particles furnish another similar example of conduction, in this case by microscopic smoke particles or liquid droplets. In these cases the particles cannot become charged at the condenser plates; for they remain clinging to these plates when they come into contact with them. The particles must therefore be charged before entering the field. In the experiments quoted the particles were electrified during their original production—the smoke particles by heating and the liquid droplets in the manner explained on p. 106. Observation showed that both positively and negatively charged particles were present. Such particles are attracted towards the oppositely charged condenser plates, with which they finally come into contact. In this way the charge of the condenser is eventually neutralized, provided that there is a sufficiently large number of particles.

The case is essentially similar to that of electrolytic conduction, except that there the charges of the ions, being determined by chemical properties, are associated with quite definite quantities of matter in each particular case, whereas in the present case of conduction in gases any particle may carry a charge of any amount and of either sign.

The neutralization of opposite electric charges through a gas by means of charged particles which are either brought in from outside or charged *in situ* by some external influence may be called **induced conduction.** The charged bodies or particles to which the conduction is due will be referred to as **carriers of electricity** or merely **carriers.***

Our reason for referring to this type of conduction as *induced* conduction is that the current is not produced by the simple application of a potential difference, but only after carriers have been brought into the field or produced there by some external influence.

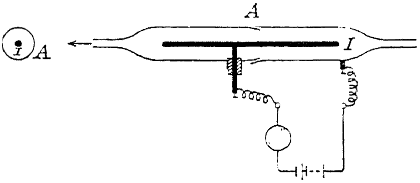

Fig. 1.—Electrical conduction due to particles of fine powder

**Production of Carriers.**—Many methods are known by which carriers are produced in a gas. Of course the carriers may be so small as to be invisible even under the microscope; indeed we must expect that charged molecules or molecular complexes will be able to act as carriers. On p. 304 we shall become acquainted with a method of calculating the size of carriers even when they can no longer be seen under the microscope.

The following are some methods of producing carriers.

1. *Friction.*—When two different substances are brought into contact (especially when the contact is very intimate—friction), the opposite charges are separated (p. 105) and the bodies become charged. By using fine powders charged carriers of both signs can be readily produced in large quantities.

For the production of positive carriers a suitable substance (after KUNDT) is red lead; for negative carriers, flowers of sulphur (see also p. 95). When a mixture of these powders is blown gently through a metal vessel and the charged dust allowed to pass through a condenser charged to (say) 220 volts, the current (about $10^{-8}$ ampere) due to transport by the carriers can be detected directly with the help of a sensitive galvanometer. A suitable type of condenser for such an experiment is shown in fig. 1. The one electrode consists of a long metal tube whose cross-section decreases gradually near its ends. The other is a metal rod with rounded ends, lying along the axis of the tube and supported by an insulated metal rod to which connexion can be made.

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* In many quarters it has become customary to use the name *ions* for the charged bodies or particles which transport the electricity through a gas. In this book, however, the name *ions* will be reserved for charged atoms or groups of atoms whose charge corresponds (in magnitude and sign) with their chemical character in the manner with which we have already become acquainted in dealing with electrolysis.

When a current of air containing the mixed powders is blown through such a condenser, the carriers are attracted towards the electrodes, which they strike with relatively high velocity (see below), becoming discharged and remaining for the most part adhering to the surface. Thus when red lead and flowers of sulphur are used, the former is deposited and held upon the negative pole and the latter upon the positive pole.

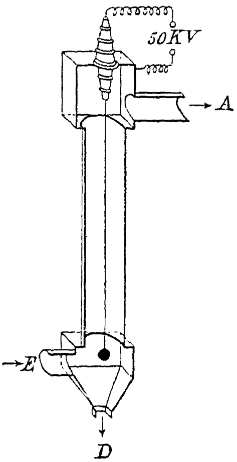

Fig. 2.—Electrical removal of dust (after Cottrell-Möller)

E, gas inlet; A, gas outlet; D, dust deposit.

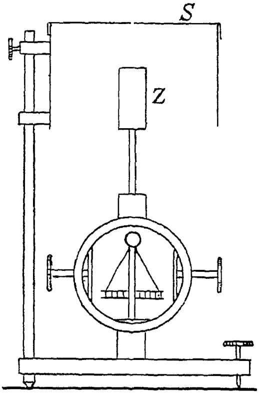

Fig. 3.—Elster and Geitel electroscope. S, screen to cut off earth's field.

[Günther & Tegetmeyer, Brunswick.]

*Electrical Removal of Dust.*—This method is used on a large scale for the removal of dust from the waste gases in mills, cement factories, mines, &c. Since the dust particles are not all originally charged, they are charged up as they pass through the condenser by a normal discharge between the plates (corona, p. 349). The quantities of dust precipitated in large-scale plants are very considerable. Thus in a lead mine, for example, where the light suspended dust was collected in this way, about 3 tons of material (90 per cent content of lead) were obtained daily for an energy expenditure of 190 kilowatt-hours. The principles of the construction of such a commercial plant are shown in fig. 2.

2. *Waterfall Electrification.*—The air in the neighbourhood of waterfalls is noticeably conducting. As LENARD has shown, this is due to the presence of charged water droplets formed by the breaking

up of larger uncharged drops. The fundamental nature of this kind of electrification has already been discussed in detail in § 18, p. 106.

The arrangement shown in fig. 3, devised by Elster and Geitel, is often used to detect the very small conductivity of air in the open. It consists of a leaf electrometer carrying a cylinder Z in conducting connexion with the leaves. The electrometer is given a charge and the potential fall is observed as a function of the time.

3. *Glowing Bodies.*—When a glowing match is brought between the plates of a charged condenser (fig. 4), there is an immediate fall in the potential difference. The air between the plates has therefore been rendered conducting. A special case of this kind is the conductivity of flame gases, which may easily be demonstrated by repeating the experiment with a small Bunsen flame (see also p. 23.)

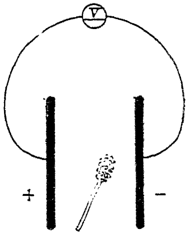

Fig. 4.—Conduction in air due to the presence of a glowing body.

4. *Light and X-rays.*—The potential of a charged condenser also falls off when the negatively charged plate K (fig. 5), which in this case should be a freshly amalgamated zinc disc, is illuminated through a quartz lens Q by an intense beam from an arc lamp L with iron electrodes. The radiation from such an arc is very rich in short wave-length ultra-violet. In the arrangement shown in the figure the condenser is a leaf electrometer, the other plate being the earthed casing A and the surroundings.

A relatively large conductivity is also obtained by passing a beam of X-rays (p. 347) between the plates of a condenser (fig. 6). Observations of the motion of the carriers produced in these ways have already been described in § 8, p. 48.

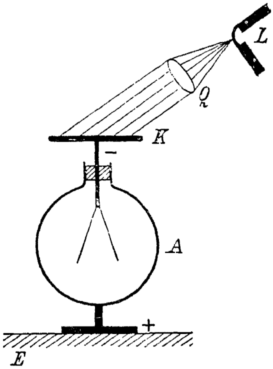

Fig. 5.—Conduction due to light

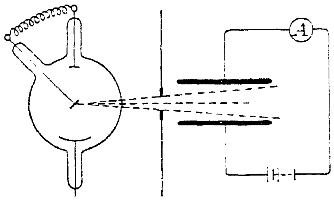

Fig. 6.—Conduction due to X-rays

5. *Radioactive Substances.*—Radioactive substances also give off rays which render the air conducting. Thus when an intense radium preparation is brought near to a charged electroscope, the potential difference falls off very rapidly even when the radioactive substance is still at a considerable distance. This experiment can also be carried

out, as in fig. 7, by placing a quantity of pitchblende (naturally occurring uranium mineral), uranium salt, or old gas mantle (active on account of the thorium content) upon one of the condenser plates. A very sensitive galvanometer or electrometer must be used to indicate the current.

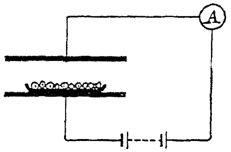

Fig. 7.—Conduction due to presence of rays from a radioactive substance.

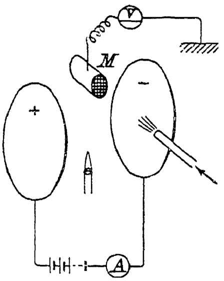

Fig. 8.—Capture and removal of carriers out of the field.

**Removal of the Carriers out of the Field.**—The following experiment provides elegant proof that in all these cases the conductivity is due to electrically charged particles, even though there are no visible carriers.

We place a small Bunsen flame midway between the plates of a condenser (fig. 8). The current read off upon the galvanometer A indicates the conduction through the air between the plates when a potential difference is applied. Near to the condenser we place a small metal cylinder M with one end open and the other covered with wire gauze. This cylinder is connected through an electrometer V to earth. We now blow a current of air transversely through the field into the cylinder on the right of the flame. The electrometer V is deflected, indicating a positive charge upon the cylinder. We now repeat the experiment with an air current on the left of the flame, and find that the cylinder becomes negatively charged. Thus we can blow away and capture the carriers as they are drawn out of the flame by the electric field—the positive carriers to the right towards the negative plate and the negative carriers to the left towards the positive plate.

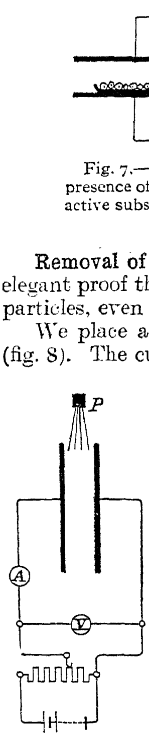

Fig. 9.—Investigation of the cur-strength as function of the E.M.F. in conduction through a gas.

**The Variation of Current Strength with Applied Potential Difference.** *Ohm's Law.*—In order to investigate the current strength as a function of the applied potential difference, the air between the plates of a condenser may be rendered conducting by means of a radium preparation P (fig. 9). The circuit also includes a galvanometer A and an arrangement for applying any desired E.M.F. The latter may come from a battery of accumulators or a potentiometer (p. 233) fed from the direct-current mains. By increasing the number of accumulators in circuit or by altering the slider position of the potentiometer the E.M.F. between the condenser plates may be increased at will. Beginning with low pressures, say 20, 40 volts and so on, we find proportionality between current and pressure, i.e. Ohm's law is obeyed. This corresponds to the initial portion, from A to B, of the curve in fig. 10. Since the conduction is due to the motion of charged particles, the considerations put forward in § 3, p. 274, are also applicable in this case. Thus the validity of Ohm's law proves that the carriers move against a frictional resistance, so that a constant force produces a constant velocity. It proves further that the number of carriers is not altered in the process, i.e. that the number of carriers which give up their charges to the plates is small compared with the total number of carriers present.

*Saturation Current.*—Further increase of applied potential difference causes the current strength to increase more slowly than would correspond to direct proportionality (portion B to C of the curve, fig. 10). At sufficiently high pressures (perhaps above 100–150 volts according to the strength of the radioactive preparation) the current remains constant (portion of the curve from C to D). This maximum current in a conducting gas is called the **saturation current.**

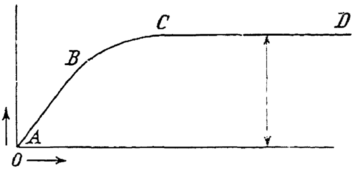

Fig. 10.—Current E.M.F. curve for conduction through air. (Abscissæ, E.M.F.s; ordinates, current strengths; CD corresponds to saturation current strength.)

The explanation of this behaviour is as follows. The deviation from Ohm's law between B and C on the curve must be due either to a modification of the velocity-force law at high velocities, or else to a considerable reduction of the number of carriers by discharge at the plates. The existence of a saturation current (C to D on the curve) decides definitely in favour of the latter explanation. As the E.M.F. is increased, more and more carriers are discharged per second, until finally the rate of discharge is equal to the rate at which the carriers are produced (in this case by the rays from the radioactive preparation). The current strength must then remain constant, there being no more carriers available for an increase. Thus the quantity of electricity flowing per second in the saturation current is equal to the sum of all the charges of the carriers generated per second, in the space through which the current flows.* The correctness of this explanation is shown by the following experiments.

The saturation current produced by two radium preparations acting together is equal to the sum of the saturation currents produced by the same preparations acting independently in the same positions relative to the electrodes.

The following remarkable phenomenon is also explained by the same theory. It is found that the current strength *increases* when the plates are moved farther apart; whereas we should ordinarily expect the opposite effect owing to the greater resistance which the larger air column would presumably offer to the passage of the current. The reason for the observed increase of current strength lies in the increase of the number of carriers generated between the electrodes by a radioactive preparation when the volume of the condenser is made larger.

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* Neglecting the recombination of oppositely charged carriers (p. 309).

The existence of a saturation current proves, therefore, that all the carriers can be removed from a gas by the application of a sufficiently strong field.

This can be demonstrated with the apparatus shown in fig. 11. The carriers, produced in this case by a glowing platinum wire P, are blown along in an air current so as to pass successively through two cylindrical condensers $Z_1$ and $Z_2$. When no potential difference is applied to $Z_1$, a potential difference applied to $Z_2$ causes a current which may be read off upon the galvanometer A. As now an increasing potential difference is applied to $Z_1$, the current across $Z_2$ decreases and finally stops altogether. The explanation is that a sufficiently high E.M.F. across $Z_1$ captures all the carriers there, leaving none to transport a current in $Z_2$.

**Measurement of the Velocity of the Carriers in an Electric Field.**—In order to obtain information as to the nature (especially as to the size and mass) of the carriers produced by the above methods, it is necessary first to measure the velocity with which they move in an electric field. Since in most cases the particles are invisible, indirect methods must be employed. The simplest is the determination of the current-E.M.F. curve for the case of a gas flowing through a cylindrical condenser.

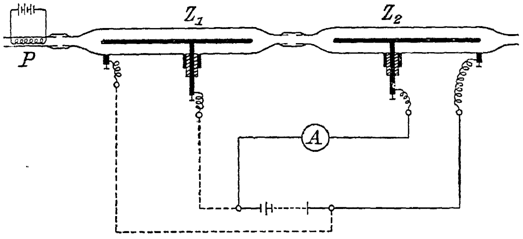


Fig. 11.—Removal of carriers by a sufficiently strong electric field

In order to arrive at quantitative relationships, we will suppose that the concentration of carriers entering the condenser is constant, and that all the carriers are charged with the same sign. Let V be the volume of gas passing any cross-section of the cylinder in unit time. Let $n$ be the number of carriers per unit volume, and $e$ the charge carried by each. Then when there is no potential difference between the outer cylinder A and the inner cylinder I (see fig. 12), no carriers are captured in the condenser and the quantity of electricity passing per second through any cross-section of the condenser in the direction of the gas current is $Vne$. Now let a field be applied between A and I, the sign of the outer cylinder being the same as that of the carriers. The carriers will be drawn inwards towards the cylinder I and will move along curves like those shown in fig. 12, the velocity of a carrier at any instant being given in magnitude and direction by combining the axial velocity of flow with the radial velocity produced by the electric field. Let $l$ be the velocity imparted to a carrier by an electric field of unit strength, i.e. the *mobility*. The corresponding velocity in a field of strength E will be $w = l\mathrm{E}$. If $r_a$ and $r_i$ are the radii of the outer and inner cylinders respectively and U the potential difference between them, the field strength $\mathrm{E}_r$ at a distance $r$ from the axis is

$$\mathrm{E}_r = \frac{\mathrm{U}}{r \log_e \dfrac{r_a}{r_i}}$$

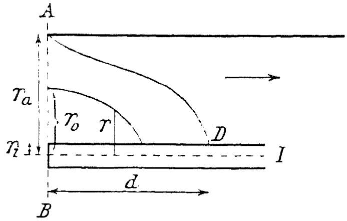


Fig. 12.—Motion of a carrier in passing through a cylindrical condenser

(see § 13, p. 76). The radial velocity of a carrier at this distance from the axis is therefore

$$l\mathrm{E}_r = \frac{l\mathrm{U}}{r \log_e \dfrac{r_a}{r_i}}.$$

Let the velocity of flow of the gas in the direction of the axis be $u$. Then the axial distance $x$ travelled by a carrier while moving inwards from its initial radial distance $r_0$ (i.e. its distance from the axis on entry) to the radial distance $r$ is given by

$$x = \frac{\mathrm{V}_r^{r_0}}{2\pi l \mathrm{U}} \log_e \frac{r_a}{r_i},$$

where $\mathrm{V}_r^{r_0}$ (dependent upon U) is the volume of gas flowing per second through a cross-section of the space bounded by two cylinders of radii $r_0$ and $r$. (The derivation of this equation need not be given here.)

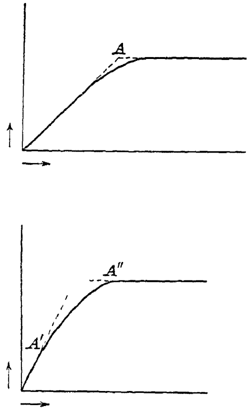
Putting $r_0 = r_a$ and $r = r_i$, we obtain a value of $x$ ($d$ say) given by

$$d = \frac{\mathrm{V}}{2\pi l \mathrm{U}} \log \frac{r_a}{r_i}.$$

The distance $d$ is thus the maximum axial distance traversed by any carrier in the given field before arriving at the inner electrode. The carriers which actually travel the axial distance $d$ must have entered the condenser at the maximum radial distance $r_a$ from the axis. All carriers entering nearer to the axis than this arrive at the inner electrode after a shorter axial distance. Beyond a distance $d$, therefore, there are no more carriers in the gas (compare the experiment of fig. 11).

Under the conditions assumed above, namely when the conduction is due to carriers of definite mobility all charged with the same sign, the variation of current strength with increasing potential difference is of the type shown graphically in fig. 13. When the E.M.F. is small, only a few carriers are caught by the electrode, namely, those that are very close to it on entering the condenser. As the E.M.F. is increased, more and more carriers are caught, until finally all of them (even those entering the condenser near the outer electrode) are discharged at the inner electrode. This corresponds to the point A of the curve of fig. 13. The potential U is then obviously such that the value of *d* calculated from the above equation is just equal to the length of the inner electrode. This involves the assumption that the field within the condenser is accurately radial and not affected by the conduction process itself. The value of V in the equation can be measured by suitable gas meter. All the quantities are then known with the exception of *l* (the velocity of a carrier in unit field), which can be calculated.

When the carriers have different mobilities, as is usually the case, we get a curve of the type shown in fig. 14. The mobility of the most mobile carriers can then be calculated from the point A′ where the first change of direction occurs, and the mobility of the least mobile carriers from the point A″ where the direction changes finally.



Figs. 13 and 14.—Mobility curves. (Abscissæ, E.M.F.s; ordinates, current strengths.)

We almost always have to do with gases containing carriers of both signs. In general, therefore, a galvanometer cannot be used to investigate the carriers of one particular sign. The method employed instead is to earth the outer electrode and connect the inner electrode with an electrometer. Measurement of the rate of charging of the electrometer then gives the current due alone to the carriers of opposite sign from the inner electrode.

Let us represent the mobility of a positive carrier by $l^+$ and the mobility of a negative carrier by $l^-$. Experiments upon air at atmospheric pressure and room temperature have shown that $l^+ = 1{\cdot}37$ and $l^- = 1{\cdot}89$ cm./sec. per volt/cm. The ratio $l^-/l^+$ is thus approximately equal to 1·38. Very slow carriers, whose mobility is of the order of $10^{-2}$ to $10^{-4}$ cm./sec. per volt/cm., are also frequently observed. In hydrogen the rough values of the mobilities are $l^+ = 6$ and $l^- = 8$ cm./sec. per volt/cm.

**Relation between the Velocity of a Carrier in an Electric Field and its other Properties.**—Every carrier is subjected to collisions with the molecules of the gas in which it is situated. Let *m* and M be the masses of the carrier and gas molecule respectively, both of which are assumed to be spherical. Further let $v$ be the component velocity of the carrier in the direction of the electric field just before its collision with a gas molecule. For the present we may assume the gas molecules to be *at rest* on an average. Then after the collision the component of the carrier's velocity in the direction of $v$, i.e. in the direction of the electric field, will on the average be $mv/(m+\mathrm{M})$. (See Vol. I, p. 215.) Thus one collision produces on the average a diminution $\mathrm{M}v/(m+\mathrm{M})$ in the component of the carrier's velocity in the direction of the electric field. In the stationary state this velocity diminution is balanced by the acceleration imparted to the carrier between its collisions by the electric field of strength E. If $\lambda$ is the mean free path and $v_m$ the mean velocity of the molecular motion of the carrier, then $\lambda/v_m$ is the mean time between two collisions and therefore

$$\frac{e\mathrm{E}\lambda}{mv_m}=\frac{\mathrm{M}v}{m+\mathrm{M}},$$

where $e$ is the charge of the carrier. The observed velocity $w$ in the direction of the field is the mean velocity in the periods of uniformly accelerated motion between the collisions, i.e.

$$w=\frac{mv}{m+\mathrm{M}}+\frac{1}{2}\frac{e\mathrm{E}\lambda}{mv_m}.$$

Substituting the value of $v$ from the previous equation, we have

$$w=\frac{\lambda}{v_m}e\mathrm{E}\left(\frac{1}{\mathrm{M}}+\frac{1}{2m}\right).$$

Now the mean free path $\lambda$ of a foreign molecular species present in small quantity in a gas (i.e. of the carriers in the present case) is

$$\lambda=\frac{1}{n'\pi s^2}\sqrt{\frac{v_m{}^2}{v_m{}^2+v_g{}^2}}$$

(compare Vol. II, p. 50), where $v_g$ is the mean molecular velocity of the gas, $n'$ the number of molecules per unit volume, and $s$ the sum of the radii ($r$ and R respectively) of a carrier and a gas molecule. Substituting this in the above equation, we obtain

$$w=\frac{e\mathrm{E}}{n'\pi s^2\sqrt{v_m{}^2+v_g{}^2}}\left(\frac{1}{\mathrm{M}}+\frac{1}{2m}\right).$$

Putting $m/(m+\mathrm{M})=\mu$ and remembering that the density of the gas is given by $\rho=n'\mathrm{M}$ and that $v_m=v_g\sqrt{\mathrm{M}/m}$, we obtain

$$w=\frac{e\mathrm{E}}{\rho\pi s^2 v_g}\cdot\frac{1+\mu}{2\sqrt{\mu}}.$$

Taking account of the Maxwell distribution of velocities and the Clausius distribution of paths (instead of calculating, as above, with mean velocities and mean paths), we obtain an additional factor $\Omega$ in the above expression for $w$. The values of $\Omega$ when $\mu = 0, \frac{1}{2}, 1$ are respectively $\Omega = 4, 1\cdot4, 1$.

The value for $w$ requires further correction on account of the error introduced by assuming that the gas molecules are (on an average) at rest. This gives another factor $3/(3 + \mu)$.

On the whole, therefore, we obtain the result:

$$w = \frac{eE}{\rho \pi s^2 v_g} \cdot \frac{1 + \mu}{2\sqrt{\mu}} \cdot \frac{3}{3 + \mu} \cdot \Omega.$$

*Special Cases.*—(*a*) When $\mu$ is practically equal to zero, i.e. when the mass of the carrier is very small compared with that of a molecule, we have

$$w = \frac{eE}{\rho \pi s^2 v_g} \cdot \frac{2}{\sqrt{\mu}}.$$

(*b*) When $\mu = \frac{1}{2}$ (carriers are simple gas molecules), we have

$$w = \frac{eE}{\rho \pi s^2 v_g} \cdot \frac{9 . 1\cdot4}{7\sqrt{2}}.$$

(*c*) When $\mu = 1$ (very large carriers), we have

$$w = \frac{eE}{\rho \pi s^2 v_g} \cdot \frac{3}{4}.$$

In *actual cases* trial values of $s$, and hence of $r$, are calculated from the measured value of $w$, using each of the above three special formulæ in turn. The right value of $\mu$ can then be ascertained and employed for the accurate calculation of $r$.

**Properties of the Carriers in Atmospheric Air.**—The *charge of the* carriers has been found in most cases to be equal to that carried by a univalent ion in electrolysis, i.e. *one elementary or electronic charge.* This was shown directly (MILLIKAN) by observations of the variations of the charge of visible particles using the suspension method of EHRENHAFT (p. 48). Carriers were generated in the neighbourhood of the particle under observation, and the collisions of the carriers with the particle were followed by measurement of the variations of the charge on the latter. Comparatively very large though still microscopically invisible carriers also appear in general to bear only *one* elementary charge. Hence in the above formula for the velocity in an electric field we may substitute for $e$ the charge of an electron. In the case of visible carriers, however, very high multiples of the elementary charge are sometimes observed.

*Size of the Carriers.*—The "normal" carriers with a mobility of the order of magnitude of 1 cm./sec. per volt/cm. prove to have a radius about 3 to 5 times that of a molecule. Hence they doubtless consist of molecular aggregates or complexes. We must assume that the original carrier, consisting probably of a single charged molecule, very rapidly attaches to itself a few other neutral molecules (especially water molecules) and then remains in this state, provided that the gas contains no large suspended particles. But when there are also large suspended particles, either solid or liquid (so-called "condensation nuclei"), the carriers soon become attached to these, thus becoming relatively large. These are the carriers of very low mobility already referred to above.

The increase in size ("ageing") of carriers has been observed in many cases by the corresponding decrease in mobility. Abnormally *high* mobilities are often observed in the case of negative carriers. It is very probable that these very mobile negative carriers are not material particles but electrons (see below), to which the conduction (e.g. in flames) is at least partially due.

**The Recombination of Carriers.**—As is proved at once by the fact that a conducting gas can be passed through a cylindrical condenser (see, for example, the experiment of fig. 1, p. 299), oppositely charged carriers can remain together in a gas for a considerable time without combination. This is because the mean distance of separation of the carriers, whose number is at most only about $10^{-11}$ of the number of gas molecules, is very great in comparison with the charges they bear, so that the mutual electrostatic attraction is probably practically negligible. Recombination is occasioned almost exclusively by collisions due to thermal agitation. In a gas containing a mixture of $n$ positive and $n$ negative carriers per unit volume, the rate of decrease of the number of carriers is given by

$$\frac{dn}{dt} = -\,an^2.$$

It is thus proportional to the square of the instantaneous carrier concentration. The constant $a$ is called the *coefficient of recombination*. Its dimension is [sec.$^{-1}$]. For air under normal conditions its value is about $1{\cdot}6 \,.\, 10^{-6}$ sec.$^{-1}$.

**Diffusion of the Carriers.**—The carriers share the thermal agitation of the gas, thus acting like a small addition of another gas. Hence they show the phenomena of diffusion, moving on the average from regions of greater concentration to regions of lesser concentration. The diffusion coefficients for "normal" carriers in air have been found to be $D^+ = 0{\cdot}03$ and $D^- = 0{\cdot}04$ cm.$^2$/sec. Since these figures give the excess of carriers moving in the direction of a concentration gradient of unit magnitude, the greater mobility of the negative carriers is once more apparent. Experiments upon neutral gases show that those with large molecular weights have diffusion coefficients of about the same magnitude as that given above for the carriers. This furnishes additional evidence that the carriers are molecular complexes.

**Condensation Nuclei.**—(Compare Vol. II, p. 67.) Supersaturated water vapour will condense upon relatively large suspended particles. Such particles are formed principally in chemical reactions, e.g. when air is irradiated with short wave-length radiation with formation of nitrogen compounds, ozone, hydrogen peroxide, &c., and during combustion (smoke particles). Such condensation nuclei are also formed in the neighbourhood of concentrated sulphuric acid by the clustering of water molecules around the $H_2SO_4$ molecules in the vapour. Since the electrically charged carriers readily cluster in this way or attach to themselves clusters of neutral gas molecules, they too can form condensation nuclei. A current of steam issuing from a nozzle is very sensitive to the presence of such nuclei (Helmholtz). Thus the previously almost invisible current of steam becomes intensely white (owing to the rapidly condensing water droplets) when a glowing match or a rod moistened with concentrated sulphuric acid is held near it (fig. 15). From the colour of the cloud of steam it is possible to draw conclusions as to the size of the nuclei.

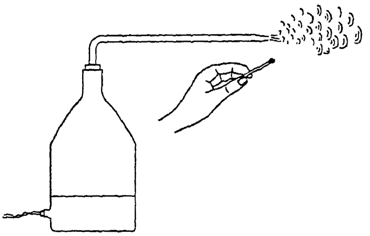


Fig. 15.—Condensation of a jet of steam upon nuclei

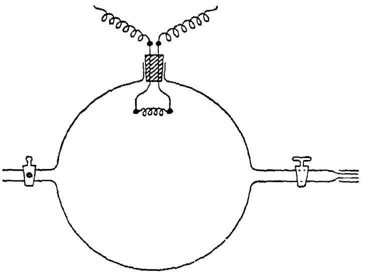


Fig. 16.—Action of condensation nuclei

The condensation of saturated water vapour only takes place in presence of condensation nuclei. This can be demonstrated with the apparatus shown in fig. 16, which consists of a glass bulb fitted with two wide-bore taps and containing a spiral of platinum wire which may be heated to redness by an electric current. The interior of the bulb is well wet with water, so that the air within it is saturated with water vapour. The platinum spiral is first left cold and the bulb put into communication with the outer atmospheric air by opening the left-hand tap, the other tap, which is connected to a suction pump, being kept closed. The first tap is now shut and the second one opened for a moment. A thick cloud, which can be seen very well when illuminated strongly from the side, is formed within the bulb. The sudden fall of pressure on opening the second tap caused a corresponding sudden fall of temperature (see Vol. II. p. 96) and a consequent condensation of the already saturated water vapour. The experiment is repeated several times, the cloud being allowed to settle before each repetition. It is observed that the cloud becomes less and less dense, until finally no cloud is formed at all; only when the rarefaction is great is there still a slight condensation of relatively large drops (see below). The platinum wire is now heated to redness for a short time and, after a sufficient wait for the temperature of the air to resume its normal value, the experiment is repeated once more. A thick cloud is again formed. The condensation nuclei given out by the platinum wire and the carriers adhering to them (see also the experiment of fig. 11) have once more made condensation possible.

The *Wilson cloud-chamber* is an extremely fruitful application of this phenomenon for the detection of carriers by means of their attachment to condensation nuclei. The principle of the cloud-chamber is the same as that of the experiment just described; but the magnitude of the pressure fall can be more exactly controlled, and the apparatus is designed to eliminate as far as possible the formation of vortices during the expansion of the gas, so that the spatial distribution of the nuclei may be determined. Fig. 17 shows a practical form of Wilson cloud-chamber. Wilson * found that there is no visible condensation when the gas is expanded to 1·25 times its original volume, provided that the nuclei originally present have been removed. When carriers are present, however, condensation begins at this expansion ratio, only the *negative* carriers being effective as nuclei. Above the expansion ratio 1·31 the positive carriers also become effective. Above 1·38 cloud formation takes place upon the water nuclei even when there are no carriers. According to Lenard the difference in the behaviour of the negative and positive carriers is due to their difference in size, as indicated by their difference of mobility (p. 309). Figs. 53 to 55 (pp. 336 to 337) are reproductions of photographs obtained by the Wilson cloud-chamber method.

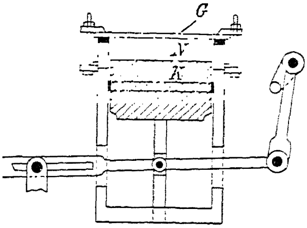


Fig. 17.—Wilson cloud-chamber (after Shimizu). N, cloud chamber; G, glass plate; K, piston.

## 2. Conduction of Electricity in High Vacua.

**Spontaneous Conduction in a Vacuum.**—If the air be removed as completely as possible (with modern pumps—see Vol I. § 1. p. 351—pressures of $10^{-6}$ mm. Hg can readily be attained) from between two electrodes, *very high* field strengths may be applied without a breakdown of the insulation. A vacuum is the best insulator known.

The application of extremely high field strengths, however, produces spontaneous conduction. The tension of the lines of force is then so great that the electrons are torn right out of the negative electrode. Field strengths of about a million volts per centimetre at the cathode, produced either by using a cathode of very small radius of curvature or by placing the electrodes very close together, are necessary in order to give this effect. It is found that the discharge takes place at certain points of the cathode, probably at very small irregularities of great curvature where the effective field strength may be estimated at $100 \,.\, 10^6$ volts/cm.

**Induced Conduction in a Vacuum.**—It is possible by certain methods to induce conduction between electrodes in a vacuum even at very low field strengths, the most important of these methods (compare p. 301) being (i) illumination with ultra-violet light and (ii) heating.

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* C. T. R. Wilson (1869– ), British physicist, Jacksonian professor of natural philosophy, Cambridge, since 1925; Nobel prizeman, 1927.

When a potential difference (say 220 volts) is applied between the electrodes of the evacuated tube shown in fig. 18 and the *negative* electrode K is illuminated with ultra-violet light from a source L through a quartz window F, a current is observed in the galvanometer G (**photoelectric effect**, see Vol. V).

A simpler method, used almost exclusively in modern applications, is to generate the carriers by means of an incandescent solid. The arrangement commonly employed is shown in fig. 19. The one electrode consists of a loop of wire (made of tungsten, often containing a very small amount of thorium or coated with a layer of certain oxides, see p. 319), which can be heated to incandescence within the highly evacuated tube by means of an external heating battery HB carried upon an insulated support. A potential difference is applied (say from the direct-current lighting mains) between this filament and the other electrode. When (and *only* when) the incandescent filament is the *cathode*, a current passes through the tube and causes a deflection of the milliamperemeter A (**thermionic emission**, p. 365).

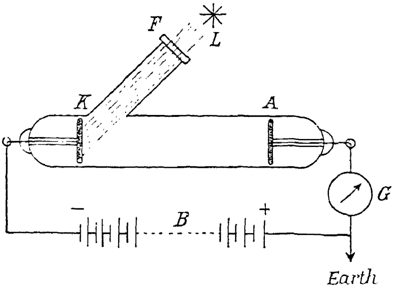


Fig. 18.—Conduction through a vacuum by illumination of the cathode

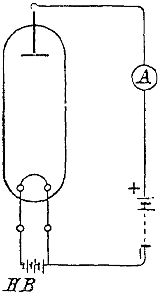


Fig. 19.—Conduction through a vacuum by carriers emitted from an incandescent cathode.

In these experiments it is found that a current only passes across the vacuum when the process inducing it takes place at the cathode. From this we conclude that the conduction is due here to *negative* carriers emitted from the cathode (*cathode rays*, p. 323).

We must expect an essential difference between the processes of conduction in a vacuum and in gases at relatively high pressures or in electrolytes. For in the latter cases the carriers are subjected to the frictional resistance of the surrounding medium, whereas in a vacuum there is no such resistance. Thus in gases at relatively high pressures and in electrolytes a constant electric force causes the carriers to move with a certain *uniform* velocity, but in a vacuum the motion of the carriers under the influence of a constant electric force is *accelerated*, like that of a body falling through a vacuum under the influence of gravity.

In the case of conduction by carriers moving with friction the velocity of migration $w$ is proportional to the field strength E, i.e. $w = El$ where $l$ is a constant (the mobility). But in the present case of conduction by carriers moving through a vacuum the velocity attained will also depend upon the distance $d$ over which the electric field E acts, i.e. upon the product E$d$, which is equal to U, the potential difference. Thus the terminal velocity $v$ and hence also the kinetic energy $W_k = \frac{1}{2}mv^2$ of the carriers ($m$ = mass of carrier) is determined only by the potential difference between the electrodes. The work corresponding to the motion of a charge $e$ (the charge on a carrier) through a potential difference U is $e$U (p. 29). Hence for a carrier moving freely we have

$$eU = \tfrac{1}{2}mv^2 = W_k.$$

*Thermal and Luminous Effects produced by the Carriers at High E.M.F.s.*—The carriers bombard the anode with this kinetic energy $W_k$, which is thereby transformed to a large extent (p. 349) into heat. At sufficiently high E.M.F.s (800–1000 volts) the anode is observed to become red hot.

In consequence of their high velocities the carriers are also able to render certain substances luminous, e.g. zinc silicate containing manganese. With the help of a screen covered with such a substance it can easily be shown that the carriers are propagated in straight lines from the cathode K. An anode A with a hole in it is used (fig. 20). The carriers fly through the hole and produce a sharp image of it upon the screen Fl set up behind the anode (U = 500–1000 volts).

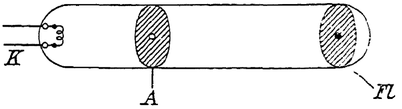


Fig. 20.—Rectilinear propagation of the cathode particles in a vacuum

**Determination of the Nature of the Carriers causing Conduction in a Vacuum.**—The carriers are characterized by their charge $e$ and their mass $m$. Now we have already derived an equation connecting these two quantities with the easily measurable quantity U. This equation can be re-written in the form

$$2U = \frac{v^2}{(e/m)}.$$

In addition to $e$ and $m$ this contains a third unknown, namely $v$.

The following is a slightly different method. Under the assumption (which will be shown later—p. 321—to be justified) that all the particles have the same $e$ and $m$, the total kinetic energy of N particles is N times that of one particle, i.e. $NW_k$. Assuming further (as is also justified, see p. 349) that this kinetic energy is transformed practically completely into heat energy, we can calculate $NW_k$ from the heating effect at the anode. The quantity of heat evolved being H, we have the equation

$$H = NW_k = \tfrac{1}{2}Nmv^2.$$

Now the product N$e$ is equal to the total quantity Q of charge transported, which can be measured by observation of the current. Substituting $N = Q/e$ in the last equation we obtain $H = \tfrac{1}{2}mv^2Q/e$ or

$$\frac{2H}{Q} = \frac{v^2}{(e/m)},$$

i.e. the same quantity as in the first equation above.

It is also possible to determine the velocity of the particles directly (Wiechert, 1899) by measuring the time they take to travel a certain distance when accelerated by a momentarily applied E.M.F. In this way an additional though not very accurate value is obtained for $v^2 = 2Ue/m$. But we require three independent equations for the determination of $v$, $e$ and $m$. There are several ways of obtaining further experimental relationships between these quantities. We will go on to consider the two most important of these.

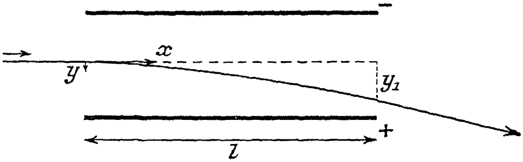


Fig. 21.—Deviation of cathode particles in a transverse electric field

**The Behaviour of Moving Carriers in Electric and Magnetic Fields.**—1. *Longitudinal Electric Field.*—As has already been stated on p. 307, a particle of mass $m$ and charge $e$ acquires in an electric field of strength E an acceleration $a = Ee/m$ in the direction of the field. Consider now a negative carrier, to which a certain final velocity $v$ has been imparted by the field between the cathode K and the anode A (fig. 20). This is the velocity with which the particle arrives at the anode. Suppose that the anode has a hole in it at the point where the particle would otherwise strike it. The particle flies through the hole and, since the space behind the anode is free from electric field, continues to move on with constant velocity $v$ (compare the analogous experiment with Attwood's machine, Vol. I, p. 69).

2. *Transverse Electric Field.*—Let such a uniformly moving particle enter a homogeneous electric field with its lines of force at right angles to the direction of motion of the particle (fig. 21). The particle is then subjected to the same kind of force as a horizontally projected stone in the earth's gravitational field. Taking the direction of the electric force E$e$ as the $y$ direction and the initial direction of the

particle as $x$ direction, the co-ordinates of the particle at a time $t$ after entering the field, measured from the point of entry, are

$$x = vt \quad \text{and} \quad y = \tfrac{1}{2}at^2$$

(see Vol. I, p. 44). Eliminating $t$ from these equations we obtain

$$2yv^2 = ax^2.$$

Substituting the value of $a = \mathrm{E}e/m$ obtained above we have

$$2yv^2 = \mathrm{E}ex^2/m,$$

whence
$$\frac{v^2}{(e/m)} = \frac{\mathrm{E}x^2}{2y}.$$

Thus under the influence of the transverse electric field the particle traces out a parabola whose equation is

$$x^2 = \frac{2v^2}{\mathrm{E}\,(e/m)} \cdot y.$$

Let $l$ be the length of the condenser plates between which the transverse field is applied. Then the value $y_1$ of $y$ corresponding to $x = l$ is the distance which the carrier has been deviated towards the positive plate during its passage through the condenser. After emerging from the electric field, the particle will move on in a straight line, i.e. along the tangent to the parabola at the point $(l, y_1)$. Suppose that a screen is set up at a distance $d$ beyond the edge of the condenser (fig. 22) and that the material of the screen is such that the point of incidence of the particle can be observed (e.g. as a luminous spot). This spot is then the intersection of the tangent to the parabolic path at the point $(l, y_1)$. Let $s$ be the observed deflection of the spot from its position when there is no electric field. Then applying the equation of the tangent to the parabola at the point $(l, y_1)$ we have

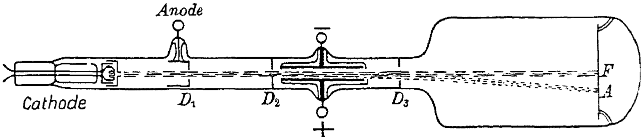


Fig. 22.—Measurement of electrostatic deviation

$$l(d + l) = \frac{v^2}{\mathrm{E}\,(e/m)}(y_1 + s).$$

But from the equation of the parabola

$$l^2 = \frac{2v^2}{E(e/m)} \cdot y_1.$$

Elimination of $y_1$ from these two equations gives

$$s = \frac{l(2d + l)\,\mathrm{E}\,(e/m)}{2v^2}.$$

From this it follows at once that the observed deviation $s$ is directly proportional to the field strength E and inversely proportional to the square of the velocity $v$, i.e. to the accelerating E.M.F.

The values of E, $s$, $l$, and $d$ can be obtained by direct measurement. The magnitudes $v^2$ and $e/m$ are unknown. Hence an experiment of this kind again gives us a value of the same quantity as before (p. 313), viz.

$$\frac{v^2}{(e/m)} = \frac{l(2d + l)\,\mathrm{E}}{2s}.$$

The deviation of moving electrified particles by a transverse electric field can be seen very beautifully in the case of the luminous streaks of vapour from an alkali salt placed in a Bunsen flame burning between charged condenser plates (LENARD, see fig. 23).

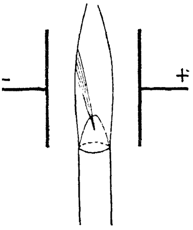


Fig. 23.—Lenard streaks in a Bunsen flame

3. *Transverse Magnetic Field.*—Like every electric current, the convection current borne by a stream of electrified carriers has a magnetic field and can be influenced by other magnetic fields. The stream of carriers is deflected by an external magnetic field in the same way as a conductor carrying a current, namely, at right angles both to the direction of the current and to the direction of the magnetic field.

On p. 440 it will be shown that the force P exerted upon a conductor running at right angles to a magnetic field is directly proportional to the strength I of the current flowing in the conductor, the length $l$ of the conductor and the induction B (or the strength H) of the magnetic field. We may express this by the equation

$$\mathrm{P} = k\mathrm{I}l\mathrm{H},$$

where $k$ is a constant (equal to 0·1257, if P is in dynes).

The convection current at present under consideration consists of a rapid succession of carriers each with the charge $e$. Suppose that there are $n$ carriers in an element of length $\Delta s$, and that the time each takes to cover this length is $\Delta t$. The velocity $v$ is then $\Delta s/\Delta t$. The number of carriers passing a given point per second is $n/\Delta t$ and therefore the current strength is $\mathrm{I} = ne/\Delta t$. Hence

$$\mathrm{I}\,\Delta s = ne\frac{\Delta s}{\Delta t} = nev.$$

The force P upon the $n$ carriers of the element of distance $\Delta s$ is therefore

$$\mathrm{P} = knev\mathrm{H},$$

where $k$ is the numerical constant mentioned above. It follows that *each individual carrier* is subjected to a force

$$\mathrm{P}' = kev\mathrm{H}$$

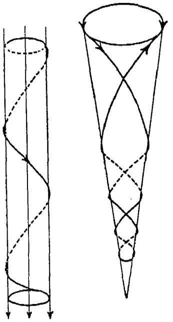
at right angles both to its direction of motion and to the magnetic field. (See p. 442.)

Under the action of this force the particle will describe a circular path (compare Vol. I, § 10, p. 78). The radius of this path being $r$, the centripetal force is given by

$$\frac{mv^2}{r} = kev\mathrm{H}.$$

Hence $$\frac{v}{(e/m)} = k\mathrm{H}r.$$

Now the magnetic field strength H and the radius $r$ of the path of the cathode particles can be measured directly (p. 320) and hence the value of the expression on the left-hand side of the equation can be obtained.



Figs. 24 and 25.—Paths of cathode particles in a longitudinal magnetic field.

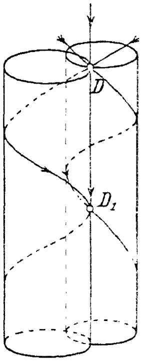


Fig. 26.—Focusing of a beam of cathode particles by means of a longitudinal magnetic field.

4. *Longitudinal Magnetic Field.*—Electrically charged particles are not affected by a homogeneous magnetic field when their direction of motion coincides with that of the magnetic lines of force. But if they enter the field obliquely, then in analogy to the experiment of fig. 61, p. 204, they will describe spiral paths around the lines of force (fig. 24). This can be understood by resolving the velocity of the entering particle into two mutually perpendicular components, one parallel to the lines of force and the other at right angles to them. The first component remains unaffected by the field, but the second gives rise to a circular motion, as described above. The spiral path is obtained by the combination of the uniform motion in the direction of the lines of force and the circular motion at right angles to them. The greater the initial inclination of the path of the particle to the field direction, the smaller its velocity component along the lines of force but the greater the circle described at right angles to them. The complete calculation shows that carriers entering the field at one point D (fig. 26) in different directions but with the same velocity are deviated in

such a way that they all pass through another point $D_1$. Thus by means of a longitudinal magnetic field it is possible to focus carriers of one velocity as light is focused by a lens.

Mention must also be made of the case of a carrier entering a magnetic field whose strength increases with the distance from the point of entry. The path of the carrier is then a spiral upon the curved surface of a cone (fig. 25). After proceeding a certain distance the particle turns back, as though reflected. Concerning other types of path, see p. 320.

The focusing of a beam of carriers in a longitudinal magnetic field can be employed to obtain a relation between $e$, $m$, and $v$ (H. BUSCH). If $v$ is the velocity parallel to the lines of force and the distance $DD_1$ (see fig. 26) is $l$, then

$$\frac{v}{(e/m)} \quad \frac{kHl}{2\pi},$$

where H is the magnetic field strength and $k$ the numerical constant referred to above.

We have now learned three different methods of determining the value of

$$\frac{}{(e/m)},$$

and two different methods for determining the value of

$$\begin{array}{c} v \\ (e/m) \end{array}.$$

Thus we only have two equations involving the three unknowns $v$, $e$ and $m$. The quantities $e$ and $m$ cannot therefore be calculated separately; the above methods enable us only to calculate $v$ and the ratio $e/m$. Nor does the method of WIECHERT (p. 314) give an independent equation, as it also involves only the quotient $e/m$.

In order to obtain $e$ and $m$ separately, one or other of them must be determined independently. Different methods must be employed in different cases (see, for example, p. 345).

**Conduction by Cathode Particles in a Vacuum.**—The above methods can be applied to study the carriers generated in a vacuum by photoelectric or thermionic emission. The apparatus employed is shown in figs. 27 and 28 and in fig. 22, p. 315.

Fig. 27 shows an arrangement for investigating the carriers produced in a vacuum by the photoelectric effect (LENARD, compare fig. 18, p. 312).

A known E.M.F. is applied between the anode A and cathode K. The other electrode B is connected to an electrometer. The dotted circle M represents an electromagnet with its field perpendicular to the plane of the figure. The cathode is illuminated and the current strength in the electromagnet (i.e. the strength of the magnetic field) is varied until the rate of charging of the electrometer connected to B is a maximum. The carriers then strike B in the way indicated in the figure. From the applied E.M.F., the magnetic field strength and the geometrical dimensions of the apparatus it is then possible to calculate the value of $e/m$ (see p. 317).

One method of investigating the carriers emitted from a glowing cathode, namely, by their deviation in a transverse electric field, has already been given on p. 314 (apparatus shown in fig. 22, p. 315). The most suitable cathode to use in this experiment is a glowing spiral of platinum or tungsten wire coated with certain oxides (see below). To investigate the behaviour of the particles in a magnetic field use is often made of the arrangement shown in fig. 28. The two thick leads for the current to heat the cathode also serve as supports for the tube. Upon the platinum foil cathode is fixed a speck of calcium oxide, or more often barium oxide with an admixture of cerium oxide (**incandescent oxide cathode** of WEHNELT). When the cathode is red hot, a potential difference of a few hundred volts (e.g. the pressure of the lighting mains), applied between it and the rod-shaped anode at the side of the tube, is sufficient to produce a considerable current. In order to render the path of the particles visible, a little air is left in the tube. This air is made luminous by the rapidly flying particles. What is normally observed is a bright, luminous streak normal to the cathode surface.

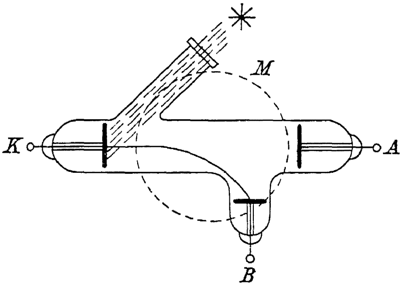


Fig. 27.—Determination of $e/m$ for photo-electrically emitted particles in a vacuum

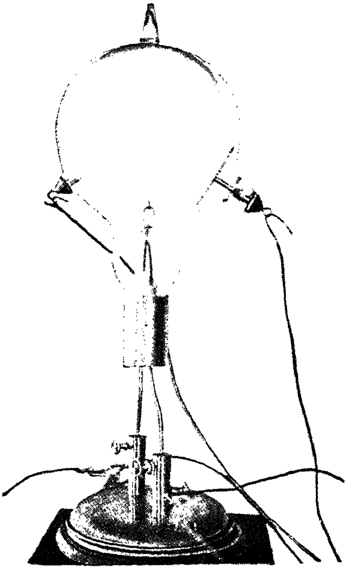


Fig. 28.—Wehnelt tube

A large plane plate B can be seen opposite the oxide cathode to the right in fig. 28. A beam of particles emitted from the cathode impinges upon this plate at a certain angle. When now the plate is connected to the negative pole of an independent battery whose positive pole is earthed, the cathode particles are repelled. If the electrostatic field in the neighbourhood of the plate be homogeneous, the beam will be bent into a parabola with its convex side towards the plate. Fig. 28 shows the effect actually observed (reproduced from a photograph by Wehnelt).

The circular path of the carriers in a magnetic field can be demonstrated by placing the Wehnelt tube between the poles of an electromagnet with a sufficiently nearly homogeneous field. The tube is so arranged that the cathode beam is emitted at right angles to the lines of force. By a suitable choice of magnetic field strength and tube E.M.F. the luminous path of the carriers can be bent round into a complete circle. Fig. 29, which is reproduced from a photograph, gives an idea of the appearance of this effect.

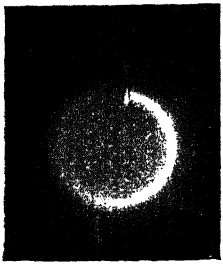


Fig. 29.—Deflection of the cathode rays in a Wehnelt tube by a magnetic field.

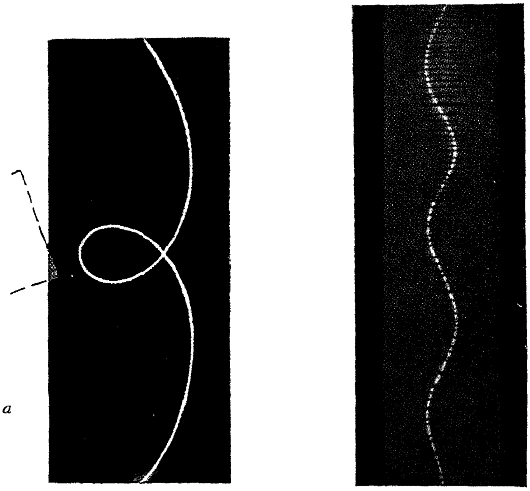


Fig. 30 *a* and *b*.—Paths of cathode particles in a magnetic field (narrow beams of 200 volts) after Brüche. *a*, Plane deflection in the field of a steel magnet; *b*, three-dimensional deflection in the internal field of a solenoid.

[From *Zeitschrift für Physik*, Vol. 64, published by J. Springer, Berlin.]

Determinations of the velocity $v$ and the ratio $e/m$ of the cathode particles by the methods given above are greatly facilitated by the use of the Wehnelt tube with incandescent oxide cathode. With its aid rough values can be rapidly obtained for instructional purposes.

**Narrow Beams of Cathode Particles.**—By means of a small cylindrical tube placed in front of the incandescent oxide cathode and kept at a relative potential of about +100 volts it is possible (at suitable emission strength and gas pressure in the tube) to confine the otherwise divergent beam of cathode particles into a narrow luminous streak with a constant cross-section of only about $\frac{1}{4}$ mm.$^2$ and a length which may amount to several metres (see fig. 30). The visibility of this streak is due to the luminosity of the gas molecules when struck by the flying carriers. The fact that it does not diverge is probably due to the action of the space charge produced by collisions with the gas molecules (p. 334).

**The Electron.**—The results obtained from the above kinds of experiments upon conduction in high vacua are as follows:

1. The particles carry only *negative* charges.

2. The luminous points of incidence of the deviated beams are just as sharp as those of the undeviated beams. From this it follows that *all* the carriers have the same velocity $v$ and the same ratio $e/m$. We may conclude that they all have the same charge and the same mass. Thus we are dealing only with *one kind of carrier.*

3. Provided that the E.M.F. is not too high (up to about 1000 volts, see p. 322), all the methods agree in giving the value of $e/m$, the **specific charge, as $(1{\cdot}769 \pm 0{\cdot}002) \cdot 10^8$ coulombs per gramme.***

Important conclusions can be drawn from a comparison of this value with that of the ions in electrolyte solutions. Each gramme-equivalent of an ion transports 96,494 coulombs. In the case of hydrogen, the equivalent weight of which is 1·008, we therefore have $e/m = 96{,}494/1{\cdot}008$ coulombs per gm. The value of $e/m$ is smaller for all other substances, since they all have greater equivalent weights than hydrogen. The quantity of electricity carried by 1 gm. of the cathode particles is thus $(1{\cdot}77 \cdot 10^8)/95{,}800 = 1850$ times greater than that carried by 1 gm. of hydrogen ions.

This result can be interpreted in two ways. Firstly, the carriers may have a mass of the order of magnitude of that of ordinary atoms. If this is so, each particle must carry a charge about a thousand times greater than the unit of charge occurring in electrolytic conduction, and in conduction by carriers through gases. This is extremely improbable; for we have seen (p. 49) that carriers generated by the same methods in air, like the ions in electrolytic conduction, usually carry *one* elementary charge (p. 308). The second possible interpretation is that the cathode particles carry *one* elementary charge, but that their mass is only 1/1850 of that of a hydrogen atom. From what has been said above it is clear that this possibility appears at once to be the more probable. It is further supported by the fact that the negative cathode particles give rise to normal carriers with one elementary charge when they pass out of the vacuum into the air (for the corresponding experimental arrangement see p. 332). The main support for the second interpretation is to be found, however, in atomic physics, which is the subject of Vol. V.

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* It is remarkable that this value is not in exact agreement with that obtained from spectroscopic measurements, viz. $e/m = (1{\cdot}761 \pm 0{\cdot}001) \cdot 10^8$ coulombs per gm. (Vol. V). Recent determinations by deviation methods (KIRCHNER, 1931) appear, however, to give the value $1{\cdot}760 \cdot 10^8$ coulombs per gm.

Since, therefore, all the observed phenomena tend to show that the negative carriers in a vacuum are charged with *one* elementary charge, we are justified in concluding that the mass of these carriers is only 1/1850 of that of an atom of hydrogen, the lightest of all known material substances.* The mass of a hydrogen atom is $1{\cdot}66 \,.\, 10^{-24}$ gm. (Vol. II, p. 53); the mass of a cathode particle is therefore $8{\cdot}98 \,.\, 10^{-28}$ gm. It follows that the particles cannot consist of any material substance; for this reason we introduce a special name for them and call them **electrons.**

**An electron is thus a non-material particle with a negative charge of $e = 1{\cdot}59 \,.\, 10^{-19}$ coulomb and a mass of $m_0 = 8{\cdot}98 \,.\, 10^{-28}$ gm.**

We must regard electrons as **actual particles of negative electricity.** *Thus in the electrons we have pure electricity, unassociated with matter.*

It was formerly supposed that mass was a property of matter alone. Hence the fact that mass had to be ascribed to a non-material substance necessitated an important revolution of thought. In Vol. V we shall obtain information as to the nature of the mass of the electron. It is found quite generally that all energy, e.g. the energy of light, possesses mass, so that nowadays we are accustomed to the idea of mass without material substance.

**Variation of the Mass of an Electron with its Velocity.**—Determinations of the values of the ratio $e/m$ for very rapidly moving electrons, such as were first observed in radioactive processes (Vol. V), led to the very surprising and remarkable result (KAUFMANN †) that $e/m$ decreases as the velocity of the electrons becomes greater.

KAUFMANN found the following values:

| Velocity | Value of $e/m$ |
|---|---|
| $1{\cdot}00 \,.\, 10^{10}$ cm./sec. | $1{\cdot}7 \,.\, 10^{8}$ coulombs/gm |
| 1·50 ,, | 1·52 ,, |
| 2·36 ,, | 1·31 ,, |
| 2·48 ,, | 1·17 ,, |
| 2·59 ,, | 0·97 ,, |
| 2·72 ,, | 0·77 ,, |
| 2·83 ,, | 0·63 ,, |

Since the charge $e$ of an electron is to be regarded as a constant, we must conclude that the mass $m$ varies with the velocity $v$ (see Table X, p. 323). Very exact measurements, especially by A. H. BUCHERER (1908) and G. NEUMANN (1914), have shown that this variation is in accordance with the equation

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}},$$

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* The term *material substance*, or more shortly *matter*, refers to all composed of the chemical elements or their compounds (Vol. I, p. 53).

† WALTER KAUFMANN, Professor of Physics at Königsberg since 1908.

where $c$ is the velocity of light, and $m_0$ the mass of the electron at relatively small velocities. Up to velocities of about $10^9$ cm./sec., where the increase of mass is less than 1 per cent, we may regard $m_0$ as a constant. A more thorough treatment of this relationship, which is of fundamental importance for the development of modern physical theories, will be given in Vol. V.

**Cathode Rays.**—The experiments described above have been concerned with electrons moving from the cathode towards the anode under the influence of a more or less homogeneous electric field. Electrons moving in this way can fly through holes in the anode, overcoming by their mass and inertia the slight deformation of the field in the neighbourhood of the holes (figs. 20 and 22, pp. 313 and 315). Thus the moving electrons behave like rays propagated in straight lines from the cathode. They are therefore often known as *cathode rays*.

From the equation $\frac{1}{2}mv^2 = Ue$, where U is the potential difference between anode and cathode, we can calculate the velocity $v$ of the rays. To get $v$ in cm. per sec. we must first express the electrical energy in mechanical units. Now 1 volt-coulomb = $10^7$ ergs (§ 2, p. 255). Hence we have $\frac{1}{2}mv^2$ ergs $= Ue \, . \, 10^7$ ergs, where U is expressed in volts and $e$ in coulombs. Using the numerical values given above we obtain (correct to 1 per cent for potential differences up to 10,000 volts)

$$v = 5{\cdot}95 \, . \, 10^7\sqrt{U} \text{ cm. per sec.}$$

*The velocity of the free electrons is proportional to the square root of the potential difference through which they have moved.*

A potential difference of 1 volt thus imparts to the freely moving electrons a velocity of 595 km. per sec. After passing through a potential difference of 10,000 volts, their velocity is about 60,000 km. per sec., i.e. $\frac{1}{5}$ of the velocity of light.

TABLE X.—VELOCITIES OF CATHODE RAYS AFTER PASSING THROUGH DIFFERENT POTENTIAL DIFFERENCES

| Volts | Velocity | | Mass in gm. |
|---|---|---|---|
| | km./sec. | $\beta = v/c$ | |
| 0·01 | 59·5 | — | $8{\cdot}98 \, . \, 10^{-28}$ |
| 0·1 | 18·8 . 10 | — | 8·98 „ |
| 1 | 59·5 . 10 | — | 8·98 „ |
| 10 | $18{\cdot}8 \, . \, 10^2$ | — | 8·98 „ |
| $10^2$ | $59{\cdot}5 \, . \, 10^2$ | 0·02 | 8·98 „ |
| $10^3$ | $18{\cdot}7 \, . \, 10^3$ | 0·06 | 8·99 „ |
| $10^4$ | $58{\cdot}5 \, . \, 10^3$ | 0·2 | 9·16 „ |
| $10^5$ | $16{\cdot}5 \, . \, 10^4$ | 0·55 | 10·8 „ |
| $10^6$ | $28{\cdot}3 \, . \, 10^4$ | 0·95 | 28·8 „ |
| $3{\cdot}1 \, . \, 10^6$ | $29{\cdot}7 \, . \, 10^4$ | 0·99 | 63·6 „ |

The velocity of electrons is often expressed as a number of volts (the **volt-velocity**), i.e. instead of being stated in cm. per sec. it is given as the number of volts through which the electron would have had to pass in order to acquire its velocity. Thus cathode rays with a velocity of 10,000 volts are a stream of electrons moving $\frac{1}{5}$ as fast as light. Table X gives the exact velocities for a range of E.M.F.s. For quite fortuitous reasons (namely, on account of the contrast effect in photographic exposures of body tissues by means of X-rays generated by cathode rays) fast cathode rays are sometimes described as "hard", slow cathode rays as "soft". It is better, however, not to use such terms.

The greater the velocity of cathode rays, the less they are susceptible to deviation by a magnetic field. Table XI gives the strengths of the magnetic fields required to bend cathode rays of different velocities into a circle of 1 cm. radius.

TABLE XI.—MAGNETIC FIELD STRENGTHS REQUIRED TO IMPART A RADIUS OF CURVATURE OF 1 CM. TO THE PATH OF CATHODE RAYS

| Volt-velocity | Magnetic Field Strength | |
|---|---|---|
| | Ampere-turns/cm. | Gauss |
| 0·01 | 0·27 | 0·34 |
| 0·1 | 0·88 | 1·06 |
| 1 | 2·7 | 3·4 |
| 10 | 8·8 | 10·6 |
| $10^2$ | 27·1 | 34 |
| $10^3$ | 88 | 106 |
| $10^4$ | 273 | 343 |
| $10^5$ | 889 | 1,120 |
| $3{\cdot}1 \,.\, 10^6$ | 9500 | 11,940 |

**Applications of Tubes with Incandescent Cathodes as Rectifiers and Oscillographs.**—1. As was shown by the experiment of p. 312, no current passes through an evacuated tube with a cold electrode and a glowing wire electrode unless the latter be the cathode. The incandescent filament has the property of emitting electrons into the surrounding space (see further p. 365). If the negative ends of electric lines of force terminate upon the filament, the electrons emitted will follow the tension of these lines and will be accelerated as cathode rays in the direction of the field. But if the glowing filament is made the anode, there is no conduction through the vacuum; for the field cannot extract electrons from the cold electrode, and the incandescent anode furnishes practically no positive carriers. The electrons emitted by the glowing filament are driven back at once by the field and thus prevented from escaping to the cathode. This behaviour makes such a tube suitable for use as a *rectifier*. When an alternating potential difference is applied to it, a current will only flow while the glowing filament is the cathode. Thus a continuously alternating potential difference produces an intermittent uni-directional current. The tube acts like a one-way valve, hence the name *thermionic valve*. Fig. 31 shows a four-valve rectifying high-tension unit for X-ray purposes.

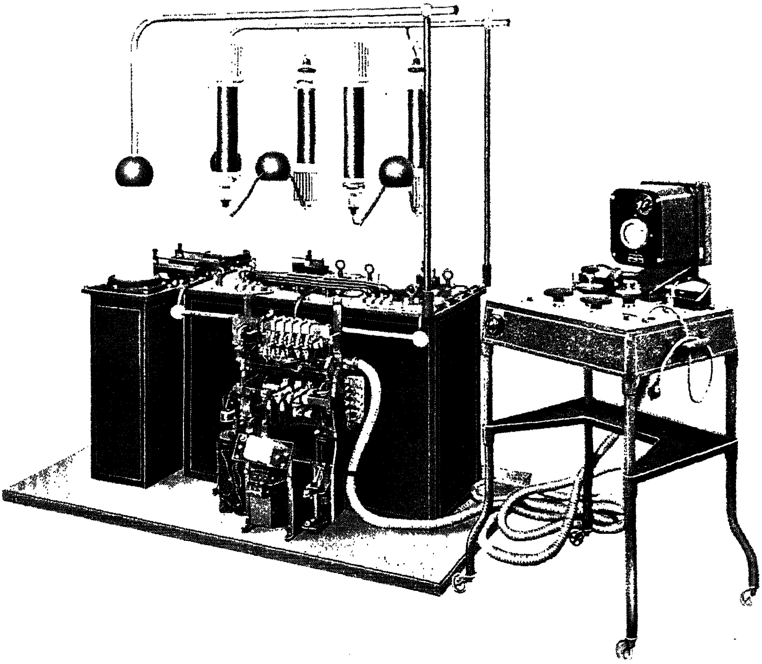

Fig. 31.—Philips Metalix rectifying valves
[The Solus Electrical Co., London.]

2. The deviation of cathode rays by electric and magnetic fields can be used with advantage to indicate the nature of rapid periodic variations of E.M.F. or current. On account of their extremely small mass the cathode rays respond with no appreciable inertia to the most rapid variations of a deviating electric or magnetic field. Hence it is possible by the deflection of cathode rays to observe periodic variations of current or E.M.F. with a frequency as high as a milliard ($10^9$) per second. Fig. 32 shows such a cathode ray oscillograph (Braun tube) with incandescent cathode. Compare the diagram of fig. 22, p. 315, and see also fig. 48, p. 333. The luminous spot where the rays strike the end of the tube is viewed from outside in a rotating mirror or

photographed (see figs. 43 and 44, p. 499, and fig. 56, p. 504).

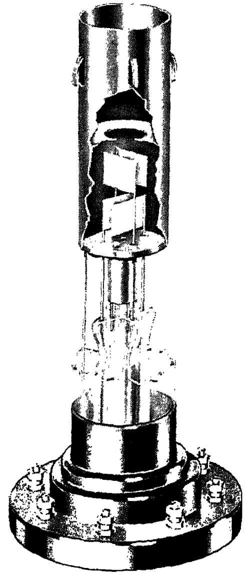

Fig. 32.—Cathode ray oscillograph with incandescent cathode

[A. C. Cossor, Ltd., London.]

## 3. Spontaneous Conduction in Gases at Reduced Gaseous Pressures

In order to understand the spontaneously occurring discharges (sparking, action of points, &c.) in gases at approximately atmospheric pressure, we must first consider the processes of conduction in gases at reduced pressures.

For the purposes of experimental investigation the gases are enclosed in glass tubes and the pressure is reduced by means of an air pump. Aluminium wires or discs are usually used as electrodes. These are sometimes mounted upon platinum wires and are fused into the glass walls. Aluminium is employed because it is only slightly affected by the discharge, whereas most other metals disintegrate (p. 340). (**Geissler tubes.***)

A unidirectional potential difference (about 1000 volts or more) is applied between the electrodes.

**Discharge Phenomena as the Pressure is Reduced.**—We use a cylindrical tube about 40 cm. long and 4 cm. wide filled with pure dry atmospheric air (or better pure nitrogen) at atmospheric pressure. The anode consists of a single aluminium wire, the cathode of a small circular disc of the same metal. We apply a potential difference of several thousand volts between the electrodes. So long as the pressure within the tube is one atmosphere there is no discharge. We now reduce the pressure gradually and observe that at a pressure of about 40 mm. a thin snake-like streak of light (fig. 33) appears. This streak is similar in appearance to the discharge which takes place in air at ordinary pressure and at correspondingly greater potential difference, but is thinner and not so bright.

As the pressure is further reduced the streak becomes thicker (fig. 34). The appearance at the two electrodes becomes strikingly different. The column of light begins close up to the anode and is a beautiful pink colour throughout its whole length. But there is a dark dividing space between the other end of the column and the cathode, while at the cathode itself there is a small luminous blue spot.

Continued reduction of pressure causes the column of light to thicken more and more, until at 2 to 4 mm. pressure it fills the whole cross-section of the tube. The dark space between the cathode and the luminous column (**the Faraday dark space***) becomes larger and the blue spot of light at the cathode increases in extent, covering a part of the cathode with a glowing layer.

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* GEISSLER, Vol. I, p. 324. This type of tube was made about 1858 by GEISSLER for the physicist Prof. PLÜCKER at Bonn for his investigations of spectra. For this reason they are also frequently referred to as Plücker tubes.

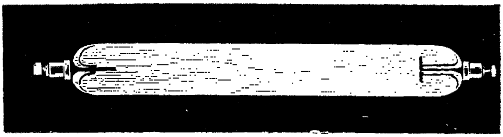

Fig. 33.—$p = 40$ mm.

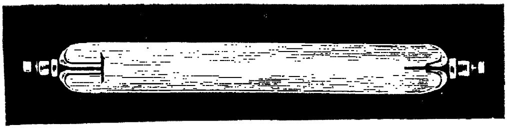

Fig. 34.—$p = 10$ mm.

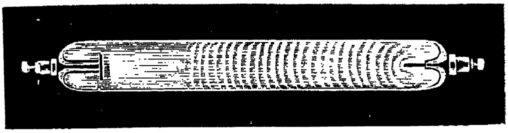

Fig. 35—$p = 1$ mm.

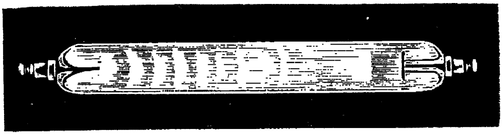

Fig. 36.—$p = 0{\cdot}5$ mm.

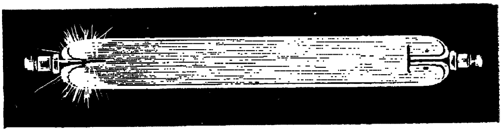

Fig. 37.—$p = 0{\cdot}02$ mm.

Figs. 33–37.—Discharge through air at reduced gaseous pressures

At about 1 mm. pressure (fig. 35) the luminous column loses its beautiful pink colour and, when mercury vapour or hydrocarbons are present (as is usually the case on account of the mercury of the pump and the grease upon ground joints and taps), becomes dull white. In addition to this it usually breaks up into peculiar striæ, which are concave towards the anode and are arranged at equal distances in the neighbourhood of the anode. The last stria is separated from the cathode by a wide dark space. In the meantime the blue luminous layer at the cathode has extended so as to cover this electrode completely. It is not, however, in contact with the surface of the cathode, but is separated from it by a dark space, known as the **first dark space** or more generally as the **Hittorf or Crookes dark space.***

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* Discovered about 1838.

At about 0·5 mm. (fig. 36) the blue cathode glow has extended further towards the anode. It passes over gradually and indefinitely into the Faraday dark space (also known as the *second dark space*), but on the other side it is sharply bounded by the first dark space. At the same time the positive column has grown shorter and the [illegible] decreased in number and become farther apart. Upon the surface of the cathode appears a very thin, faintly pinkish layer of light, the **cathode surface glow.** The extent of this surface layer increases with the current strength, as can be seen particularly well in the case of cathodes consisting of a long wire (see p. 335). Fig. 38 is a diagrammatic representation of the appearance of such a discharge.

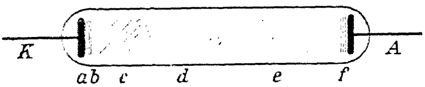

Fig. 38.—Diagram of the different parts of a normal discharge through a rarefied gas.

*a*, Cathode surface glow; *b*, Hittorf or Crookes dark space; *c*, negative glow; *d*, Faraday dark space; *e*, positive column; *f*, anode glow.

At 0·1 mm. pressure the striæ vanish altogether. The *negative glow*, now rather feeble, extends almost to the anode. The Hittorf or Crookes dark space becomes larger and larger.

At 0·02 mm. pressure the negative glow has also disappeared and the interior of the tube is practically free from light: only a short and feeble streak remains (fig. 37). But the glass walls of the tube, especially opposite the cathode, are now luminous with a greenish glow.

We will now follow out the whole range of phenomena once again in a tube of the shape shown in fig. 39 to 41.

---

* Sir William Crookes, 1832–1919, at first professor at the Training College at Chester, later for many years President of the Society of Chemical Industry. He publishe[illegible] investigations of electrical discharges in rarefied gases about 1879, ten years la[illegible] than Hittorf. Most of his results had already been observed by Plücker, Hittorf, and Goldstein, but had remained comparatively unknown. Crookes used much higher potential differences and more completely evacuated tubes than the previous investigators, and his observations were therefore more striking. Moreover, the attractive and popular style of his publications accounted in some measure for the rapidity with which his experiments became known. Crookes always maintained the view that the cathode rays were a material radiation. He spoke of them as radiant matter, or matter in a fourth state. The Crookes dark space had been observed and described by Plücker in 1858. The phenomena at the cathode at pressures below 2 mm. were first investigated systematically by Hittorf in 1884. For further notes on Hittorf and Goldstein see pp. 272 and 341 respectively.

In the earlier stages, i.e. at the higher pressures, the phenomena are essentially the same as those already described above. The thin streak of light and then the pink column fill the tube from anode to cathode by the shortest route (fig. 39), the bulb at the top of the tube remaining quite dark. The discontinuities appear in the cylindrical part of the main tube and in the side tube containing the anode. As the pressure is further reduced the negative glow separates as before from the cathode and extends to the point at which the side tube is joined on to the main tube (fig. 40).

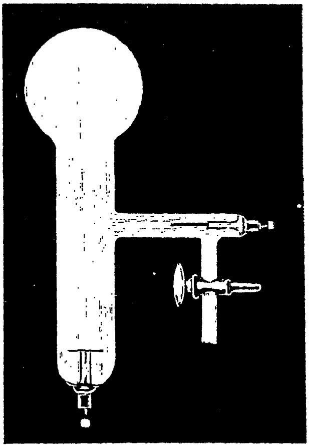

Fig. 39

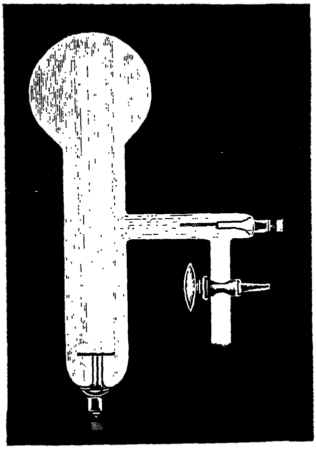

Fig. 40

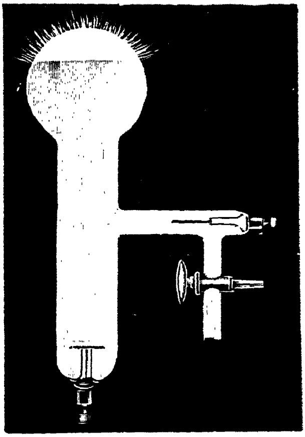

Fig. 41

Figs. 39, 40, and 41.—Discharge at reduced gas pressure

From this stage onwards, however, the phenomena are essentially different from those previously described. When the striæ disappear in the anode tube, the negative glow does not extend into this tube, but continues in a straight line along the extension of the main tube. Then this glow also disappears gradually until only a short thin streak of light remains. This streak runs straight along the axis of the main tube. The first dark space then increases in length in the same direction and not along the shortest path to the anode. Finally we observe a brilliant green luminosity on the glass wall of the bulb at the opposite end of the main tube from the cathode (fig. 41). This luminous region has a sharply defined boundary: the cylindrical walls of the main tube cast definite shadows.

**Cathode Rays.**—We will now investigate the properties of the rays emanating from the cathode. They leave the cathode surface normally and obviously travel in straight lines. We will at once introduce the name *cathode rays* to describe them, as we shall find that they are identical with the rays dealt with on p. 323.

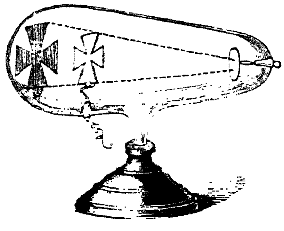

Fig. 42.—Crookes tube

*Excitation of Luminosity.*—A screen of some suitable substance (such as zinc silicate containing a trace of manganese) glows very brightly when placed in the path of the rays.

*Rectilinear Propagation.*—An object placed in the path of the rays throws a shadow of corresponding shape. See, for example, fig. 42, which shows the shadow cast by a metal cross upon the end of the tube.*

*Emission Normal to the Cathode Surface.*—In the experiment of fig. 42 we observe a characteristic difference from the case of shadow formation by ordinary light. If the cathode disc were emitting light, the shadow of the cross would be surrounded by a region of half shadow (penumbra, see Vol. IV). This effect is not observed in the case of cathode rays. It follows that the rays are not emitted in all directions from every point of the cathode surface, but only in one direction, namely normally. This can be seen even more clearly from the following experiment.

The cathode surface is made spherical and concave towards the anode (see fig. 43). At the centre of curvature of the cathode surface is placed a piece of platinum foil. During the discharge the platinum foil is seen to become incandescent. Thus the cathode rays produce a considerable heating effect.

*Cathode Ray Furnace.*—The heating effect of cathode rays is employed for the production of very high temperatures. Since the substance to be heated is in a vacuum, it is possible to use crucibles of materials which are permeable to air and flame gases at high temperatures. The working conditions are therefore much more satisfactory than with other types of furnace.

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* Hittorf called attention as early as 1869 to "sharp shadows" cast by cathode rays, and recognized that the rays, which were unable to follow the curves of his tubes, are propagated in straight lines.

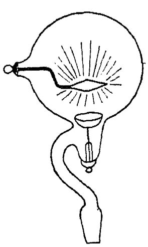

Fig. 43.—Discharge tube with concave cathode.

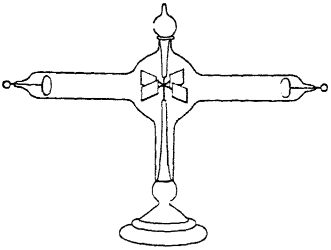

Fig. 44.—Discharge tube with small paddle wheel

*Mechanical Effects of Cathode Rays.*—A small, lightly mounted paddle-wheel placed in the path of cathode rays (as in fig. 44) is made to rotate.

*Negative Electric Charge.*—Considerable time elapsed before it could be proved satisfactorily that cathode rays carry a negative charge. The proof was at last obtained (Perrin, 1895) by allowing the rays to fall into a Faraday cage (fig. 45). When a simple disc was employed, the charge collected was variable—a phenomenon which is understandable now that the behaviour of the rays has been more fully investigated (p. 336). It was also a relatively long time before the rays could be deflected by an electrostatic field (J. J. Thomson, 1895). The earlier attempts failed owing to the fact that it had not previously been possible to produce a sufficiently high vacuum with certainty, nor to maintain a sufficiently high field at right angles to the rays. (For the work of Sir J. J. Thomson, see Vol. V).

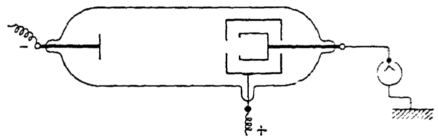

Fig. 45.—Determination of the negative charge of cathode rays

The most important advance in the investigation of the nature of cathode rays was furnished by the experiments of P. Lenard, who was the first to attain satisfactory experimental conditions. When an ordinary discharge tube is used, any alteration of gas pressure, E.M.F. or current strength has a simultaneous effect upon the process of production of the cathode rays. Now H. Hertz had observed that the rays are capable of passing through thin foils. By providing a highly evacuated tube with a window of aluminium foil, $\frac{1}{1000}$ mm. thick and free from holes (**Lenard window**), Lenard succeeded in making the rays pass right out of the apparatus (fig. 46). They could then be investigated under conditions independent of those governing the process of their production. The far-reaching consequences of these classical researches cannot be discussed until Vol. V. Fig. 47 shows an actual discharge tube with Lenard window for very high E.M.F.s.

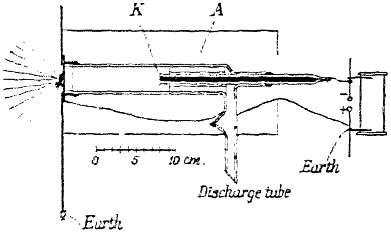

Fig. 46.—Discharge tube with Lenard window. A, Anode; K, cathode; around the tube is an opaque screen; the window is on the left.

[After Lenard, *Über Kathodenstrahlen*, published by J. A. Barth, Leipzig.]

The Faraday cage and electrostatic deflection experiments indicate that the cathode rays consist of *a stream of negatively charged particles*. From the magnitudes of the electrical and magnetic deflections it must therefore be possible (p. 317) to determine the specific charge $e/m$ of the cathode particles.

*Electrical and Magnetic Deflection of Cathode Rays produced by a Discharge through a Rarefied Gas.*—When a strong bar magnet is brought up to a Crookes tube of the kind shown in fig. 42, p. 330, the deflection of the cathode rays can be observed by the displacement of the shadow. Hittorf discovered (1869) that the deflection is the same as would be produced in a flexible conductor attached to the cathode but otherwise free to move, and carrying a negative current in the direction from cathode towards anode.

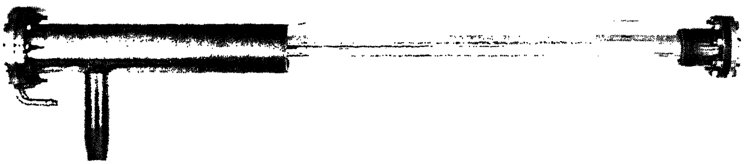

Fig. 47.—Lenard tube for 180,000 volts

[C. H. F. Müller A.-G., Hamburg.]

For quantitative determinations a tube of the form shown in fig. 48 is employed.* This is a Braun tube, similar to that shown in fig.

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* For the investigation of the magnetic deflection the poles of an electromagnet are placed, one in front of the plane of the figure and the other behind it, in a position corresponding to that of the plates of the condenser (see figure) used for the electrostatic deflection.

22, p. 315, but containing gas at about 0·001 mm. pressure and having a simple cathode disc instead of an incandescent filament.

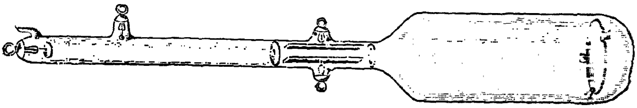

Fig. 48.—Braun tube with gas filling

The deflection of the beam of cathode rays can be observed and measured by the motion of the spot of light upon the screen on the right of the figure.

In order to obtain satisfactory measurements of electrostatic and magnetic deflections, it is necessary to take precautions to prevent disturbing electric charges, derived from the high E.M.F. leads to the discharge tube, from collecting upon the outer surface of the walls. For this purpose the tube is provided with a coating of tin-foil, which is connected to earth.

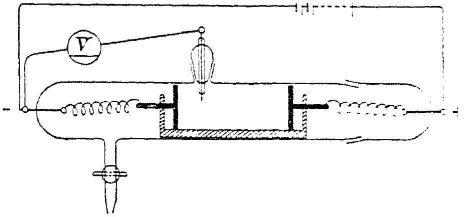

Fig. 49.—Measurement of the field distribution in a discharge tube

Determinations of $e/m$ by various methods give the numerical value $1{\cdot}77 \,.\, 10^8$ coulombs per gramme. The particles constituting the rays are thus *electrons* and the rays are identical with those investigated on p. 321.

*Cathode rays from discharges through gases consist of moving electrons.*

Two facts stand out, however, in contrast to the experiments described on p. 319. Firstly, the cathode rays in the present experiments are always emitted normally from the cathode surface (compare, for example, the experiment with concave cathode, p. 331). Secondly, their path is independent of the position of the anode. In order to understand these peculiarities, we must consider the nature of the field during the discharge.

**The Variation of Field Strength along a Discharge Tube.**—The field strength at different points within a discharge tube can be determined by means of corresponding thin test wires sealed through the walls. Alternatively one single test wire may be employed, and the anode and cathode shifted bodily along the tube (see fig. 49) so as to bring different parts of the discharge successively opposite it.

The variations of potential, field strength and space charge along a tube during a spontaneous discharge are shown diagrammatically in fig. 50. We see at once that the greatest fraction of the potential fall occurs near the cathode. The difference of potential between the cathode and the negative glow is known as the **cathode fall.**

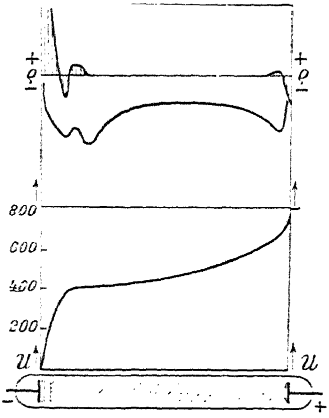

Fig. 50.—Diagram of the distribution of potential U, field strength, and space charge ρ in a normal discharge.

The smallest fall of potential is in the Faraday dark space and in the positive column. Then in the neighbourhood of the anode there is again a more rapid fall (the anode fall), which only amounts in general, however, to a fraction of the cathode fall.

So long as the cathode is not completely covered by the negative glow, the cathode fall is independent of the current strength and the pressure of the gas in the tube (HITTORF, 1883). It is then known as the *normal cathode fall.* Its magnitude depends upon the nature of the gas and of the electrodes. In the case of air at low pressure and electrodes of platinum the normal cathode fall is approximately 400 volts. For helium and electrodes of potassium the value is especially low, namely about 70 volts.

In general the cathode fall is smaller for the more electropositive gases (see p. 283) and for cathode materials which give off electrons more easily.

The cathode fall may be diminished and in certain cases reduced to zero by the introduction of electrons at the cathode, e.g. by using a glowing cathode (WEHNELT, p. 319). It is then possible to maintain a discharge through the tube with a potential difference of a few volts, whereas in the ordinary way the applied E.M.F. must be higher than the normal cathode fall. The reason for this diminution of the cathode fall is as follows. In an ordinary condenser with plates corresponding to the electrodes of the tube shown in fig. 50 the field would be homogeneous, i.e. the field strength E would be constant. If, as in actual fact, E decreases in the neighbourhood of the cathode, this means that the density of lines of force must decrease as we pass farther out from the cathode. Now the lines of force run in the main normal to the cathode, as is proved by the direction of the cathode rays. Hence their density can only fall off if a certain number of them come to an end in the space in front of the cathode, i.e. there must be *a positive space charge* in this region. The same considerations also apply to the anode fall, which must be due to a negative space charge (see fig. 51). These space charges (p. 338) are thus the causes of the observed abnormalities of the potential fall. If the positive space charge in front of the cathode be neutralized by the introduction of negative electrons into this region from an incandescent cathode, the cathode fall also disappears.

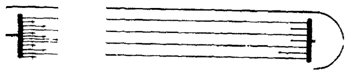

Fig. 51.—Diagrammatic representation of the lines of force at a certain instant during a normal discharge

It is remarkable that the negative glow, which (as mentioned above) at first only covers a small portion of the cathode, spreads out more and more over the cathode as the current strength is increased. If the cathode is a wire, the glow covers it like a luminous tube, the length of which is found to be proportional to the current strength. An arrangement of this kind can therefore be used as a current measurer.

Practical use is made of this in the *negative glow oscillograph* of GEHRCKE.* This consists of a discharge tube with two long electrodes of nickel wire. When a direct current of variable strength is passed through the tube, the blue negative glow covers the cathode wire to a length proportional to the instantaneous current strength. The variations in the length of the glow occur without any time lag. In the case of an alternating current the glow covers first one electrode and then the other. The nature of the current alternations can be conveniently investigated by viewing the glow in a rotating mirror.

As soon as the cathode is completely covered by the glow, the cathode fall increases (*abnormal cathode fall*) and ceases to be independent of the gas pressure and current strength. So long as the cathode fall is normal, the length of the Hittorf or Crookes dark space remains constant; but as soon as the cathode fall is increased above its normal value by reduction of gas pressure or increase of current strength, this dark space begins to extend out into the tube. The abnormal cathode fall corresponds to an increased field strength, so that at very low gas pressures potential differences of many thousands of volts are required to maintain the discharge.

We can now understand the remarkable behaviour of the cathode rays previously mentioned on p. 330. Almost the whole of the potential fall occurs near to the cathode. Hence the electrons emitted from this electrode attain practically their full velocity in its immediate neighbourhood. The relatively small potential fall in the remaining space between the electrodes is then insufficient to affect the rapidly moving particles to a noticeable extent. Hence they continue by virtue of their inertia to fly in their initial direction, namely the direction of the lines of force near the cathode, i.e. normal to the cathode surface. Thus the position of the anode is of no importance.

**The Mechanism of the Discharge.**—In order to arrive at an understanding of the processes underlying the spontaneous discharge in gases, we will consider what happens when a potential difference of increasing magnitude is applied to a condenser.

We have seen above that air is an insulator: hence at low E.M.F.s there is practically no conduction. The current is not strictly zero, however, for the air always contains carriers (though only in extremely small quantities) from ever-present sources such as radioactive radiations, cosmic rays, &c.

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* E. GEHRCKE, Professor at the German National Physical-technical Laboratory.

On the average there are only a few of these carriers per cubic centimetre. Under the influence of the electric field they migrate and produce a current whose magnitude may be measured by a very sensitive instrument. This current soon reaches a saturation value, however, after which it remains constant for a time as the E.M.F. is increased (fig. 52). But though the current strength remains constant, the velocity of the carriers increases in proportion to the field strength. As the E.M.F. between the electrodes is still further increased, a sudden rapid rise of current strength is eventually observed, the value rising to a million or more times that of the saturation current. At the same time the characteristic phenomena of the discharge make their appearance. There can be no doubt that this point is the beginning of a new process in which carriers are produced. The cause of this new production of carriers can only lie in the increased velocity of those carriers which were present originally. A large number of other facts to be discussed later also point to the following conclusion:

*Very rapidly moving carriers are capable of knocking electrons out of the atoms and molecules with which they collide, thus giving rise to new carriers.*

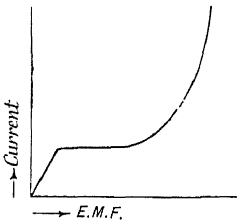

Fig. 52.—Current E.M.F. curve for a normal discharge through a gas.

This process is called **ionization by collision.** It will be treated in greater detail in Vol. V.

This phenomenon can be demonstrated directly by making use of the fact (p. 310) that water vapour condenses most readily upon the molecular complexes of the carriers.

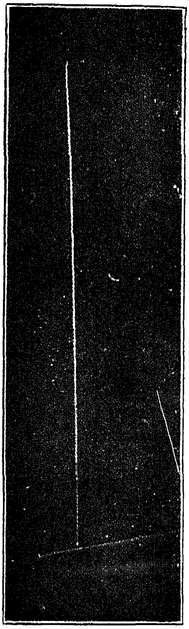

Fig. 53.—Production of carriers along the path of a rapidly moving He$^{++}$-particle (α-ray).

When an electrically charged particle is allowed to enter with very great velocity into a space which is supersaturated with water vapour, a streak of cloud is produced along the track of the particle. Fig. 53 shows the track of a very rapidly moving carrier, namely a helium atom with two elementary positive charges (an α-particle). The formation of the cloud streak is due to the production of carriers along the whole track and the condensation of the water vapour upon them (compare the experiment of p. 310). Rapidly moving electrons also show this behaviour. Fig. 54 shows ionization produced by an X-ray beam about 5 mm. in diameter. The X-rays traversed the air before its expansion in the Wilson chamber. The positive and negative ions have been separated by the electric field before losing their mobility by condensation of water on them. Fig. 55 represents α-ray tracks obtained in air in a magnetic field of 39,400 gauss, with an air expansion of 1·4.

Photographs of this kind demonstrate very clearly the phenomenon of ionization by collision.

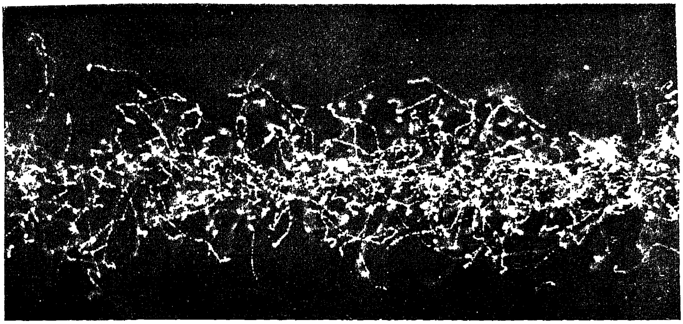

Fig. 54.—Ionization by X-ray beam
[C. T. R. Wilson.]

But the principal value of this elegant method, from whose application we shall derive a number of important results (see, in particular, Vol. V), lies in the fact that it registers the path traced out by a rapidly moving carrier in passing through a gas.

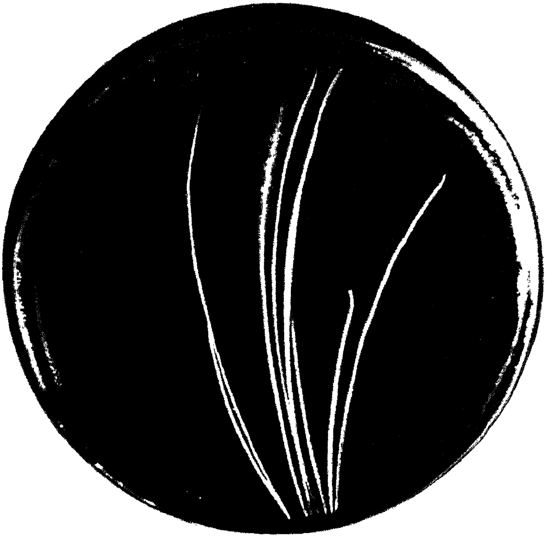

Fig. 55.—Tracks of $\alpha$-rays in high magnetic field
[P. L. Kapitza.]

Ionization by collision occurs not only in spaces containing a gas, but also by the impact of rapidly moving carriers upon solid surfaces.

This is the reason, for example, why a Faraday cage must be employed to collect the cathode rays (p. 331) in the determination of their charge. For when the rays impinge upon the surface of a simple plate, they knock electrons out of the atoms (secondary electrons, Vol. V), which are thus left with a positive charge. Since the secondary electrons are often more numerous than the primary, the observed anomalous reversals of the charge on the collector are at once understandable.

*The mechanism of the processes leading up to the discharge* are therefore to be pictured as follows.

The initial state of the tube is represented diagrammatically in fig. 56 I. Amongst the neutral gas molecules are a few positive and negative carriers (actually in the proportion of about 1 : $10^{17}$).

When the field is applied (fig. 56 II) the carriers begin to move. If the field strength is sufficiently great, their velocity may become so large that they will knock electrons out of the cathode. These electrons, being of small mass, attain a great velocity in the field (p. 323), and are consequently able to produce new carriers (see fig. 56 III) all along their paths. As is seen from fig. 54, p. 337, each rapidly moving electron can give rise to a large number of carriers (for further details see Vol. V). These newly produced carriers are in turn set in motion by the field, so that the whole process is maintained on an ever increasing scale. Thus the number of carriers increases in the manner of an avalanche and the current strength may attain an extremely high value. Further large numbers of carriers are also produced, as in the experiment of p. 312, by the photoelectric effect of the light emitted by the discharge. The principal process acting in opposition to the increase in the number of carriers is their recombination (p. 309). The mechanism outlined above also provides an explanation of the appearance of space charges in the neighbourhood of the electrodes.

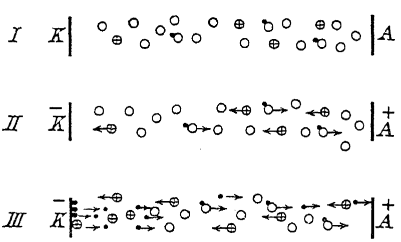

Fig. 56.—Production of a discharge in a tube containing gas (diagrammatic). ● electron, ○ atom, ⊕ positive ion, •○ negative ion.

The interaction between electrons and material atoms and molecules will be treated in greater detail in Vol. V, when the luminous phenomena associated with the discharge will become more readily understandable. For the present it suffices to notice that the luminosity of the first cathode layer is obviously produced by the impacts of the positive carriers upon this electrode, and that it is in the cathode fall of the Hittorf or Crookes dark space that the electrons attain the high velocity which makes them able to produce the new carriers necessary to maintain the discharge.

*Thus the cathode fall is the most important part of the spontaneous or normal discharge through a rarefied gas.*

The importance of the cathode fall may be demonstrated by the following experiment, originally performed by Hittorf. The apparatus is shown in fig. 57. The two electrodes of fairly stout aluminium wire pass through two comparatively large glass bulbs into the short connecting tube, where their ends are only 1 to 2 mm. apart. The two bulbs are also joined by a relatively long detour tube. At sufficiently low air pressure the discharge does not pass directly between the ends of the electrodes but through the long alternative tube. Between the ends of the electrodes the space is too small and the molecules capable of being ionized by collision are too few to yield enough carriers to start and maintain the discharge. Very high E.M.F.s must be used to produce a discharge when the normal cathode processes cannot occur. This can be easily demonstrated by progressively reducing the gas pressure within a small tube in which there is little space between the electrodes. As the pressure decreases the negative glow goes on extending until at last there is not enough space for it. From this moment onwards there is an extremely rapid rise of the E.M.F. (normally only a little more than the cathode fall) necessary to maintain the discharge through the tube. That the disappearance of the normal discharge is due to the lack of space is shown very elegantly by the following experiment. A Hittorf tube of the kind shown in fig. 57 is exhausted until the discharge just stops between the electrodes and begins to pass through the long alternative tube. When now a strong bar magnet is brought up near the electrodes, the discharge once more passes directly between them. The reason for this lies in the increase in the length of the paths of the carriers owing to the curvature imparted to them in the magnetic field. Were it not for the fact that the electrodes must be a certain distance apart before a discharge can pass, it would be impossible to measure the electrostatic deflection of particles within a discharge tube by the method shown in fig. 48, p. 333. The plates between which the electrostatic field is applied must be placed so close together that no discharge can take place between them at the E.M.F. employed.

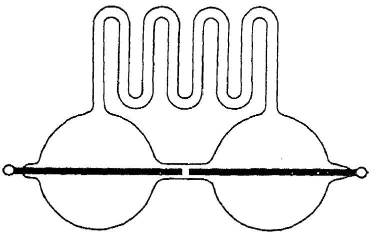
Fig. 57.—Hittorf's experiment on cathode fall

A small object placed just in front of the cathode throws a distinct shadow in the negative glow. This proves that the negative glow is produced by the cathode rays.

**Applications of the Normal Discharge through Rarefied Gases.** One application of the negative glow has already been mentioned on p. 335, namely the *negative glow oscillograph* of Gehrcke. A second example is the *neon discharge lamp*, which is used for illumination.

No solution has yet been found of the problem of constructing lamps of very small candle-power (economy lamps of say one candle-power) acting on the same principle as ordinary incandescent lamps and suitable for use at the normal pressures of 110 or 220 volts. The smallest commercially practicable candle-power for 220 volts is about 15. Neon discharge lamps (fig. 58) represent a solution of the problem by another method. Since the normal cathode fall for air is about 300 to 400 volts, it is necessary to employ some other gas and electrodes of such a material as to reduce the cathode fall as much as possible. The gas usually chosen is neon and the electrodes are usually made of iron coated with metallic barium. The lamps can then be used at the normal pressures of 110 or 220 volts. In the lamps for use with direct current the cathode is made as large as possible, whereas the anode is usually only a short wire. In the lamps for use with alternating current both electrodes are of the same size. An auxiliary resistance is required and is usually incorporated in the base of the lamp itself. The maximum brightness is about 0·3 candle-power and the consumption about 15 watts per candle-power. The colour of the light varies from pink to orange according to the nature of the gas filling. The normal running pressure of the 220-volt lamp is 140 to 150 volts. The initial pressure required to light the lamp is, however, somewhat higher than this, viz. 160 to 170 volts. On the other hand the pressure must drop some 10 volts below the normal running figure before the lamp goes out.

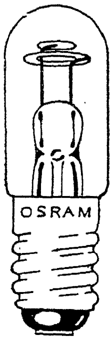

Fig. 58.—Neon discharge lamp

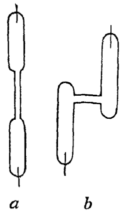

Fig. 59 *a* and *b*. — Geissler tubes for the investigation of spectra.

In addition to its use for advertising purposes, the positive column is used as a *source of light in spectrum analysis* (see Vol. IV). Discharge tubes of the shapes shown in fig. 59*a* are usually employed. The intensity of the light is greatest in the capillary part of the tube. A still greater intensity can be obtained with a tube of the kind shown in fig. 59*b*, the capillary part of which is viewed “ end-on ”.

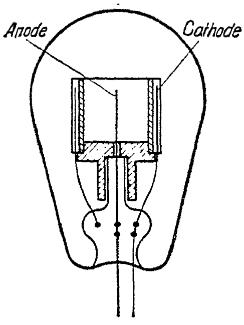

Fig. 60.—Negative glow rectifier for E.M.F.s up to about 500 volts.

[Osram Co., Ltd., Berlin.]

*Application for Rectifying Alternating Currents.*—The difference between the normal and the abnormal cathode fall is widely used nowadays in rectifiers, i.e. devices which only allow current to pass through them in one direction.

The tube (see figs. 60 and 61) has one large and one small electrode, the latter being surrounded to a great extent by glass in the type shown in fig. 61. When the large electrode is cathode and the small one anode, the resistance of the tube is small; but in the reverse case it is very great, owing to the high cathode fall. Hence the current can pass practically only in the one direction. The electrodes may also be made of different materials with different cathode falls.

**Positive or Canal Rays.**—If a disc with holes (“ canals ”) in it be used as *cathode* in an ordinary glow discharge, a luminosity is observed on the opposite side from the anode. It appears (fig. 62) to belong to rays passing through the holes in the cathode. In hydrogen the streaks of light are pink, in nitrogen reddish yellow. They were discovered by

E. Goldstein,* who gave them the name *canal rays*. On account of their electric charge (see below) they are also frequently referred to as *positive rays*.

*Properties*.—Like cathode rays, the positive or canal rays can render certain substances luminous. In the case of glass, however, this effect is very feeble. Another common property of both kinds of rays is that of affecting a photographic plate. Positive rays also appear to possess the peculiar property of disintegrating metals upon which they fall ("sputtering"). With gold this effect is particularly marked; with iron and aluminium, on the other hand, it is relatively slight.

*Cathode Sputtering*.—Use is made of the sputtering of the cathode metal for coating articles with a thin layer of the metal, e.g. for the silvering, platinizing, and gilding of mirrors and the like. The circular, well cleaned cathode is set up at a distance of 1 to 5 cm. from the surface to be coated. The sputtering is carried out in $H_2$, $N_2$, or still better in A at a pressure of a few tenths of a millimetre of mercury. The pressure is regulated so that the dark space occupies from $\frac{1}{2}$ to $\frac{2}{3}$ of the distance between the electrodes, but does not extend up to the cathode. E.M.F.s of 1000 to 3000 volts are employed and the current strength is made as high as possible.

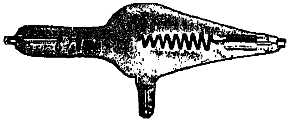

Fig. 61.—Rectifier for high E.M.F.s

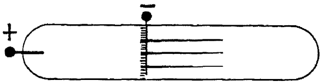

Fig. 62.—Positive or canal rays

*The region of origin* of the positive rays lies chiefly in the negative glow. This is shown by the following experiment.

A small object, say a rod, is placed in the Hittorf or Crookes dark space. It is seen to cast a shadow not only in the negative glow but also in the cathode surface glow. If the shadow falls upon some of the holes in the cathode, no positive rays emanate from these holes behind the electrode.

This experiment makes it probable that the canal rays consist of positively charged particles which have attained high velocities in passing through the cathode fall in the opposite direction from the electrons, and whose inertia then causes them to fly in straight lines through the holes in the cathode. This is also in harmony with the fact that the light from the cathode surface glow has the same composition (Vol. IV) as that from the canal rays. The cathode surface glow is connected, through the holes in the electrode, with the luminosity of the positive rays on the other side, so that the latter appear

* Eugen Goldstein, born in 1850 at Gleiwitz, physicist at the Potsdam Observatory from 1888, died in 1931. Performed many valuable experiments upon discharges through gases. The name "cathode rays" is also due to him (1876).

as a mere extension of the former. From this we conclude that the light must be produced in both cases by the same process. Thus positive rays are present in *every* discharge tube immediately in front of the metallic surface of the cathode. The normal disintegration of the cathode is due to them.

Strict proof that the rays consist of positively charged particles is obtained by collecting them in a Faraday cage.

*The Nature of Positive or Canal Rays.*—In order to determine the charge, mass, and velocity of the particles of which the rays are composed, it is necessary (as in the case of the cathode rays, p. 332) to measure their deviation in an electric and a magnetic field. These experiments, which have been performed very thoroughly by W. Wien,* Sir J. J. Thomson, and others, are attended by much greater difficulties than the corresponding experiments with cathode rays.

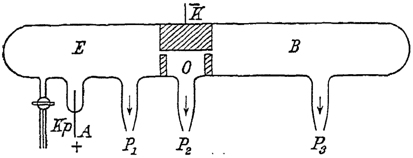

Fig. 63.—Diagram of apparatus for the investigation of positive rays in a high vacuum

Thus, for example, the deflection by a magnetic field is so small that the rays were originally thought to be incapable of deflection by this means. But if strong enough magnetic fields be employed, the effect can easily be observed (W. Wien, 1900). Further, the results obtained in the deflection experiments are indefinite, in that the luminous spot produced by the rays is not merely displaced when the field is switched on (as is the case with cathode rays), but is spread out into a band. This may be due to the particles having very different charges, very different masses or very different velocities. The difficulty of deciding between these possibilities arises from the fact that, in order to obtain the rays with sufficient intensity, it is necessary to employ considerable gas pressures in the discharge tube. This, however, disturbs the experimental conditions. The nature of the positive rays could not be satisfactorily determined until it became possible, as in the case of the cathode rays by means of the Lenard window (p. 331), to separate the region of production of the rays from the region in which the observation is made. This was done by employing long "canals", which in combination with the improved pumping technique made it possible to maintain a very high vacuum (under $10^{-4}$ mm. Hg) in the space behind the cathode, even though the pressure was about 0·01 mm. Hg on the other side of the electrode.

Fig. 63 shows the construction of such a tube. The cathode K consists of a block of metal several centimetres thick, through which is bored an extremely narrow "canal". It is also provided with an outlet O connecting the "canal" with a high-vacuum pump $P_2$. The discharge region E and the observation region B are completely separated by the cathode, except for the narrow "canal". The space B is connected to a high-vacuum pump $P_3$, by means of which the pressure is maintained at a very low value, which can be still further reduced by absorption vessels cooled with liquid air. Thus the positive ray particles passing through the "canal" enter a space in which they are practically free from disturbances and absorption due to collisions with gas molecules. Only in this way can the electrostatic and magnetic deviations be measured with sufficient precision. Since gas is being sucked continuously out of the discharge space E, the pressure must be kept up by a corresponding supply through an adjustable capillary K*p*.

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* Wilhelm Wien (1864–1928), professor at Würzburg and (from 1920 onwards) at Munich. He received the Nobel Prize in 1911. Well known also for his work upon hydrodynamics and heat radiation (see Vol. V).

*The deflection of positive rays by electric fields* is in the opposite sense from that of cathode rays. For the same applied E.M.F., however, the deflection is of the same order of magnitude in both cases. This means that both types of rays have been accelerated by the same potential fall U, a result which is in harmony with what has been said above (p. 334). For according to p. 315 the deflection is proportional to $(e/m)/v^2$, and $v^2$ is itself proportional to $(e/m)\mathrm{U}$ (p. 323); hence the deflection is dependent only upon $\mathrm{U}.(e/m) \div (e/m) = \mathrm{U}$, i.e. only upon the potential fall, being independent of the value of $(e/m)$.

The amount by which positive rays are deflected is not the same at all points of their path. With the usual arrangement (fig. 62, p. 341), in which the gas pressure is the same through the whole tube, the deflection of the rays is greater at great distances behind the cathode. The particles must therefore have lost part of their velocity and kinetic energy. This is explained as due to encounters with gas molecules, in which the positive ray particles give up a part of their kinetic energy in causing ionization by collision (p. 336). Another result of these encounters is that the positive ray particles are scattered and deviated from their original direction; hence the indistinctness of their track.

Deflection experiments have also shown that positive rays contain a considerable number of electrically neutral particles (i.e. such as are incapable of being deflected), and even particles with negative charges. This must be ascribed to interaction with electrons knocked out of gas molecules by collisional ionization. These electrons may become attached to the initially positive particles, thereby rendering them neutral or even negative. Moreover the positive ray particles may themselves lose electrons in further collisions. These changes of charge within the rays make their precise investigation extremely difficult.

In general, therefore, positive or canal rays contain particles with different charges and also with different velocities and kinetic energies. The velocities of a large proportion of the particles are smaller than that corresponding to the potential U (see above).

According to the formula of p. 317, the radius of curvature of the rays produced by deflection in a magnetic field is proportional to $v/(e/m)$, i.e. to $\sqrt{2\mathrm{U}/(e/m)}$, since $v = \sqrt{2\mathrm{U}(e/m)}$. As mentioned above, positive rays are deflected to a very much smaller extent than cathode rays in a magnetic field. Hence, assuming the charge $e$ of the positive ray particles to be the same as that of the cathode ray particles, the mass $m$ of the former must be very much greater than that of the latter. The magnetic deflection of positive rays, as eventually obtained by W. WIEN in 1900, supports this conclusion.

Magnetic deflection confirms the result obtained above by electrostatic deflection, namely that positive rays are very unhomogeneous, i.e. that they generally contain particles with very different velocities and also particles which are electrically neutral or have had their charges reversed to negative.

Combination of electrostatic and magnetic deflection experiments permits of the determination of the *specific charge* ($e/m$) of the moving particles and also of their *velocity*, without involving a knowledge of the E.M.F. U (see § 2, p. 318 *et seq.*). The following important result was obtained:

*The masses of positive ray particles are never smaller than those of chemical atoms and molecules.*

Even when care is taken to ensure that all the positive ray particles have the same volt-velocity (p. 324), a magnetic field sorts out different discrete fractions with different deflections (under the same conditions there would be no such resolution in an electric field). This can only mean that positive rays always contain particles with different values of $e/m$, and (making allowance for the different charges) with different masses.

*There is no single kind of carrier characteristic of positive rays.*

The positive ray particles have different masses according to the nature of the gas in the discharge tube: these masses are always those of atoms or molecules capable of existence in this gas. The particles carry either one or a relatively small number of elementary charges.

*Positive ray particles are therefore singly or multiply charged atoms or molecules.*

Although determination of the charge $e$ independently of the velocity $v$ may show the particles to carry several elementary charges, it is found that the velocity $v$ (as determined by combined electrostatic and magnetic deflection) can only be identified with the volt-velocity U (see p. 324) by assuming that the majority of the particles have just *one positive elementary charge.* From this it follows that the particles must have been singly charged at the time when their velocity was imparted to them, i.e. as they were passing through the cathode fall towards the cathode. The changes of charge by loss or gain of electrons must, therefore, have occurred later.

The *velocity* of the positive rays can also be determined (J. STARK) by a very elegant optical method, namely from the Doppler effect (Vol. II, p. 280) of the emitted light waves. The results of these investigations (Vol. V) agree with those obtained by electrostatic and magnetic deflection. They show further that a certain fraction of the light from the positive rays is sent out from particles at rest, these being excited to emit light by the impacts of the rapidly moving positive ray particles.

*Note.*—The methods outlined above for the determination of the mass of a positive ray particle involve a fundamental ambiguity (p. 321). Deflection experiments only give the *ratio* $e/m$ of charge to mass. Suppose now that in a certain case this ratio is found to be half as great as for hydrogen (p. 321). This may mean either that the particle carries a single elementary charge $e$ and has a mass twice as great as that of a hydrogen atom, or that it carries a charge of $2e$ and a mass four times as great as that of a hydrogen atom. In the first case the particle would have to be identified as a hydrogen molecule $H_2$ which had lost an electron and thus become charged with one elementary positive charge; in the second case it would have to be identified as a helium atom (atomic weight 4) with two elementary positive charges.

Methods have been found, however, of removing this ambiguity, either by very refined measurements (Aston, Vol. V) or by direct counting and determination of charge, i.e. by measurement of $e$ (see, for example, Regener, Vol. V).

**Electrical Determinations of Atomic and Molecular Weights.**—The determination of the mass of a positive ray particle by deflection experiments provides a new method of determining atomic and molecular weights, in which the individual atoms and molecules are sorted out according to their mass and, as it were, separately weighed. This method has led to new results of chemical interest, namely to the discovery (according to the gas in the tube) of particles which had to be identified as $H^+$, $H_2^+$, $H_3^+$, $H_3^{++}$, $He^{++}$, $He^+$, $C^{++}$, $N^{++}$, $O^+$, $O_3^+$, $O_6^+$, $C_3^+$, $C_6^+$, $Hg^+$ up to Hg with 8 elementary positive charges, $Hg_4^+$, $(CN)^+$, $(C_6H_6)^+$, $H^-$, $O^-$, &c.

Such molecules or ions, which were not previously known in chemistry, are obviously capable of existence under the conditions obtaining in the discharge tube and for the short duration of the observation. In the hands of Aston the determination of atomic weights by the positive ray method has recently reached a degree of accuracy far surpassing that of chemical determinations, and has led to a remarkable extension of our knowledge (see Vol. V, mass spectrograph).

**The Smallest Particle with a Positive Charge.**—In spite of all attempts no positive ray particle has ever been found with a mass smaller than that of a hydrogen atom. This has led to the conclusion that:

*The smallest masses which can be associated with positive elementary charges are hydrogen ions.*

**"Positive electrons", i.e. elementary positive charges unassociated with matter have never yet been observed.** *

Hence although a body may be given a negative charge of any magnitude by the addition of electrons, it is impossible to charge it positively beyond a certain point; for there is no such thing as pure positive electricity. The body can only be charged positively by the removal of electrons from it. Now it is obviously impossible to remove from the body more electrons than it contains. The extent to which the body can be positively charged is therefore limited. Thus, for example, it is clear that only one electron can be removed from a hydrogen atom. The resulting particle, the $H^+$ ion, contains (as far as we know) no more negative electricity. Since the $H^+$ ion is the smallest material mass which is ever associated with an elementary positive charge, it has been given the special name **proton.** As will be shown in Vol. V, the proton (like the electron) appears to play a fundamental part in the structure of matter. The data for the proton are: $m = 1{\cdot}66 \,.\, 10^{-24}$ gm.; $e = +1{\cdot}59 \,.\, 10^{-19}$ coulomb; $e/m = 95{,}800$ coulombs/gm.

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* Recent (1933) work, however, at Cambridge makes it practically certain that the positive electron has been discovered.

**Anode Rays.**—In 1906 GEHRCKE and REICHENHEIM discovered that positively charged rays are emitted directly from the anode of a discharge tube, when this electrode consists of a glass tube open at one end and filled with a fused salt. The iodides and bromides of lithium, sodium and potassium are particularly suitable. In order that they may conduct better, they are mixed with powdered graphite before being fused. When the current has been passing long enough for the anode to have become sufficiently hot, intensely luminous rays are emitted from this electrode, giving the appearance shown in fig. 64.

The rays can be deflected both by an electrostatic and by a magnetic field. Hence they are not mere streams of vapour, but real discharge rays like the canal rays. Determinations of the mass of the particles by deflection (see above) gave in each case the mass of the atoms of the corresponding metal. The alkali metal atoms were found to carry one elementary positive charge; strontium gave ions with two elementary positive charges.

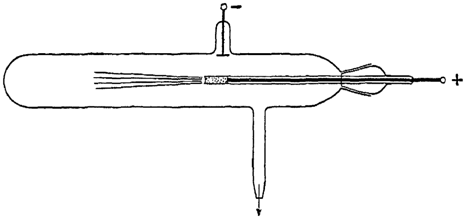

Fig. 64.—Anode rays

*The anode rays therefore consist of a stream of positive metal ions flying away from the anode.*

It is probable that the anode rays are not derived directly from the salt itself, but from the layer of metal vapour at the surface of the hot anode. The production of the rays is due to the fact that the use of a salt as anode and more especially the presence of a small quantity of iodine in the gas cause the anode fall to assume an abnormal magnitude. The positive ions are therefore accelerated away from the anode in the same way as the electrons are accelerated away from the cathode by the cathode fall. Thus the anode rays are the positive analogue of cathode rays.

As far as is known, the alkali metals and alkaline earth metals do not form gaseous chemical compounds. They cannot therefore be investigated by the positive-ray method. Hence anode rays provide for experimental purposes a valuable supplement to ordinary positive rays.

**Röntgen Rays or X-rays.**—Using a cathode ray tube of the Lenard type, RÖNTGEN * observed that new rays are emitted from the parts of the tube which are struck by the cathode rays. These new rays had the power of rendering luminous certain substances † placed in the neighbourhood of the tube; they also affected a photographic plate, even though this was wrapped in black paper. Whereas cathode rays are completely absorbed by quite thin layers of matter, the new rays showed a great power of penetration. The fact that they could not be deflected by magnetic or electrostatic fields proved that they could not consist of moving charged particles. Finally, their property of showing interference phenomena (Vol. II, p. 211), such as are characteristic of all wave motions, provided proof of their undulatory character.

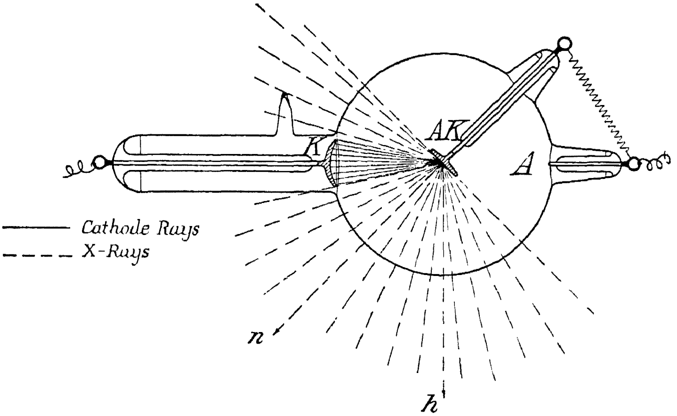

Fig. 65.—Gas-filled X-ray tube (old form). K, cathode; AK, anti-cathode or target; A, anode (no longer employed); *n*, normal to the surface of the target; *h*, direction of observation

**Röntgen rays or X-rays consist of waves analogous to those of light** (p. 646).

Their remarkable properties will be discussed more fully in Vol. V. It suffices here to give a brief description of the methods of production of the rays. They are emitted (like light) in all directions from the point of impact of cathode rays upon matter. In practice cathode rays are concentrated as far as possible upon a piece of metal, the so-called **anti-cathode or target,** which thus becomes the source of Röntgen rays. Fig. 65 shows a form of X-ray tube in which the ex-

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* WILHELM KONRAD RÖNTGEN (1845–1923), then Professor of Physics at Würzburg, later at the University of Munich, retired from office in 1920. He received the first Nobel Prize for physics in 1901.

† Nowadays zinc silicate containing traces of manganese is generally used; even better, but not so durable, is barium platinocyanide.

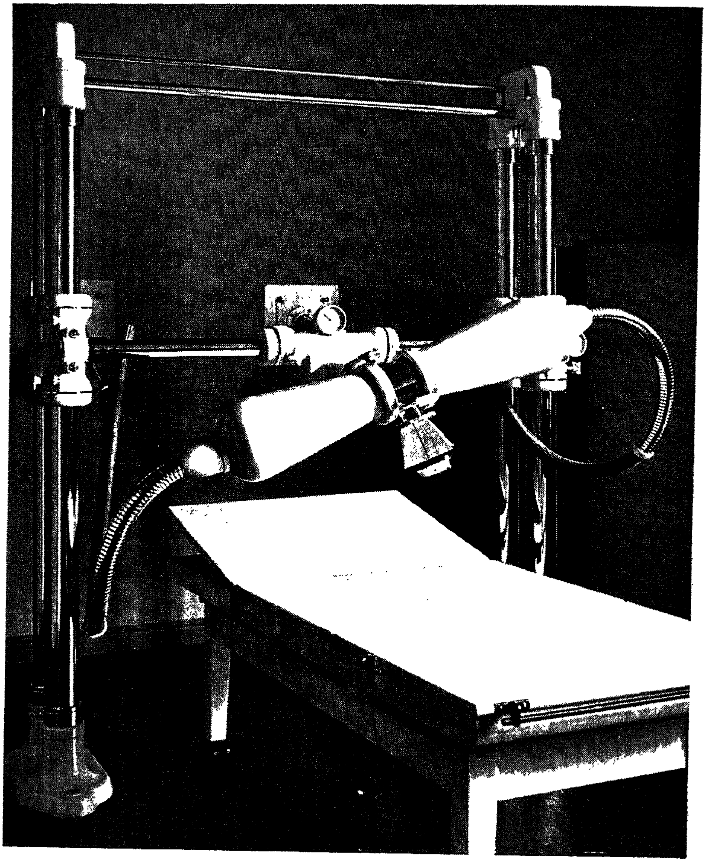

Fig. 66*a*.—Modern X-ray tube, for use up to 200,000 volts
[Philips " Metalix " deep therapy tube, by Philips Industrial, London.]

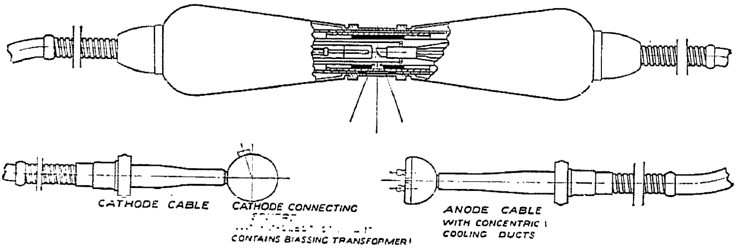

Fig. 66*b*.—Section of " Metalix " tube

citing cathode rays are produced by a discharge through a gas. The tube shown in fig. 66*a*, *b* has a glowing cathode. On account of the great evolution of heat (99·9 per cent of the incident energy) the anti-cathode or target must generally be specially cooled. See further Vol. V.

## 4. Normal Discharge in Gases at Higher Gas Pressures.

**Corona Discharge and Brush Discharge.**—As the E.M.F. between two electrodes is increased in a gas at relatively high pressure (say 1 atmosphere), the electrodes (when viewed in the dark) are often observed to become covered with a luminous layer or a number of luminous points. At the same time the leads are surrounded by a feebly glowing sheath (*corona discharge*). These phenomena are accompanied by a humming or buzzing sound. In this kind of discharge, which is dependent to a great extent upon the shape of the electrodes, we are again concerned with processes which are produced by collisional ionization in the neighbourhood of the electrodes (p. 336), but which are unable on account of the relatively high pressure to extend right across the gap between the electrodes. As the potential difference is still further increased, luminous branching streaks are seen to stretch out into the gas (*brush discharge*). Those extending from the anode are longer than those from the cathode, a fact whereby it is easy to see which electrode is which. Corona discharges are very undesirable in electrical engineering, as they may cause considerable losses at the high E.M.F.s (up to 200,000 volts in overhead mains) which are nowadays employed.

**Discharge at Points.**—This is a special form of corona discharge occurring when one electrode is in the form of a sharp point. It appears as a luminous tuft at the point. It can often be observed in the open air (on masts of ships, &c.), when it is popularly known as **St. Elmo's fire.** The luminous tuft is divided into three zones; in the case when the point is the cathode these correspond to the cathode surface glow, the first dark space, and the negative glow.

The current strength of such a discharge depends very largely upon the sharpness of the point and the nature of the surrounding gas. The discharge is due to ionization by collision in the strong field in the neighbourhood of the point. As the E.M.F. is increased it eventually changes over into a glow discharge with a marked positive column.

In the case of a point and a flat plate with gas between, the E.M.F. required to produce a discharge is usually smaller when the *point* is made the *positive pole* and the *plate* the *negative pole.* In the case of gases free of oxygen, however, the current strength of the discharge is greater, for a given potential difference, when the point is the negative electrode.

Discharge at a point does not begin until a certain minimum potential difference has been exceeded. Thus it is observed that a silver wire which has been dissolved away at its end to the very small thickness of 0·005 mm. must be raised to a potential of +2500 volts before there is any demonstrable escape of electricity from the point into the air. When the wire is charged negatively, the minimum potential is several hundred volts lower.

The reasons for the differences of behaviour corresponding to the differences of the sign of the point are as follows. When the point is negative the whole of the cathode processes, which constitute the most important part of a glow discharge, can develop normally; but when the point is positive, the cathode must be formed in the surrounding gas. Hence in the latter case the discharge usually occurs along narrow, variable streaks.

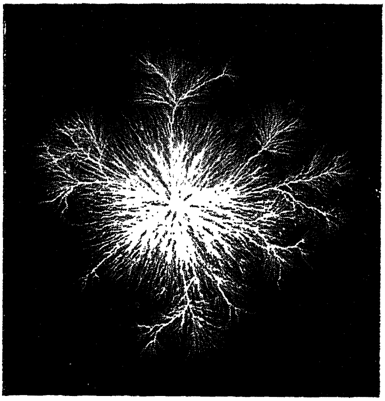

Fig. 67.—Lichtenberg figure formed by the discharge from a positively charged point

These differences can be demonstrated very elegantly in the case of a point mounted upon an insulating plate whose surface has been sprinkled with lycopodium powder. The point is raised to a high potential for a short time, e.g. by connecting it to one pole of an influence machine, and a discharge figure (*Lichtenberg* figure*) of the kind shown in fig. 67 is obtained. The nature of the figure is different according as the point is positively or negatively charged. These differences are due ultimately to the fact that negative electricity exists in the pure form as electrons, whereas positive electricity is always associated with matter.

*The Geiger Counter* (1913).—When the E.M.F. applied to a sharp point is lower than the minimum E.M.F., no discharge occurs because the field strength is not great enough to give rise to the cumulative increase in the number of carriers by collisional ionization (see fig. 56, p. 338). But if a large number of carriers is introduced for a moment near the charged point, a discharge may be started even though the E.M.F. lies below the critical value. Such an introduction of carriers may occur, for example, when a very rapidly moving charged particle flies past. The large number of carriers thus produced by collision (see figs. 53 and 54, pp. 336 and 337) causes a glow discharge of short duration, the passage of which may be observed with the help of a current meter or a telephone. In order that the current shall not automatically increase too much in strength, but shall stop again as soon as the particle has flown past, a high resistance is inserted in the lead to the point. In fig. 68 the inner electrode D (e.g. a gramophone needle) is insulated from the metal tube A (diameter about 2 cm.) by an ebonite block E. The inner electrode terminates in a sharp point of easily oxidizable steel or in a small platinum sphere. The chamber is provided with an opening B through which the particles can enter. The potential of the tube A may be either positive or negative. With the help of this counter the entry of single rapidly moving electrons or positive particles can be very elegantly demonstrated (see Vol. V).

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* Georg Christoph Lichtenberg, born in 1744 near Darmstadt, died 1799, Professor of Physics at Göttingen, also well known as a satirical writer.

**Spark Discharge.**—Either of the above types of discharge may occur, according to the shape, capacity and distance apart of the electrodes; or another type, the electric spark, may be observed.

The latter consists of a sudden leaping of the discharge right across the gap between the electrodes. It is preceded by an intense production of carriers by one of the processes described above. The spark advances out of a corona discharge in a series of distinct jumps until, uniting with the discharge from the opposite electrode, the final spark discharge occurs, i.e. there is a momentary enormous increase of current strength due to a sudden cumulative production of carriers. The heating of the cathode at the sparking point plays a decisive part in this production of carriers (Arc Discharge, p. 352). The sharp noise of the spark is due to the sudden intense heating of the air surrounding the path of the discharge. For further details concerning spark discharges see p. 567 (oscillations) and Vol. V (lightning).

In the case of electrodes of well-defined size and shape, e.g. spheres of definite diameter, the E.M.F. required to produce a spark is uniquely determined by the distance across which it has to pass. The E.M.F. can then be determined approximately from the maximum length of the spark. Some numerical values are given in Table XII.

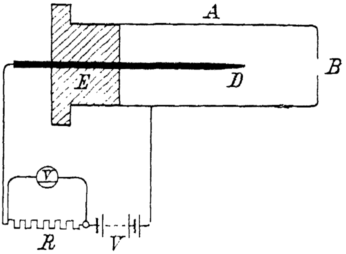

Fig. 68.—Geiger counter

TABLE XII.—SPARKING POTENTIALS (IN VOLTS) FOR SPARK LENGTHS $s$ BETWEEN EQUAL SPHERES OF DIAMETER $d$

| $d$ in cm.<br>$s$ in cm. | 1 | 2 | 10 | 25 |
|---|---|---|---|---|
| 0·1 | 5,000 | — | — | 4,600 |
| 0·2 | 8,700 | 8,500 | — | 7,000 |
| 0·5 | 17,900 | 17,900 | — | 17,400 |
| 1·0 | 29,300 | 32,000 | 32,100 | 31,800 |
| 5 | — | 76,800 | 128,000 | 138,000 |
| 10 | — | — | 200,000 | 247,000 |
| 40 | — | — | — | 555,000 |

*Spark Lag.*—Since the production of the enormous number of carriers necessary for the spark discharge takes an appreciable time, there must be a certain interval between the application of the potential difference and the actual passage of the spark. This lag may be reduced by the previous introduction of a large number of carriers between the electrodes.

**Arc Discharge** *.—This is related to the glow discharge. Its peculiarity lies in the fact that the cathode is very strongly heated by the current process itself, thus becoming an extremely intense source of electrons (p. 312).

Two carbon rods connected to the poles of a battery giving a potential difference of 110 volts may be brought very close together, almost to touching, without any discharge. When they are made actually to touch each other a current passes, the strength of which depends upon the E.M.F. of the battery and the resistance of the whole circuit.

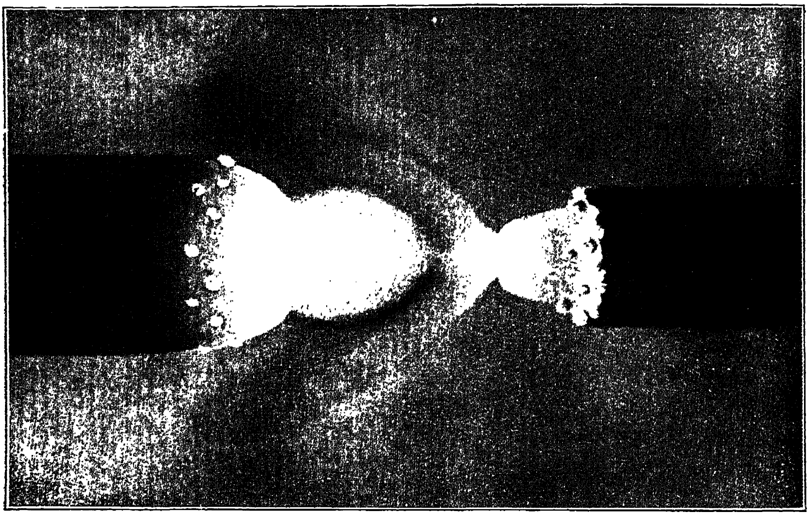

Fig. 69.—Electric arc. Left-hand carbon +, right-hand carbon −

[From Marx, *Handbuch der Radiologie*, Vol. IV, Part V, published by Akad. Verlagsges., Leipzig.]

Separation of the carbon rods so as to leave a gap of about 3 mm. between their ends does not stop the current. On the contrary, the space between the carbons is bridged over by an intensely luminous arc which maintains the conduction. The ends of the two carbon rods become white hot, but the arc itself gives out only little light. If the current be switched off after a few minutes, it is seen that the positive rod has become much hotter than the negative one; the former goes on glowing for a considerable time after the latter has become quite dark.

By projecting a real image of the arc upon a white screen with the help of a convex lens we obtain the result shown in fig. 69. The positive carbon (on the left in the figure) burns out into a deep crater. The actual sources of the intense white light are the interior and edges of this crater, the arc itself emitting only a relatively feeble bluish-violet.

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* Sir Humphrey Davy (1778–1829), see footnote to Vol. II, p. 27. He first produced an electric arc between carbon points in the year 1812, using a battery of 2000 cells. The length of the arc so obtained was 4 inches.

The arc discharge is essentially similar to the discharge with artificially heated cathode (Wehnelt tube, p. 319), but in the case of the arc the heating of the cathode is due to the current process itself. That the high temperature of the cathode is mainly responsible for the production of the arc is proved by the following experiments.

(*a*) Attempts to produce an arc between a carbon rod and an aqueous electrolyte solution are unsuccessful when the latter is the cathode.

(*b*) It is only possible to light an arc again with a small E.M.F. and without bringing the electrodes again into contact, provided that the cathode is still sufficiently hot. This can be demonstrated by using as electrodes a rod of carbon and a rod of some metal (e.g. copper) with a high thermal conductivity, and making first the one and then the other the cathode.

(*c*) If one electrode is a metal plate and the other a carbon rod which can be moved about over the plate by hand, it is found that, when the carbon is the cathode, the arc can be drawn around over the plate along with the carbon rod without being extinguished. But when the plate is the cathode, this is impossible.

The arc discharge consists of the following parts (see fig. 69): at the cathode the very hot spot at which the arc terminates; in front of the cathode the cathode brush; then after a darker dividing space the arc column or positive column; and often also a luminous anode layer. The cathode brush corresponds to the negative glow. The first dark space is absent, the cathode fall being very small on account of the enormous electron emission from the cathode.

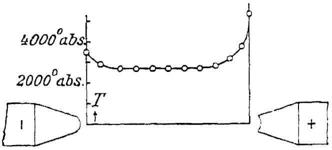

Fig. 70.—Temperatures in a direct-current arc

The preliminary contact of the electrodes serves to produce a sufficient initial heating of the cathode by the great increase of resistance just before separation.

The *temperatures in a carbon arc* are extremely high, that of the positive crater being 4000° C. and that of the negative point somewhat lower. At greater gas pressures temperatures are still higher. In this way LUMMER was able to exceed the temperature of the sun (about 6000° C.). As is seen from the example of fig. 70 (after MATTHIESEN), the temperatures of certain parts of the space between the electrodes are higher than that of the cathode.

In the case of a direct-current arc between pure carbon electrodes, about 85 per cent of the light is emitted by the anode and only about 10 per cent from the cathode. Hence the arc should always be arranged in such a way that best use can be made of the light from the anode (e.g. horizontal position of the anode in projection apparatus). Because of its higher temperature the positive carbon burns away faster than the negative one. For this reason the diameters should be in the ratio 9 : 5 (positive to negative), so as to ensure equal rates of consumption (see fig. 69, p. 352). As the carbons burn away, the gap between them tends to widen. Hence arc lamps require continuous regulation. A large number of devices exist whereby this regulation is automatically performed; they cannot, however, be discussed here.

The potential difference between the electrodes is only 35 to 40 volts, so that unless two arc lamps are connected in series when the mains pressure is 110 volts (or four arc lamps in series when the mains pressure is 220 volts), a large fraction of the available energy must be lost as heat in a resistance.

*Relation between Current Strength and Potential Difference in the case of Arc Discharge.*—In parallel with the *pure* carbon electrodes (see p. 355) we connect a voltmeter; in the main circuit we include an ammeter and a rheostat. When the resistance is varied, the current strength and potential difference also vary.

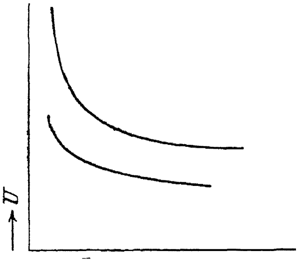

Fig. 71.—Characteristics of carbon arcs. (Longer arc above.)

Plotting as ordinates the E.M.F. U between the carbons and as abscissæ the current strength I for a constant distance between the electrodes, we obtain hyperbolas of the kind shown in fig. 71. These curves may be called the *characteristics* of the arcs. Thus as the current strength *increases*, the potential difference *decreases*, a behaviour opposite to that shown when Ohm's law holds. We have seen (fig. 52, p. 336) that Ohm's law is not obeyed in the case of glow discharges. Now in the case of an arc between pure carbon electrodes we encounter a third kind of relation between current strength and potential difference, namely the **descending characteristic.**

It is not difficult to obtain at least a qualitative insight into this behaviour in the light of what has been said above concerning the nature of arc and glow discharges. Calling the quotient U/I the resistance, the course of the characteristic curves of fig. 71 indicates that the resistance of the arc falls off rapidly as the current strength increases. Two of the possible causes of this are a very great increase in the number of carriers and an increase in their velocity of migration. Now from what has been said above we can understand at once that the number of carriers must increase very rapidly as the current strength is increased; for the moving carriers, especially the electrons, produce secondary electrons, and the number is still further increased by the photoelectric effect (p. 312) of the light emitted, so that the whole process would go on becoming more and more intense were it not subject to some kind of external restriction. The alteration of the velocity of the carriers, which would normally decrease with decreasing potential difference, also plays an important part; but we cannot discuss here these very complicated and by no means completely explained phenomena. Concerning the processes when an alternating potential difference is applied to the arc, see p. 585.

Neither a glow discharge (see neon discharge lamp, p. 340) nor a carbon arc can run without an additional resistance connected in series, by means of which the current strength is limited to a certain maximum value. Hence the available E.M.F. must always be higher than that which actually exists between the electrodes. This is also necessary for another reason, as may be seen from the characteristic curves of fig. 71. For if the current strength decreases (e.g. owing to cooling of the gas between the electrodes by a draught), the E.M.F. must rise. Unless this is possible, the arc goes out.

Under certain conditions the necessary limitation of the current strength may be effected by heat radiation.

This is the case, for example, in carbon filament incandescent lamps. Since the resistance of carbon decreases with rise of temperature (p. 243), the current passing through the filament at constant E.M.F. tends to go on increasing indefinitely. But the greater the temperature difference between the filament and the surroundings, the more rapid the heat radiation (Vol. II, p. 179). A state is eventually reached in which the rate of increase of radiation with further rise of temperature becomes greater than the corresponding rate of decrease of resistance.

The current strength must then stop increasing; for any further rise of temperature would result in heat being radiated at a greater rate than it could be supplied by the current processes. Thus were it not for the heat radiation, it would be quite impossible to use a carbon filament for lighting purposes at constant E.M.F. and without a resistance in series.

All liquid electrolytes also show the same behaviour, namely, a decrease of resistance with rise of temperature. Equilibrium is not reached in all cases. This is shown by the following striking experiment (after POHL).

At relatively high temperatures glass conducts electrolytically (p. 158). A glass rod about as thick as an ordinary pencil is fixed between two metal electrodes, between which is applied a potential difference of 220 volts. The circuit also includes an ammeter and a protecting resistance of about 5 ohms. When the glass rod is heated with a Bunsen burner, a measurable current is observed even before the rod becomes red hot. Then, although the Bunsen burner is immediately removed, the current strength goes on increasing and the rod gets hotter and hotter. Finally the rod melts and the current is stopped.

With the help of a voltmeter connected in parallel with the electrodes it can be observed that the E.M.F. falls off as the current strength increases.

Such occurrences are frequently met with in practice, e.g. in using the electrolytically conducting Nernst lamp.

*Flame Arcs.*—When volatile salts are introduced into an arc, the latter is coloured by the light emitted by the metal vapour (Vol. V). In this way, making use not only of the light emitted by the hot carbon electrodes but also of that emitted by the gases between, a better distribution of intensity and also a greater illuminating power for a given current can be attained.

The salts formerly used were $CaF_2$ and NaF, which give a yellowish red and yellow light respectively. Nowadays use is generally made of the fluorides of magnesium and of the rare earth metals, which give a white light.

The salts are pressed into a hole bored through the axis of the carbons. The specific consumption of such flame arc lamps is about 0·2 to 0·3 watt per candle-power, as against about 1 watt per candle-power for an ordinary carbon arc lamp. It is also remarkable that the characteristic of these lamps is not a descending one, but corresponds more or less to Ohm's law.

*Mercury-vapour Lamp.**—In this type of lamp use is made of the incandescence of mercury vapour in an arc (usually between mercury electrodes). The amount of light given out by the electrodes themselves is only small. Fig. 72 shows a practical form. The electrodes are of mercury, and are cooled by external metal fins. The leads of invar steel are sealed into the tube, which is usually made of quartz, a substance which admits of high arc temperatures and hence great luminous intensity and relatively small dimensions. In order to light the lamp, the tube is tilted so as to produce a continuous thread of mercury between the electrodes, and is then righted again, whereby the thread of mercury is broken and the arc is formed. The colour of the light (see Vol. IV) is bluish green and contains only a few spectrum lines in the visible, the strongest being in the green and blue. Since the

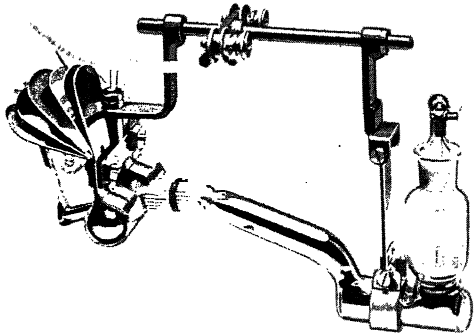

Fig. 72.—Mercury vapour lamp
[Kelvin, Bottomley and Baird, Ltd., Glasgow.]

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* The mercury-vapour lamp was first described by LEO ARONS (1860–1919) in 1892. In 1904 COOPER HEWITT chose a very wide glass tube and also added special cooling chambers to prevent the walls being too strongly heated, thus giving the lamp a practical form for illuminating purposes. The firm HERAEUS of Hanau later succeeded in constructing the lamp of quartz, which is transparent to ultra-violet as well as to visible light.

light from the mercury-vapour lamp contains no red, all red materials appear a dirty brown-grey when illuminated by it. The lamp emits a large amount of ultra-violet light; this is stopped by glass but transmitted by quartz. On account of the biological action of these rays quartz lamps of this type are in wide use as "artificial sunlight" and for the sterilization of drinking water, &c.

In the neighbourhood of a recently lit quartz mercury-vapour lamp there is always a smell of ozone. This is formed by the irradiation of the oxygen of the air with ultra-violet light. It is extremely injurious to the eyes to look directly at a quartz mercury-vapour lamp: very dangerous inflammations of the eyes are produced by the ultra-violet rays. Hence in working with quartz lamps it is necessary to protect the eyes, e.g. with ordinary glasses.

The temperature of the mercury arc under normal load is about 2000° C. Under four atmospheres pressure it has been possible to raise it locally to something like 8000° to 10,000° C.

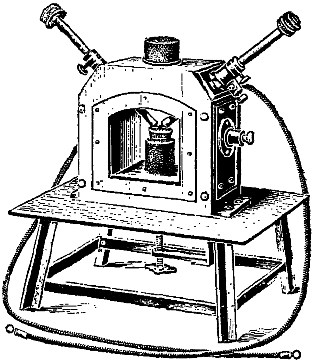

Fig. 73.—Small arc furnace

*Industrial Applications of the Electric Arc.*—In addition to the use of arc lamps for purposes of illumination (see above), the chief applications are (1) for the production of high temperatures and (2) for rectifying alternating currents.

1. *Carbon arc furnaces* consist of a housing of clay into which two carbon rods project through holes in the sides. The carbon rods are connected to a powerful source of current, and an arc produced between them with a current strength of several hundred amperes. The substances to be heated are exposed directly to the arc. A furnace of this kind for experimental purposes is depicted in fig. 73. Fig. 74 shows a diagrammatic cross-section. In some cases the crucible itself is made one pole of the arc, the other being a carbon rod or bundle of carbon rods. This type of furnace is especially suitable in cases where the substance in the crucible is to be simultaneously heated and electrolysed. Frequent use is also made of a combination of arc furnace and resistance furnace (see fig. 75).

As an example of an industrial process carried out on this principle we may quote the large-scale production of *calcium carbide* ($CaC_2$) from lime and coke. On account of the use of $CaC_2$ for the preparation of calcium cyanamide (fertilizer) and the rapidly increasing consumption of acetylene (prepared from $CaC_2$ by the action of water) for welding and for the synthesis of organic substances, the annual output of calcium carbide by this method had risen in 1927 as high as 1½ million tons. The furnaces are built in units consuming up to 25,000 kilowatts.

Also interesting from the point of view of physics is the process in which air is passed through an electric arc at about 3000°, whereby the nitrogen is burned to nitric oxide in accordance with the equation

$$N_2 + O_2 \qquad 2NO$$

The nitric oxide is then further oxidized to $NO_2$ and this is then converted into nitric acid by treatment with water.*

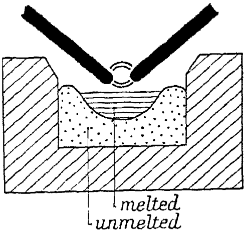

Fig. 74.—Indirect heating

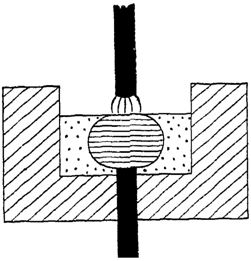

Fig. 75.—Combined arc and resistance furnace

Figs. 74 and 75.—Arc furnaces

In order to obtain rapid cooling of the products and thus prevent the NO from breaking down again, the arc is extended as much as possible. In the Birkeland and Eyde process this is done by means of a transverse magnetic field. The paths of the carriers are then curved (see p. 317), and the arc is distended into a semicircular (or if the electromagnets are fed with alternating current into a circular) sheet (see fig. 76). At 4000 kilowatts the diameter of this sheet of flame is 3 m.

Space does not permit here of more than a bare mention of the production of ozone by electric glow discharge, the electrothermal production of carborundum, artificial corundum, carbon disulphide, and phosphorus, and the manufacture of electro-iron and electro-steel.

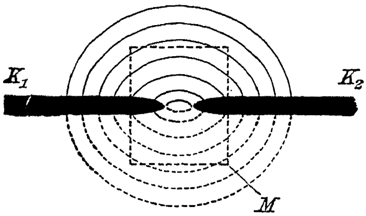

Fig. 76.—Electric arc distended by a magnetic field

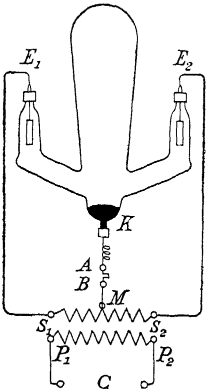

Fig. 77.—Mercury vapour rectifier. (AB, direct-current consumer; C, alternating current.)

2. *Mercury-vapour Rectifier.*—The fact that an arc can only continue to burn when the cathode is hot is made use of for the rectification of alternating currents. Fig. 77 shows the principle of the circuit. The rectifier consists of a well-evacuated vessel of glass or iron in which is a pool of mercury acting as cathode K. The iron anodes $E_1$ and $E_2$ are relatively large, so as not to become too hot. The

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* As early as 1785 and 1788 respectively Cavendish and Priestley observed the formation of nitrogen oxides when an electric discharge is passed through air.

arc concentrates itself upon the cathode in a small spot which moves about irregularly and rapidly over the surface of the mercury. The current density at this spot is about 4000 amperes per cm.$^2$ and the temperature is between 2000° and 3000° C. When once the arc is lit (the auxiliary device whereby this is performed is not shown in the figure), it can only continue to burn provided that K is the cathode. The alternating current is passed through the primary of a transformer (§ 8, p. 508), the secondary of which is tapped at the middle point M. When the direction of the alternating current is from $S_1$ towards $S_2$, it flows by way of $E_2$ and K through the current consumer AB to M. When the direction of the alternating current is reversed, i.e. is from $S_2$ to $S_1$, it flows by way of $E_1$ and K to M. Thus between the terminals AB of the consumer there is a unidirectional (variable) pressure. The choking coil (p. 483) serves to bridge over the moment of zero current by virtue of its "extra current" (p. 396), thus preventing cooling of the cathode spot.

## CHAPTER XI

# Metallic Conduction

### 1. Electrons as Carriers of Current in Metals

The detailed explanation of **conduction in metals** remains one of the most puzzling problems of physics. The reason for this lies in the fact that, as in all conduction processes, the essential factor is the behaviour of the matter through which the conduction takes place. Now, as we have seen above, a certain degree of clarity has been attained in explaining the processes of conduction in liquids and gases, because we are more or less acquainted with the internal molecular nature of these states of aggregation. But the solid state and particularly the internal state of metals are not nearly so well understood (Vol. V).

**The Validity of Ohm's Law.**—Ohm's law holds strictly for metallic conduction, i.e. the current strength is proportional to the applied potential difference.

Deviations only appear to be possible under quite extreme conditions, namely, in the super-conducting state (p. 245) where the validity of Ohm's law has not yet been proved, and also at very high current densities of about 5 million amperes per cm.$^2$ (deviation of approximately 1 per cent according to BRIDGMAN, 1922).

**The Nature of the Carriers.**—One of the most important facts about the transport of electricity in the interior of metals is that it is *not* accompanied by a transport of matter. We are familiar with this from everyday experience, and the most accurate experiments designed to detect a transport of matter have always given a negative result.*

Hence the particles which carry the electricity must be non-material. Now we have already become acquainted with such mobile particles of pure electricity, namely, electrons; and we may presume, therefore, that conduction in metals may also be due to the agency of electrons. To make this certain, it is necessary to determine the mass and charge of the carriers.

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* The case of liquid alkali metal amalgams is the only one in which it has been possible up to the present to detect any motion of matter. Here there is a very slight displacement of the alkali metal towards the cathode.

C. R. Tolman and his collaborators have succeeded in doing this (1916–26). The principle of their method is as follows. On the assumption that the charged carriers in a metal are free to move and that they possess mass and inertia, they will necessarily be displaced relative to the solid lattice of the metal when the latter is accelerated. The result will be an accumulation of excess carriers at one end of the piece of metal (i.e. the appearance of a charge of the same sign as that of the carriers) and a deficit of carriers at the other end (i.e. a charge of opposite sign). Hence between the ends of the piece of metal there will be a certain potential difference. If $e/m$ is the specific charge of the carriers, $l$ the length of the piece of metal and $a$ the acceleration, then the value E of the field strength produced is

$$\mathrm{E} = \frac{m}{e} l\, a.$$

The significance of this equation will be readily appreciated; its complete derivation need not be given here. The actual experiment employs a flat coil with many turns of thin wire made of the substance under investigation; this is rotated rapidly in its own plane and then suddenly braked. The ends of the coil are connected to a sensitive galvanometer by means of wires running along the axis of rotation.

In this way it was proved that the carriers are negatively charged. The value obtained for $e/m$ (average for Cu, Ag and Al) was $1{\cdot}5 \,.\, 10^8$ coulombs per gm., which is in very satisfactory agreement with the value $1{\cdot}77 \,.\, 10^8$ for electrons (p. 321).

*Hence the freely moving carriers in metals are electrons.*

A second way of proving this is as follows. In spite of the free charges in the interior, no external magnetic field can be detected when a metallic conductor is moved with uniform velocity; for the numbers of positive and negative charges are equal and their external effects therefore compensate each other. But if the charges of one sign move with greater velocity than those of the other sign, the magnetic effect of the former will be greater than that of the latter, and the difference between them will be observable outside the conductor. Now if we move the conductor backwards and forwards, we shall shake up the electrons rather like peas in a box, and in consequence of their motion relative to the rest of the conductor we shall observe an alternating magnetic field in the surrounding space. The magnitude of this field may be measured by means of its induction effects (p. 387).

Experiments of this kind verify the truth of the above result with even greater accuracy than the experiments previously mentioned.

Further support is also provided by the fact that it is possible to extract electrons out of metals by the application of very strong electric fields (p. 311).

**Theories of Electron Conduction in Metals.**—The most recent views concerning the nature of electron conduction in metals are closely bound up with our knowledge of the electrical structure of matter. They will therefore be discussed (as far as that is possible to-day) in Vol. V. The underlying theory, expressed quite crudely, is that in the interior of metals a certain number of electrons are actually separated from the metal atoms. That this separation may occur with ease is proved by the existence of positive metal ions in electrolysis. In metals it is probably due to the extreme proximity of the electric fields of neighbouring atoms (theory of LENARD). Thus we may imagine that in the interior of a metallic conductor there are always a large number of electrons with the capacity to move "freely" in the interstices of the lattice of atoms and ions. With certain restrictions it has been found permissible to treat these electrons in accordance with the principles of the kinetic theory, considering them as performing random motions (see Vol. II, § 28, p. 126 *et seq.*) in which (by the principle of the equipartition of energy in thermodynamical equilibrium) their mean kinetic energy is equal to that of gas molecules at the same temperature.* On account of their extremely small mass the electrons must therefore possess quite a high average velocity, namely about 100 km. per second (Vol. II, p. 47) at ordinary temperature. The velocities of the electrons, which are due to thermal agitation, are distributed at random over all directions, so that on the average equal numbers of electrons are moving in all directions of space. The positive metal ions, on the contrary, occupy fixed positions in the space lattice of the crystalline conductor. When now an electric field is produced and maintained between two points of the conductor by the application of a certain potential difference, all the free electrons have an additional component motion imparted to them in the same direction, namely in the opposite direction to the field. We must assume that the "conductivity electrons" are not seriously impeded by the atoms and ions of the space lattice (Vol. V). In most cases the effect of the field forces of these atoms and ions is probably merely to deviate the electrons more or less. Eventually, however, the moving electron will be held fast by an atom or more probably by an ion. Another electron then continues in its place, so that on the average the number of free electrons per unit volume (the *density of conductivity electrons*) remains constant. Thus the electric current in a metallic conductor consists of a stream of electrons moving in the negative direction of the electric field. In cathode rays we have already become acquainted with a similar electron stream. In the present case, however, the velocity component attained under the accelerating action of the field is always lost after a certain time, the electron either being brought to a complete standstill by combination with an atom or ion or else being successively deflected until it is moving at right angles to the direction of the field. Hence the electrons, like the thermally agitated molecules of a gas, must possess a *mean free path.* As a result of the combinations and deflections the kinetic energy of the electrons derived from the field becomes distributed amongst the atoms and over all directions in space, so that directed energy is transformed (at least partially) into random energy. But such random kinetic energy is heat. Thus by the action of the fixed ions and atoms a part of the electrical energy is transformed into heat, just as if the electrons were subjected to a frictional resistance opposing their directed motion in the electric field. Obviously the fraction of the electrical energy which is transformed into heat will increase as the velocity of the electrons is increased, i.e. *ceteris paribus* as the field strength is increased. Under the action of the field, therefore, the electrons finally attain a certain mean velocity component in the field direction, and the magnitude of this component is proportional to the field strength. This state is reached when the rate at which kinetic energy is being transformed into heat is just equal to the rate of supply of energy from the field. The larger this mean velocity of migration of the electrons, the greater also the rate of transport of electricity through the conductor, i.e. the greater the current strength. The *current density,* i.e. the current strength per unit area normal to the current direction, must depend not only upon the field strength but also upon the mean free path and the density of the conductivity electrons. These last two factors are specific constants for each conducting substance: in combination they determine the *specific conductivity.*

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* It has been shown to be probable (SOMMERFELD) that the statistical distribution of the velocities of the electrons does not correspond to the ordinary Maxwell law. According to this view the electron velocity in the interior of metals is *independent* of the temperature within the ranges which we can attain (so-called *gas degeneration*). For further information on this point see Vol. V.

**Velocity of Migration of Electrons in Metallic Conduction.**—We can deduce the approximate value of the velocity component in the field direction by the following consideration. For a conduction process obeying Ohm's law we have the relation (p. 274)

$$\frac{U}{R} = neA\,(u + v).$$

In the case of metallic conduction $u = 0$, and we have

$$\frac{U}{R} = neAv \quad \text{or} \quad v = \frac{U}{RneA}.$$

As an example we may take the case of a copper wire 1 mm.$^2$ in cross-section and 10 m. in length. According to p. 223 the resistance of such a wire is 0·17 ohm. Hence when a potential difference of 1 volt is applied between its ends, a current of about 5·9 amperes will flow through it—a current strength which will not cause a serious heating effect in the wire. In the absence of more exact knowledge we may make the assumption that the number of conductivity electrons is equal to the number of atoms. Now according to Table III, Vol. I, p. 413, the mass of 1 cm.$^3$ of copper is 8·9 gm. Since the atomic weight of copper is 63·6, this represents 0·14 gramme-atoms. Now 1 gramme-atom contains $6{\cdot}06 \,.\, 10^{23}$ atoms (Vol. II, p. 53); hence the number of atoms in 1 cm.$^3$ of copper is $85 \,.\, 10^{21}$. Further, $e = 1{\cdot}6 \,.\, 10^{-19}$ coulomb (p. 322) and $A = 0{\cdot}01$ cm.$^2$. Substituting, we obtain:

$$v = \frac{1}{0{\cdot}0017 \,.\, 85 \,.\, 10^{21} \,.\, 1{\cdot}6 \,.\, 10^{-19}} = \frac{1}{23} = 0{\cdot}043 \text{ cm./sec.}$$

The electrons therefore attain this component velocity of about $\frac{1}{2}$ mm. per sec. in the negative direction of the electric field. The reason for the smallness of the velocity lies in the smallness of the field strength that can be maintained in metals. In the above example, where the current load in the wire was about the maximum attainable under normal conditions, the *field strength* was only

$$E = \frac{U}{d} = \frac{1}{1000} = 0{\cdot}001 \text{ volt/cm.}$$

We see that the electrons move through a metal wire similarly to gas molecules moving through a narrow tube plugged with a porous wad. In both cases there is a frictional resistance (see above).

The existence of this resistance is proved by the evolution of heat. It may also be demonstrated directly by the following elegant experiment. (Compare fig. 64, p. 205, in which the spoked wheel is to be imagined as replaced by a solid disc.)

A circular disc of copper is mounted between the poles of a magnet in such a way as to be capable of rotation about a horizontal axis through its centre and perpendicular to its plane. The edge of the disc dips at its lowest point into a trough of mercury. Let a current be passed through the disc in the direction from the axis to the mercury. The negative current thus flows more or less vertically upwards. Since this electron current flows through a transverse magnetic field, it is deflected (as explained in § 2, p. 316 *et seq.*) in a direction at right angles both to its own direction of flow and to the magnetic field. Now the electrons are subjected to a frictional resistance when they move relative to the metal. Hence their lateral motion will be transmitted to the disc, which will therefore be set into rotation. In actual fact the disc is observed to turn in the expected direction.

**Hall Effect * and Related Phenomena.**—As a result of this deflection of electrons in a magnetic field we may expect that a potential difference will be produced in the conductor at right angles to the direction of the primary current. For since the electrons are deflected laterally by the magnetic field, there must be an excess of them on one side and a deficit on the other. Effects of this kind have indeed been observed. More accurate investigations have shown, however, that the crude theory of an "electron gas" does not suffice even for a qualitative explanation of the phenomena. This illustrates particularly clearly the incompleteness of our present knowledge of the processes of conduction in metals.

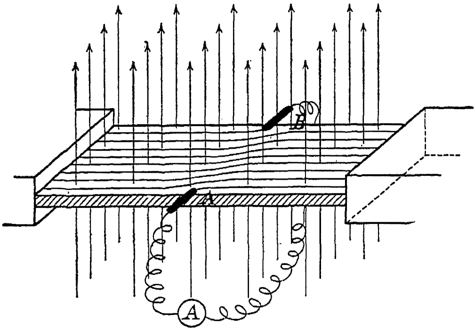

Fig. 1.—Hall effect

Consider a thin metal plate (fig. 1) through which flows a uniformly distributed electric current. Between two points A and B equidistant from the leads is connected a sensitive galvanometer A. There is no observable difference of potential. But when now a magnetic field is produced perpendicular to the plate (see the arrows in fig. 275), a current is observed to flow between A and B (Hall effect), the lines of flow being as shown in the figure. The magnitude of the effect is proportional to the product of the primary current strength and the magnetic field strength. It is especially large in bismuth, and even considerably larger, but of opposite sign, in tellurium. Since the effect may have different sign in different cases and since it is affected to a large extent by impurities, the underlying mechanism must be very complicated and dependent upon the nature of the material in which it occurs.

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* E. H. Hall, born in 1855 at Gorham, Maine, Professor at Harvard University.

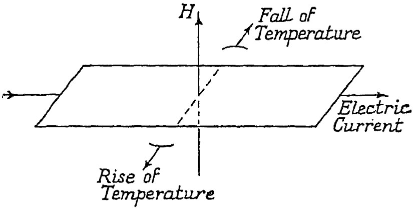

Fig. 2.—Ettinghausen effect

For a plate of bismuth 0·01 mm. thick and for a primary current strength of 0·1 ampere (i.e. for a comparatively wide plate) the potential difference produced by a magnetic field strength of 10,000 gauss is of the order of magnitude of 0·01 volt.

Along with the Hall effect there is also an observable thermal effect (Ettinghausen), a temperature gradient being produced in a direction at right angles to the primary current (fig. 2).

If a current of heat be passed through the plate in the place of the current of electricity, a transverse potential difference is again obtained (Nernst effect, fig. 3), and at the same time a transverse temperature gradient (Leduc).

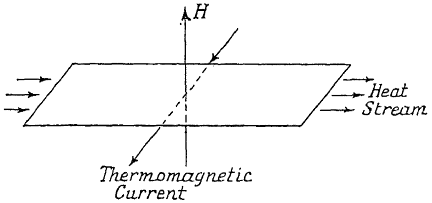

Fig. 3.—Nernst effect

Similar effects are also observed when the direction of the magnetic lines of force coincides with that of the primary current of electricity or heat (longitudinal effects).

Making use of the "electron gas" theory, we can obtain mental pictures of a few further phenomena, or at least of their main features.

**Thermionic Emission.**—As the temperature is raised, the mean kinetic energy of the electrons is increased and they become able on the average to pass through opposing fields of greater strength.* At higher temperatures, therefore, a larger number of electrons can escape from the metal, which becomes surrounded by a "cloud" of electrons like the vapour of a liquid in a closed vessel. In the present case the analogy to the closed nature of the vessel is provided by the electric forces which come into play. By removing the escaping electrons from the neighbourhood of the metal by means of an applied electric field, it is possible under certain conditions to obtain a strong stream of electrons (fig. 4, compare fig. 36, p. 593). This is the thermionic emission already mentioned on p. 312. The quantitative treatment of the theory that thermionic emission is analogous to evaporation (i.e. that the statistical equilibrium between the electrons escaping by virtue of thermal energy and those returning under the electrical forces is comparable to the equilibrium between the molecules of a vapour and its liquid) leads to the following equation (Richardson's equation) for the saturation electron current $I_s$, viz.:

---

* The mean velocity of 100 km. per second at room temperature corresponds to a volt-velocity (p. 324) of 0·03 volt.

$$I_s = aT^2e^{-b/T},$$

where $a$ and $b$ are constants characteristic of the material, and T is the absolute temperature. It may be remarked that the nature of the surface has a very great effect upon the emission and may in certain cases entirely alter its order of magnitude. (For further details, see Appendix II.)

On this theory the emission of the individual electrons must occur at random intervals on account of the statistical fluctuations of velocity. Hence there must be a corresponding statistical fluctuation of the emission current. This has actually been observed with suitable apparatus (Schottky).

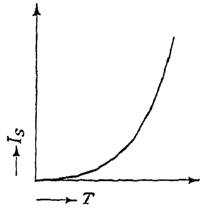

Fig. 4. — Temperature variation of the saturation current $I_s$ in thermionic emission.

**Contact and Thermo-potentials.**—The number of freely-moving electrons split off from the atoms is presumably different in different metals. Hence we can see that when two metals are put into contact there will be an equalization of electron "pressure" and a current of electrons from the metal with the higher electron "pressure" to that with the lower. In contrast to the complete pressure equalization in the case of two gas containers in communication, this transference of electrons cannot go on until the difference of "pressure" has completely vanished; for an opposing potential difference is set up at the interface, the influx of electrons to the metal of lower "pressure" charging this negatively, and the efflux of electrons from the other metal producing a corresponding positive charge. Hence the transference of electrons from the one metal to the other will be limited by the *contact potential difference* which it produces. The problem of detecting this potential difference, which probably exists between all metals in contact and which would be expected to be of the order of magnitude of a few hundredths of a volt, is a very difficult one and so far has not been solved (p. 110). The variation with temperature can, however, be followed with ease. In the case of a loop made of two pieces of wire of different metals soldered together at their ends, this temperature variation will cause a current to flow when the one junction is kept at a different temperature from the other. This is the thermoelectric current with which we have already become acquainted. Further, when an electric current is passed through the interface between two metals, it must overcome the contact potential difference; hence a positive or negative quantity of work must be performed at the junction, which will therefore be heated or cooled. This is the Peltier effect. From the magnitude of this effect (number of calories produced) and from the known current strength, it is possible to draw conclusions as to the magnitude of the contact potential difference (indirect determination of contact potential). Since the relationship between thermoelectric E.M.F. and temperature is not always a simple one, we conclude that the electron "pressure" does not obey the Gay-Lussac gas law. Thus the simple theory of a perfect "electron gas" is insufficient to account for the facts.

A consideration of the phenomena associated with *the super-conducting state* (p. 245) makes it particularly clear that essentially different laws, about which we know very little as yet, must be involved (compare footnote to p. 362). Here the frictional resistance to the electrons has obviously practically vanished. In this connexion it is worth noticing that the electrons move in the metal along quite definite paths, which they cannot leave (compare the sphere experiment of Kamerlingh Onnes, Vol. V). At these low temperatures, therefore, there can be no suggestion of a " gas-like " behaviour of the electrons. At present it is impossible to say to what extent the laws governing super-conductivity are still valid at higher temperatures.

**Electron Conduction in Non-metals.**—There are still no well-founded theories as to the "metallic" conduction in such substances as carbon and silicon. It is interesting to note that induced electron conduction can be observed in solids as well as the spontaneous metallic conduction with which we have so far been exclusively concerned. Thus whereas in metals (perhaps owing to the proximity of the neighbouring atoms) the electrons are already free, it is possible in the case of substances like diamond or zinc sulphide (in general, all substances with an optical refractive index greater than 2) to liberate them by the action of light (Gudden and Pohl—compare pp. 301 and 312).

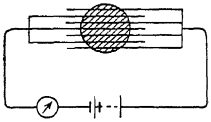

Fig. 5.—Selenium cell in circuit

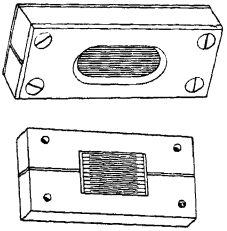

Fig. 6.—*a*, Commercial selenium cell (after Thirring). *b*, Cell opened

When a plate of crystal line zinc blende is placed between the plates of a small charged condenser, a discharge occurs as soon as the zinc sulphide is illuminated (Vol. V).

**Selenium Cells.***—The alteration of the resistance of metallic selenium when illuminated finds application in talking films and for telephony by means of a light beam. Fig. 5 shows the arrangement of a selenium cell of the so-called condenser type. One side of a small mica condenser is rubbed off until the two sets of plates, one within the other as in fig. 5, are exposed. A layer of metallic selenium about 0·02 mm. thick is then placed upon the edges of the plates so as to form a conducting connexion between the two sets. In the dark the resistance of the cell is about a megohm; but in the light it is much less. Hence if a potential difference of say 20 volts is applied between the two sets of plates, the current passing through the cell can be controlled by varying the intensity of the illumination. The greatest disadvantages of selenium cells are that they have a certain lag, that the change of resistance is not strictly proportional to the change of light intensity, and that there are certain residual effects. A few seconds elapse before the maximum alteration of resistance is attained, and after very intense illumination it may be hours before the resistance returns to its true " dark " value. For the above mentioned applications, however, it is possible to avoid these disadvantages. Special attention is drawn to the fact that good specimens of selenium cells possess a " static " sensitivity (i.e. for slow changes of light intensity) of about $50,000 . 10^{-6}$ amperes per lumen (see Vol. IV), and that they are thus about a thousand times more sensitive than good gas-filled photoelectric cells (Vol. V). Fig. 6 shows a commercial form of selenium cell.

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* Hittorf discovered in 1852 that the electrical conductivity of metallic selenium is greater when the metal is illuminated than when it is in the dark.

# PART III

# THE VARIABLE* ELECTROMAGNETIC FIELD

## CHAPTER XII

## Electromagnetic Induction†

### 1. The Induction Process

**Fundamental Experiment.**—Fig. 63, p. 205, shows an experiment in which a conductor carrying a current is observed to move at right angles to the lines of force of a magnetic field. We will reverse this experiment in the manner shown in fig. 1. We connect a galvanometer G between the stands carrying the ends of the wire conductor, and then move the wire vertically downwards (see arrows in figure) between the poles of the electromagnet so as to cut through the lines of force. We observe that the galvanometer needle is deflected during the motion of the wire across the lines of force. From this it follows that the motion of the wire in the magnetic field has produced an electric current. The deflection of the galvanometer only occurs so long as the wire is in motion across the lines of force. When the direction of motion of the wire through the magnetic field is reversed, the galvanometer throw is also reversed.

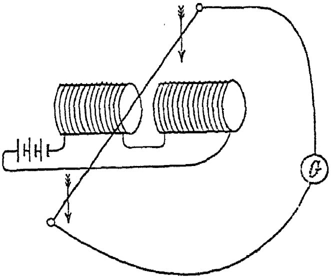

Fig. 1.—Induction by the movement of a conductor in a magnetic field

By comparing the direction of the observed galvanometer throw with that produced by a battery of known polarity, we may deduce the following rule concerning the sense of the current:

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* I.e. variable in time.

† After untiring researches directed towards the transformation of magnetism into electricity, which he was convinced must be possible, FARADAY discovered electromagnetic induction in the year 1831.

*When a conductor is moved in a magnetic field so as to cut the lines of force, an electric current is induced in the conductor. As seen by an observer looking along the magnetic lines of force in the positive sense, this current will be from left to right when the conductor is moving downwards.*

This process is known as **electromagnetic induction.*** The resulting current is called an **induced current.**

Instead of moving the conductor and keeping the magnetic field at rest, the same current can be induced by keeping the conductor at rest and moving the magnet in the opposite direction. Thus the induction process is determined only by the *relative motion* of conductor and magnetic field (p. 386).

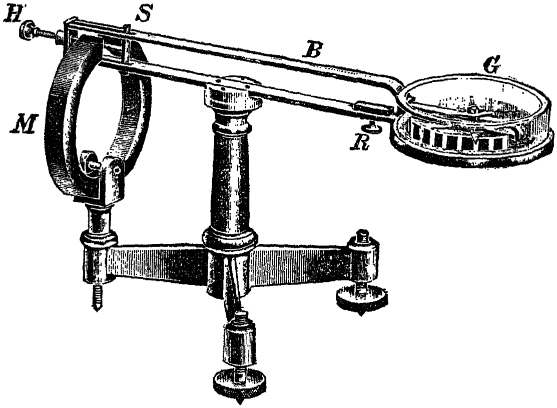

Fig. 2.—Grimsehl's apparatus for demonstrating induced currents

The relation between the directions (and senses) of the magnetic lines of force, the motion of the conductor, and the induced current are given by the **converse of the left-hand rule** (p. 206), viz.:

*Imagine the left hand to be laid upon the conductor so that the palm is turned towards the north pole (i.e. so that the magnetic lines of force are normal to the hand and directed from the palm towards the back), and so that the outstretched thumb (see fig.* 67, *p.* 206) *points in the direction of the motion of the conductor relative to the magnetic field. Then the direction of the induced current is from the finger tips towards the wrist.*

The induced current can be conveniently demonstrated with the apparatus depicted in fig. 2. The poles of the ring-shaped steel magnet M are close together (toroid with gap) and between them is a strong magnetic field. The copper strip B is bent round so as to form a simple galvanometer G, the ends on the left being insulated from one another by a distance-piece of ebonite. The galvanometer end G bears an astatic pair of magnetic needles mounted on agate cones. One of the needles of the pair is between the two limbs of copper, the other is above the upper limb. They can be adjusted by means of a small magnet R until they are parallel to the copper strip B. The ends of the copper strip (on the left in the figure) are slit longitudinally so as to form guides in which a small copper rod S can be moved along by means of the knob H. During its motion the rod S makes good contact with the copper strip. As it moves it cuts through the lines of force which run between the magnetic poles, and the corresponding induced current causes a deflection of the galvanometer. From the sign of this deflection the direction of flow of the induced current can be deduced by the corkscrew rule (p. 161), and in this way the left-hand rule stated above may be verified.

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* This phenomenon of electromagnetic induction must not be confused with the concept of magnetic induction, i.e. the vector **B** (pp. 121 and 377).

The process can also be regarded from a rather different point of view. The bent upper strip and the movable copper rod together form a closed conductor. When the rod is in the position depicted in fig. 2, no magnetic lines of force pass through the surface enclosed by the conductor. But when the rod has been moved to its extreme left-hand position, all the lines of force between the magnetic poles thread through this surface. Thus as the rod is moved from right to left, the number of lines of force passing through the surface bounded by the closed conductor is increased; conversely as the rod is moved back from left to right, this number is decreased. Exactly the same is true of the experiment shown in fig. 1, p. 369, in which the closed conducting circuit in which the current is induced (in future to be called the *secondary circuit*) is formed by the movable wire and the galvanometer with its leads. Thus taking account of the direction of the induced current we obtain the new rule:

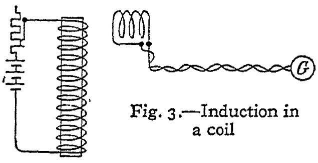

Fig. 3.—Induction in a coil

*When the number of magnetic lines of force threading through the surface bounded by a closed conductor is increased, a current is induced in the conductor. As seen by an observer looking along the magnetic lines of force in the positive sense, the sense of this current is anticlockwise. When the number of magnetic lines of force threading the surface bounded by the conductor is decreasing, the sense of the induced current, as seen by the same observer, is clockwise.*

In the above experiments the induced E.M.F. is only small. In spite of this the current strength in the apparatus of fig. 2, p. 370, is sufficient to produce a deflection of the galvanometer needle. The explanation of this lies in the extremely small resistance of the whole circuit in which the current is induced.

We will now carry out experiments of a different kind to test the validity of the above rule.

A. Near a straight electromagnet with an iron core we move a flat coil of wire, to the ends of which is connected a galvanometer (fig. 3). Keeping the plane of the coil parallel to the planes of the turns of the electromagnet winding, we bring the coil up towards the electromagnet. A current is induced in the coil, owing to the fact that the number of magnetic lines of force passing through the surface bounded by its turns is increasing. As the coil is withdrawn again from the electromagnet, an induced current of opposite sense is observed. It is an easy matter to verify that the current sense is as required by the rule. When the plane of the coil is kept inclined to the planes of the turns of the electromagnet winding, the same motion as before produces a considerably smaller deflection of the galvanometer. Practically no deflection is produced when the coil is moved about in the neighbourhood of the electromagnet, provided that its plane is kept as nearly as possible parallel to the magnetic lines of force.

B. The number of magnetic lines of force threading through the secondary coil can also be altered in the following way. We place the coil near the electromagnet with its plane parallel to the magnetic lines of force, and then turn it through 90° without altering the position of its centre. A current is induced in the coil in a direction corresponding to the increase in the number of magnetic lines of force threading through it. As the coil is turned through a further 90°, the number of lines of force threading through it decreases and an induced current of the opposite sense is observed.

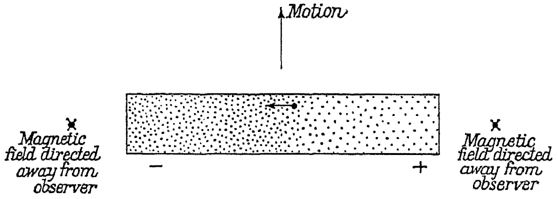
Motion
×
Magnetic field directed away from observer
−
+
×
Magnetic field directed away from observer

Fig. 4.—Diagram illustrating the induction of an E.M.F. in a piece of metal

C. There is yet another way in which we may vary the number of magnetic lines of force threading through the coil. This consists in increasing or decreasing the strength of the primary magnetic field by a corresponding variation of the current in the windings of the electromagnet. Experiment shows again that currents are induced in the expected sense.

**The Ultimate Mechanism of Electromagnetic Induction.**—In Chap. XI (p. 360 *et seq.*) we learned that the electrical conductivity of metals is due to the presence of mobile electrons in their interior. Hence in the case of a metallic conductor in motion we are actually dealing with a convection current in the direction of motion, viz. the current of the negative electrons and the current of the corresponding positive ions.

When the conductor moves in a magnetic field, both the electrons and the positive ions are subjected to forces at right angles to the direction of motion and to the magnetic field (p. 316). Under the influence of these forces the freely movable electrons are displaced relative to the fixed material lattice of the conductor. This produces a deficit of electrons on the one side of the conductor and an excess on the other side (see fig. 4), in consequence of which a potential difference is set up in the conductor at right angles both to its direction of motion and to the magnetic field. This potential difference increases until the force of the corresponding electric field upon the electrons just compensates the force of the magnetic field (see below). If electrons are allowed to enter and leave the conductor at the regions of deficit and excess (as, for example, in the experiment of fig. 1, p. 369), a current is produced and goes on flowing so long as the conductor continues to move in the magnetic field.

When the positive charges are also free to move (as, for example, in an electrolytic conductor), the process remains the same except that the current now consists of a simultaneous motion of negative and positive charges in opposite directions.

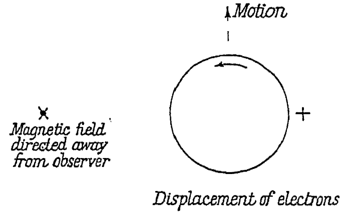
Motion
×
Magnetic field directed away from observer
+
Displacement of electrons

Fig. 5.—Induced E.M.F. in the case of a closed conductor moving in a homogeneous field

It is important to apply these considerations to the case (see above) of a closed circuit moving in a magnetic field in such a way that the number of magnetic lines of force threading through its surface is *constant*. Consider the simplest example of this, namely the uniform translation of a plane closed circuit in a homogeneous magnetic field. Here again the electric charges in the moving conductor are acted upon by transverse forces, and there results a deficit of electrons (i.e. a positive charge) on one side of the circuit and an excess of electrons (i.e. a negative charge) on the other side. This state is established during the acceleration of the circuit from rest to its final constant velocity, after which there is no further change. In this state, therefore, there is a potential difference between two opposite points of the circuit (see fig. 5); but during the uniform motion no current flows in the circuit, because the electromotive forces in the two halves are equal and opposite. For the further analysis of this instructive example see p. 383.

## 2. The Fundamental Laws of Electromagnetic Induction

**The Induced E.M.F.**—From what has been said above it is clear that the laws of electromagnetic induction can be most easily deduced from a consideration of the induced potential difference or E.M.F. This method of treatment has the further advantage that it does not involve any of the individual properties of the materials employed (see below). If the resistance of the galvanometer in the experiment of fig. 1, p. 369, is very great, the instrument may be used as a voltmeter and will then indicate (when suitably calibrated) the potential difference between the ends of the wire when the latter is moved through the magnetic field. Of course this potential difference can also be measured statically by connecting the ends of the wire to an electrometer.

*Influence of the material nature of the Conductor.*—By using wires of different materials and different thicknesses and keeping constant the velocity and direction of motion, the magnetic field strength, and the length of the wire passing through the field, it is an easy matter experimentally to prove that:

*The induced E.M.F. is independent of the material nature of the moving conductor.*

*Influence of the Direction of Motion.*—When a wire is moved through a magnetic field in the direction of its own length, no E.M.F. is observed between its two ends—a voltmeter connected between them shows no deflection. We can understand this in the light of the considerations put forward at the end of § 1, p. 372. For the displacement of the electrons is at right angles to the direction of motion and to the magnetic field; and hence the induced E.M.F. is at right angles to the direction of the length of the wire and has no component in this direction.

Similarly, no induced E.M.F. can be detected between the ends of the wire when the motion is in the direction of the lines of force. The reason for this is the same as before.

When the wire is moved with constant velocity in a direction inclined to the field, we obtain an induced E.M.F. whose magnitude is dependent upon the inclination.

*The induced E.M.F. is greatest when the wire is moved at right angles to its own length and at right angles to the field.*

*Influence of the Velocity of Motion.*—It may be shown experimentally that:

*Other things being equal (same magnetic field, same length of wire, same direction of motion), the induced E.M.F. is directly proportional to the velocity with which the conductor is moved through the magnetic field.*

**Experimental Verification.**—The above laws may be verified with the help of the apparatus depicted in fig. 6.

F is a large coil of insulated wire. The leads from a battery are plugged in at A in such a way that the current flows round the coil F in the direction of the arrow in the figure. Then in accordance with the corkscrew rule the front face of the coil assumes southern magnetic polarity and the magnetic lines of force run along the axis of the coil away from an observer looking at the figure. A brass arm is mounted so as to be able to turn about the middle point C of the coil as axis. The other end R of this arm rubs against the edge of a strip of brass bent round a circular disc of wood so as to project a few millimetres forwards.

The axis of the arm is in conducting connexion with the binding screw $K_1$; the brass ring is likewise connected to $K_2$. The binding screws $K_1$ and $K_2$ are connected by suitable leads to the terminals of a sensitive galvanometer. When now the arm CR is turned about its axis C so that its outer end R always makes contact with the brass ring, it cuts through the magnetic lines of force threading the coil. For a given sense of rotation the lines of force are always cut in the same sense. The result is that the galvanometer or voltmeter gives a deflection in a certain definite direction. Between C and R is induced an E.M.F., the magnitude of which remains constant so long as the arm is rotated with constant velocity. When the sense of rotation of the arm is that indicated by the arrow in the figure, the end C (and hence also the binding screw $K_1$ connected to it) assumes a negative potential relative to the end R and hence also $K_2$).

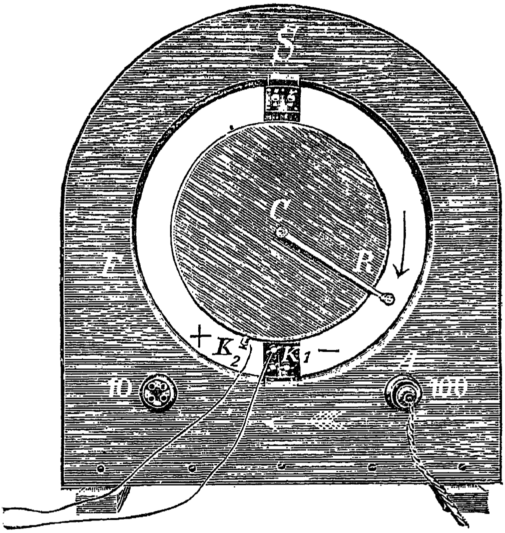

Fig. 6.—Grimsehl's apparatus for the quantitative investigation of electromagnetic induction

*Note.*—Since each point of the moving arm has the same constant angular velocity, the individual elements of the conductor move with different linear velocities relative to the field. The observed E.M.F. between the ends of the arm is therefore to be regarded as due to the sum of the E.M.F.s of the different elements connected end to end. Hence the laws for the whole conductor also hold for each element whose velocity can be regarded as constant; for the connexion of all the elements end to end gives a total E.M.F. which is the simple sum of the individual E.M.F.s.

When the arm is turned with constant velocity, the deflection of the voltmeter remains constant. When the brass is replaced by another metal or by a glass tube filled with an electrolyte (copper sulphate with copper electrodes), the induced E.M.F. is always the same for the same velocity of rotation. (In actual practice care must be taken that the internal resistance of the voltmeter is always very large in comparison with that of the arm.)

When the velocity of rotation of the arm is doubled (the magnetic field being the same as before), the induced E.M.F. is also doubled. In general the induced E.M.F. is directly proportional to the velocity (p. 374).

*Influence of the Magnetic Field Strength.*—By varying the current through the primary coil it can be shown that the induced E.M.F. for a given velocity of rotation is exactly proportional to the strength of this current. But the strength of the magnetic field threading the primary coil is also directly proportional to the current strength flowing round it. Hence:

*The magnitude of the induced E.M.F. is directly proportional to the primary magnetic field strength.*

## 3. Magnetic Flux

**Flux of Force.**—The results of the above experiment show that for one complete turn of the movable arm the product of the induced E.M.F. and the time taken is constant. Thus, for example, if the turn is completed in half the time, the induced E.M.F. is doubled; the product of E.M.F. and time, which may be called the **time-sum of the E.M.F.**, always remains constant.

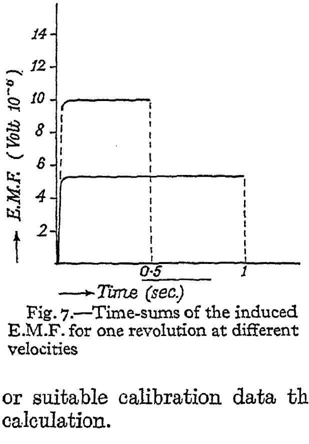

Fig. 7.—Time-sums of the induced E.M.F. for one revolution at different velocities

Fig. 7 shows two corresponding curves of the type obtained when a very rapidly acting and sensitive voltmeter (string electrometer, string galvanometer) is employed. When no such rapidly responding instrument is available, the time-sum of the E.M.F. may be obtained from the quantity of electricity (i.e. time-sum of the current) as indicated by a ballistic galvanometer (p. 218). From a knowledge of the internal resistance of the instrument or suitable calibration data the time-sum of the E.M.F. may be obtained by calculation.

Like the induced E.M.F. itself, the time-sum of the induced E.M.F. must be proportional to the field strength.

*Thus the time-sum of the E.M.F. induced in a wire is dependent only upon the strength of the magnetic field, being independent of the material of the wire and of its velocity of motion.*

For these reasons the time-sum of the induced E.M.F. has been chosen for the quantitative description of the magnetic field passing through the surface swept out by the wire. It gives a measure of the *magnetic flux of force* Φ (referred to simply as the *magnetic flux*) through this surface.

The E.M.F. being measured in volts and the time in seconds, the unit of magnetic flux is the **volt-second.** The international name *pramaxwell* has been suggested for this unit, the prefix *pra* being an abbreviation of *practical.* The unit corresponds to the volt-ampere-ohm system generally adopted for practical purposes.

**The magnetic flux through a surface is of unit magnitude (i.e. one volt-second) when an E.M.F. of one volt is induced between the ends of a wire which sweeps out the surface in one second at uniform velocity.***

**The Magnetic Induction.**—By dividing the magnetic flux by the area of the corresponding surface, we obtain the *specific flux* or the *flux density*, i.e. the flux per cm.$^2$.

This quantity is called the **magnetic induction B.**

Its unit is the volt-sec./cm.$^2$. The unit of magnetic induction used in scientific literature is the **gauss,** which is equal to $10^{-8}$ volt-sec./cm.$^2$. The derivation of the gauss as unit and its relation to the C.G.S. system have already been discussed in § 4, p. 125.

The magnetic induction **B** is a vector, for it has direction as well as magnitude. In the examples given above its direction coincides with that of the magnetic field lines; thus, as was shown by the experiments of p. 374, the induced E.M.F. is dependent upon the direction of motion of the wire relative to the field. The maximum induced E.M.F. is obtained by moving the wire at right angles to the magnetic field. The *direction* of the magnetic induction **B** may be defined quite generally as normal to that plane in which the wire must be moved in order to produce the maximum time-sum of induced E.M.F. per cm.$^2$ swept out.

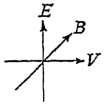

Fig. 8.—Diagram illustrating the law of induction

It remains to define the sense of the vector **B**. The positive sense is chosen so as to coincide with the positive sense of the vector **H** in space devoid of matter.

The rules of § 2, p. 370 *et seq.*, can now be stated in the following shorter form:

*To an observer looking in the direction and positive sense of the magnetic induction, the electric field strength induced in a conductor moving from left to right is directed vertically upwards.* (See fig. 8, and compare fig. 8, p. 164.)

**Relation between the Magnetic Field Strength H and the Magnetic Induction B.**—Our experiments have already shown that in air the magnetic induction is proportional to the magnetic field strength. Careful investigation has proved this to be true in air (or in a vacuum, since the effect of the air is vanishingly small) up to the highest attainable field strengths. Hence for space devoid of matter we have:

$$\mathbf{B} = \mu_0 \mathbf{H}.$$

In order to obtain the numerical value of the constant $\mu_0$ when H is expressed in ampere-turns/cm. and B in volt-sec./cm.$^2$, we may make use of the apparatus shown in fig. 6, p. 375, modifying it as follows so as to make it suitable for continuous action.

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* Historically the unit of flux in the C.G.S. system was first fixed (p. 125), and afterwards the volt was defined according to the above relationship. This accounts for the simple connexion between the gauss and the volt-sec./cm.$^2$.

Instead of one rotating arm CR (see fig. 6, p. 375), imagine a number of such arms (as in fig. 9, where six are represented). Further imagine these arms to be connected together with concentric conducting rings. In each arm there is the same induced E.M.F., and therefore no current will flow round the concentric rings. Finally imagine all the arms replaced by a solid circular disc which rotates in the magnetic field about an axis through its centre. Between this axis and the outer edge of the disc will be an induced E.M.F. of the same magnitude as that which would be produced between the ends of any one of the single arms rotating with the same velocity. This induced E.M.F. will be proportional to the velocity of rotation and to the strength of the magnetic field. It may be measured between two binding screws $K_1$ and $K_2$ (fig. 10), one connected to the axis C and the other to a metal spring or brush pressing against the edge R of the disc.

It is convenient to employ a circular disc of radius CR = 1·78 cm. The area of the disc is then exactly 10 cm.$^2$.* The disc is driven by a train of cogged wheels, so arranged that it may easily be made to rotate 100 times a second by turning a crank handle.

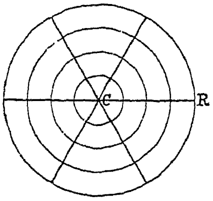

Fig. 9.—Absence of current in concentric ring elements during rotation in a homogeneous field

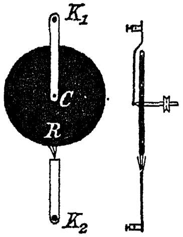

Fig. 10.—Rotation of a solid disc in a homogeneous magnetic field

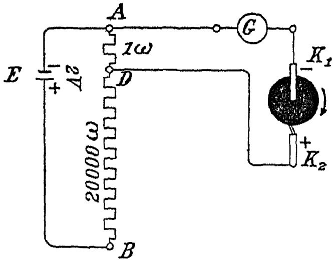

Fig. 11.—Determination of the fundamental magnetic constant

The measurement of the induced E.M.F. is performed by the following compensation method (fig. 11). The poles of an accumulator with an E.M.F. of 2 volts are connected to the ends A and B of a rheostat of 20,000 ohms resistance. A tapping is taken off at a point D such that the resistance between D and A is 1 ohm. The potential difference between D and A is then 0·0001 volt (potentiometer arrangement). The point A is connected through a sensitive galvanometer G to the binding screw K of the induction apparatus. The point D is connected to the binding screw $K_2$. The galvanometer shows a deflection corresponding to the potential difference between D and A.

The disc is now driven round at a speed of just 100 revolutions per second, there being no current in the field coil. The next operation is to switch on the primary current, at first with only a small current strength. If the direction of the primary current is correct, the deflection of the galvanometer will be diminished on account of the E.M.F. induced in the rotating disc. The current in the primary field coil is now adjusted until the galvanometer shows no deflection. When this is so, the induced E.M.F. in the disc is 0·0001 volt. Since the area swept out per second is 100 . 10 cm.$^2$, the magnetic induction is $10^{-7}$ volt-sec./cm.$^2$. If the field coil has a radius of 10 cm. and consists of 10 turns worked so as to give a very small thickness, the corresponding current through it (as read off upon an ammeter connected in the primary circuit) is found to be 15·92 amperes. The magnetic field strength near the centre is therefore (p. 192) 7·96 ampere-turns/cm.

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* In order that it may lie entirely in the homogeneous part of the magnetic field, the disc should have a radius about 5 times smaller than that of the field coil.

Hence we obtain:

$$\mu_0 = 1{\cdot}257\,.\,10^{-8}\,\frac{\text{volt-sec./cm.}^2}{\text{amp.-turns/cm.}}.$$

This constant $\mu_0$ is usually known as the **absolute permeability of space devoid of matter** (or the absolute permeability of the ether). We shall refer to it as the **fundamental magnetic constant.**

**Analogy to the Description of Electric Fields.**—For the characterization and measurement of magnetic fields it is usual to make use of two different vectors **H** and **B**. The advantage of this is the same as in the case of electric fields, where two vectors are also used, namely, that it leads to considerable simplification and increased clarity in the treatment of fields in space containing matter. On the one hand there is an analogy of physical character between magnetic field strength **H** and electrical displacement **D**: both are measured in terms of the quantities producing the field, the former in amp.-turns/cm. and the latter in coulombs/cm.$^2$ (i.e. amp.-sec./cm.$^2$). On the other hand, magnetic induction **B** is analogous to electric field strength **E**: both are measured in terms of the *force effects* upon charges at rest or in motion, the former in volt-sec./cm.$^2$ and the latter in volts/cm. We see that the corresponding electrical and magnetic quantities differ in dimensions by a velocity. Thus volt/cm. = (volt-sec./cm.$^2$) . (cm./sec.) and amp.-turns/cm. = amp./cm. = (amp.-sec./cm.$^2$) . (cm./sec.) (see also p. 196).

Considered merely as mathematical quantities in the equations $\mathbf{D} = K_0\mathbf{E}$ and $\mathbf{B} = \mu_0\mathbf{H}$, however, **D** is analogous to **B**, **E** is analogous to **H** and $K_0$ is analogous to $\mu_0$.

**Magnetic Lines of Force.**— The course of the vector **B** in space can be represented by continuous lines, the **lines of induction.** The magnitude B of the vector is given by the number of these lines of induction passing through each cm.$^2$ of a surface normal to them in the region under consideration. It is customary to refer to these lines as *magnetic lines of force* in analogy to the electric lines of force which represent the vector **D**. In space devoid of matter the direction of **B** coincides with that of **H**; the representations of the field by magnetic lines of force and by magnetic field lines differ only in the number of unit lines per cm.$^2$, i.e. in the line density. For the same field the ratio of the density of lines of force to the density of field lines is equal at every point to the ratio of the corresponding magnetic induction B to the magnetic field strength H, i.e. is equal to $\mu_0$.

We have already (p. 113 *et seq.*) become acquainted with the course and properties of magnetic lines of force. Comparison with the vector **B** at present under consideration shows that we were dealing then with lines of induction.

The result given on p. 376 can now be restated as follows:

*The time-sum of the induced E.M.F. in a conductor is equal to the number of magnetic lines of force which it cuts through during the period under consideration.*

**Eddy or Foucault Currents.**—The rotation of a solid disc of metal in a homogeneous magnetic field has already been treated on p. 378. A constant potential difference is produced between the periphery and the centre; but on account of its symmetrical distribution this E.M.F. only causes a current to flow in the disc when two points at different distances from the centre are joined by a conductor which does not take part in the rotation. In the case where the magnetic field is variable in space or in time, however, or where the motion is not uniform, eddy currents may be produced in the mass of the disc. These currents in the interior of masses of metal are generally called Foucault currents (see historical note below, p. 381).

Their presence is easily recognized by the heat they produce. Thus, for example, a stout-walled cylindrical copper vessel filled with water becomes so hot when rotated in the field of a powerful electromagnet that the water is soon brought to the boil and will blow out the cork of the vessel with a loud pop.

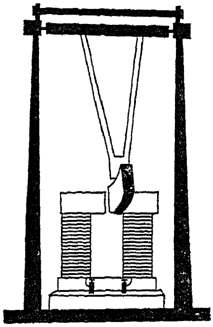

Fig. 12.—Damping due to Foucault currents

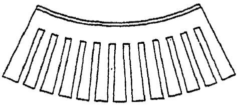

Fig. 13.—Prevention of Foucault currents by means of slits in the Waltenhofen pendulum

The currents induced in a moving conductor are so directed as to tend to stop the motion (p. 382). This is shown very well by the experiment represented in fig. 12.*

Between the poles of a powerful electromagnet hangs a pendulum, whose bob consists of a massive copper plate in the shape of a sector of a circular ring. It is suspended in such a way as to swing through the narrow space between the magnet poles. When displaced and let go before the current through the electromagnet windings is switched on, the pendulum swings backwards and forwards in the familiar way, until after many swings it finally comes to rest on account of the resistance of the air. But if the current of the electromagnet be switched on while the pendulum is still swinging, the pendulum bob is brought to rest almost instantaneously as soon as it enters the space between the magnetic poles. If an attempt be made to move the pendulum backwards and forwards in the field, a resistance is felt, the sensation being the same as if the pendulum were surrounded by a viscous medium like syrup. But if the pendulum bob has a number of slits in it (see fig. 13), no appreciable braking effect is observed. This is because this shape prevents the formation of Foucault currents of such strength as are obtained in a pendulum bob of the type shown in fig. 12.

The chief practical application of Foucault currents is for damping or braking the motion of conductors. The body to be braked is attached to a metal disc (usually of aluminium) which moves between the poles of a powerful magnet—see, for example, the damping of the vibration apparatus of Vol. II, fig. 17, p. 198, the damping of the heated wire meter (fig. 7, p. 257, and fig. 31, p. 489), and the braking of the meter shown in fig. 65, p. 558. Owing to the motion of the core or coil former of moving-coil instruments in the strong field of the magnet, these instruments have an automatic damping.

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* Due to VON WALTENHOFEN (1874).

Great commercial importance also attaches to the Foucault currents produced by variations of the magnetic field.

When a coil of wire with a multiple iron core is connected through a Wehnelt interrupter (p. 519), a rapidly varying magnetic field is produced; for the strength of the current through the coil alternates rapidly between zero and its full value. The speed of alternation may be determined from the pitch of the note emitted by the interrupter. When now a massive copper ring is held over the coil, the Foucault currents induced in it attain such a strength that the ring becomes very hot, possibly even incandescent. The great current strength is due to the low resistance of the copper. The energy given up to the magnetic field by the rapidly varying primary current is transformed into heat in the copper ring.

The iron core of the electromagnet must be *multiple*, i.e. must consist of a bundle of iron wires insulated from each other; otherwise Foucault currents will be produced in the iron core itself, which will therefore become hot. Such a heating of the core of the primary coil represents a partial loss of energy.

As will be shown in § 1, p. 523 *et seq.*, almost all electrical machines contain massive metal parts (especially iron parts) situated in variable magnetic fields. Special attention has to be paid to the prevention of energy losses by Foucault currents. This is done by subdivision of the mass of metal and by the use of substances with as low a conductivity as possible. (For further details see the descriptions of the actual machines.)

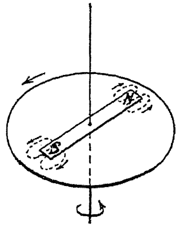

Fig. 14.—Arago's experiment

**Historical Note.**—The first observations of phenomena of this kind were made by Arago,* who discovered in 1825 that a magnetic needle freely suspended over a rotating copper disc is also set into rotation, even though the effect of air currents is entirely removed by a plate of glass between disc and magnet (fig. 14). Arago gave the name *rotational magnetism* to this phenomenon. After Faraday's discovery of electromagnetic induction (1831) it became clear that the reason for the rotation of the magnetic needle lay in the currents induced in the copper disc by the field of the needle. The sense of the direction follows from Lenz's law (see below). Currents of this kind were later investigated more thoroughly by Foucault,† and for this reason are generally known as Foucault currents.

## 4. Quantitative Formulation of the Law of Induction

**Lenz's Law.**—Comparison of the experiments of fig. 63, p. 205, and fig. 1, p. 369, reveals the fact that for a given direction of the current the directions of motion are opposite in the two cases. Similarly for a given direction of motion the current direction in the one case is the opposite of that in the other. This is expressed in the left-hand rule for the induced current (see p. 370), as compared with the rule for the motion of a conductor already carrying a current (see p. 206).

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* Dominique Arago (1786–1853) at Paris.

† Léon Foucault (1819–68). See also footnote, Vol. I, p. 168.

We may reverse the experiment of fig. 6, p. 375, by passing a current through the movable arm CR.

For this purpose the experiment may be modified in the manner shown in fig. 15. The coil producing the magnetic field lies horizontally, so that the magnetic lines of force run vertically downwards. Beneath the coil is a wooden base upon which are mounted a circular trough and a central cup, both filled with mercury. Into these dip the ends of a piece of copper wire bent into the shape shown in the figure. The wire is suspended upon a thin torsion fibre. By means of leads attached to the binding screws at the front corners of the base board a current can be passed through the suspended copper wire. Suppose that the direction of this current is from the outer mercury trough towards the central mercury cup, i.e. opposite from that in the experiment of fig. 6, p. 375.

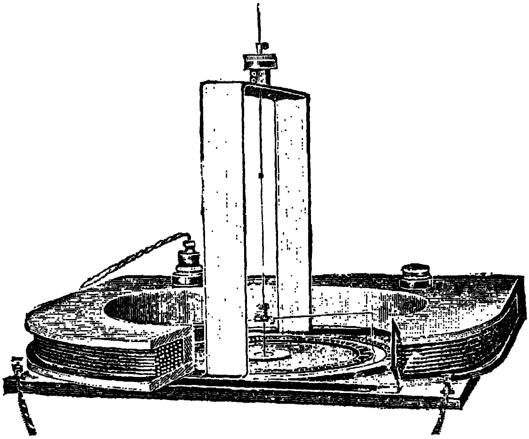

Fig. 15.—Apparatus for demonstrating Lenz's law

When now the magnetic field is switched on, the wire is observed to rotate in the clockwise sense, i.e. in the same sense as the arm CR was turned by hand in the experiment of fig. 6, p. 375. Thus the direction of the current in the present experiment is the opposite of that which would be induced by the resulting motion.

We see that the current induced in a conductor owing to its motion in a magnetic field is so directed as to tend by its electromagnetic action to give rise to a motion in the opposite direction from that in which the body is actually moving. The induced current exerts a mechanical opposition to the motion, i.e. external mechanical work must be expended in the induction process. If this were not so and if the induced current were in the opposite direction, then when once the conductor had been set into motion in the magnetic field, the result would be a continuously accelerated motion and a continuous induced current. This would mean a continuous production of mechanical and electrical energy, and a contradiction of the principle of the conservation of energy. (See also Le Chatelier's Principle, Vol. II, § 15, p. 152.) In general, therefore, we have the following law (**Lenz's**[*] **law**):

**The current induced by any change is always so directed as to tend to oppose that change.**

**Circuital Electromotive Force.**—When a closed loop of wire moves as a whole in a magnetic field, electromotive forces are induced in the different parts of the loop according to their respective velocities and directions of motion. Since all these different parts are in conducting connexion, the total induced E.M.F. is the algebraic sum of the partial induced E.M.F.s. Thus the electromotive force which is effective in determining the flow of electricity round the circuit is obtained by summing the induced E.M.F.s right round the boundary of the surface enclosed by the wire loop. The case is similar to that of the magnetic field of an electric current (p. 186). There we made use of the name "circuital potential" for the magnetic potential difference along a closed path encircling the electric current. We may introduce a similar name here and make the following definition:

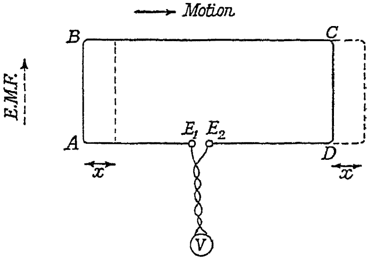

Fig. 16.—Circuital E.M.F.

(The magnetic lines of force are normal to the plane of the figure and directed away from the observer.)

**The algebraic sum of the partial induced E.M.F.s around the boundary of a surface moving in a magnetic field is called the circuital E.M.F.**

**Calculation of the Circuital E.M.F.**—In order to obtain the relationship for a closed conducting circuit in motion in a magnetic field, imagine the following experiment to be carried out. The rectangle of wire ABCD (fig. 16) is completely closed except for a small gap $E_1E_2$, across which is connected a voltmeter V, the leads being twisted around one another as indicated in the figure. Let the rectangle be moved from left to right through a magnetic field whose lines of force run normally through the plane of the paper away from the observer. The E.M.F. induced between $E_1$ and $E_2$ may be measured upon V. If the magnetic field is constant in the plane of motion of the circuit, the number of lines of force cut through per unit time by the side

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[*] H. F. E. Lenz (1804–65) deduced his law in 1834 from the principle of the equality of action and reaction.

AB is the same as that cut through by the side CD. Hence the induced E.M.F. in AB is equal to that in CD, the former being directed from A to B and the latter from D to C. The E.M.F.s induced in the sides BC and AD are at right angles to the wire and therefore have no component along it. The circuital E.M.F. in this case is therefore zero: the voltmeter remains undeflected.

It should be noticed that the number of lines of force passing through the surface of the rectangle remains constant.

Now consider the case in which the magnetic field strength decreases from left to right in the plane of motion of the circuit. In accordance with p. 376 the time-sum of the E.M.F. induced in the side DC as the rectangle moves a distance $dx$ in a time $dt$ is equal to the number of magnetic lines of force threading through the surface swept out by this side. The area of this surface is DC . $dx$. In the same way the time-sum of the induced E.M.F. in the side AB is equal to the number of lines of force threading through the area AB . $dx$ swept out by this side. Now by assumption the field strength decreases from left to right, so that the induced E.M.F. in DC must be smaller than that in AB. In this case, therefore, we observe a deflection of the voltmeter corresponding to the circuital E.M.F., i.e. the difference between the E.M.F.s in the sides AB and DC. (Between B and C or between A and D there is no induced E.M.F.)

The time-sum of the circuital E.M.F. is equal to the difference between the time-sums of the induced E.M.F.s in the sides AB and DC. It is therefore equal to the difference between the flux through the area AB . $dx$ swept out by the side AB and the flux through the area DC . $dx$ swept out by the side DC. But this is simply equal to the decrease $-d\Phi$ of the total flux $\Phi$ through the surface ABCD as this surface moves through the distance $dx$.

Hence putting the induced circuital E.M.F. equal to $E_i$, we have $E_i\,dt = -\,d\Phi$ or

$$\mathbf{E}_i = -\frac{d\Phi}{dt} = -\frac{d(\mathbf{BA})}{dt} \text{ volts,}$$

where B is the magnetic induction in volt-sec./cm.$^2$ and A is the area in cm.$^2$ enclosed by the moving circuit. (For the explanation of the minus sign see below.) In this equation $\Phi$ is expressed in volt-sec. If B be expressed in gauss instead of volt-sec./cm.$^2$, the equation becomes:

$$E_i = -\frac{d(\mathrm{BA})}{dt} . 10^{-8} \text{ volts.}$$

The expression $-\dfrac{d\Phi}{dt}$ gives the rate of decrease of the flux at the time $t$. Assuming that this rate may be regarded as constant during each unit of time, the expression $-\frac{d\Phi}{dt}$ gives the actual decrease of flux per second.

We may now express the **law of induction** in the following form:

**The circuital E.M.F. is equal to the time-rate of decrease of magnetic flux through the enclosed surface.**

The equations derived above are valid for the case in which the closed conductor encircles the magnetic flux of force only once. If the circuit consists of a coil with a number of turns of wire, these turns act like cells connected in series and the total E.M.F. between the ends of the coil is equal to the sum of the partial E.M.F.s of all of the turns. If the number of turns is $n$, the total induced E.M.F. is given by:

$$\mathrm{E}_i = -n\frac{d\Phi}{dt} \text{ volts,}$$

where $\Phi$ is expressed in volt-sec. and $t$ in sec.

If the flux changes uniformly from the value $\Phi_1$ at the time $t_1$ to the value $\Phi_2$ at the time $t_2$, we may write:

$$\mathrm{E}_i = -n.\frac{\Phi_2 - \Phi_1}{t_2 - t_1} = n.\frac{\Phi_1 - \Phi_2}{t_2 - t_1}.$$

According to p. 377 the magnetic flux $\Phi$ is equal to the product of the magnetic induction B and the area A, and gives the number of lines of force passing through this area. Hence we may express the law of induction in the following simple form:

*The E.M.F. induced in a uniformly moving conductor is equal to the number of magnetic lines of force cut through per second.*

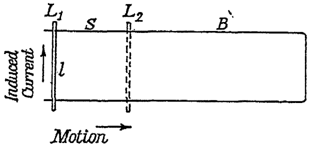

Fig. 17.—Derivation of the law of induction

(The magnetic lines of force are normal to the plane of the figure and directed away from the observer.)

The *negative sign* in the above equations for the law of induction corresponds to Lenz's law, since we have defined the direction and sense of the vector **B** as the same as the direction and sense of the vector **H**. Let us consider this a little more closely.

The circuit in fig. 17 consists of a wire bent round to form three sides of a rectangle and a sliding wire $l$ forming the fourth side. It is situated in a magnetic field whose lines of force are to be imagined as running normally through the plane of the paper away from the observer. We may imagine this field to be produced by an electric current flowing round the circuit. This current must then flow in a clockwise sense, as indicated by the vertical arrow in the figure. In order to obtain an induced current in the same sense, we must move the sliding side of the rectangle in the direction of the horizontal arrow in the figure, i.e. in the direction corresponding to a decrease of the area of the rectangle. Thus the differential coefficient of $\Phi$ with respect to $t$ must be negative. If the sliding side $l$ be moved in the opposite direction, namely from right to left, the differential coefficient $\frac{d\Phi}{dt}$ becomes positive; but the direction of the induced current is then opposite from that of the primary current producing the initial magnetic field. This is the explanation of the minus sign in the equations.

In the form $\int E_i dt = -\int d\Phi$ the previous equations express the fact, stated on p. 376, that the time-sum of the induced E.M.F. may be used as a measure of the magnetic flux.

**The Induced Current.**—If R is the resistance of the whole circuit, then by Ohm's law the *strength* I of the induced current is given by

$$I = -\frac{1}{R}\frac{d\Phi}{dt}.$$

Multiplying by $dt$ we obtain $I dt = -d\Phi/R$. Now the product $I dt$ on the left-hand side is the quantity $dQ$ of electricity flowing during the time $dt$ through any cross-section of the conductor. Hence

$$dQ = -\frac{d\Phi}{R}.$$

If the motion of the circuit is not confined to a small element of time but continues for a longer period, the quantity of electricity $dQ$ in each element of time is given by the above expression, and the total quantity Q is obtained by summation of all the elements. Similarly all the elements $d\Phi$ add together, their sum being equal to the total change of the flux. If the initial value of $\Phi$ is $\Phi_1$ and the final value $\Phi_2$, this total change is simply $\Phi_2 - \Phi_1$. Hence we obtain

$$Q = -\frac{\Phi_2 - \Phi_1}{R}.$$

This equation does not contain the time.

*Hence the total quantity* Q *of electricity flowing through any cross-section of the conductor depends only upon the change in the magnetic flux.*

This explains how it was that we were able to measure the time-sum of the induced E.M.F. (i.e. the magnetic flux) with a ballistic galvanometer (p. 376).

**Relative Nature of the Induction Processes.**—The above formulation of the induction law holds quite generally. As was proved by our introductory experiments, an E.M.F. is produced not only by the motion of a conductor in a magnetic field, but also in a conductor at rest in a moving magnetic field. Quantitative investigation shows that the value of the time-sum of the induced E.M.F. is independent of whether it is the conductor or the magnetic field which moves, provided that the relative motion is the same in both cases.

We may demonstrate this quantitatively by means of the arrangement shown in fig. 18. The two coils can be moved along independently between wooden guides. The coil P (the primary or field coil) is connected to a battery; the other coil S (secondary coil) is connected to a ballistic voltmeter. We start with the coils close together, and then slide the secondary a few centimetres away from the primary. The ballistic voltmeter shows a throw A. After the instrument has come to rest again, we slide the secondary coil back into contact with the primary. This causes a throw —A of the ballistic voltmeter. We now repeat the experiment, leaving the secondary coil at rest and sliding the primary coil through the same distance. Once again we obtain the throws A and —A.

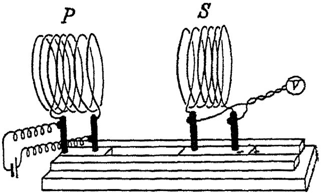

Fig. 18.—Relative nature of inductive action

The magnetic induction B decreases with increasing distance from the primary coil. In each of the four displacements of the above experiment the magnetic flux through the secondary coil was altered by the same amount, and consequently all the four throws of the ballistic instrument were equal.

From this we see that the equation of p. 384 also holds in the case where the magnetic field moves and the conductor remains at rest.

This result can be still further generalized. It is not necessary that the change of magnetic flux through the surface of the secondary coil be produced by a motion of the primary coil. As may be shown experimentally, this change may be produced by altering the current strength in the primary coil (see also p. 372).

## 5. Rotation of a Loop or Coil of Wire in a Homogeneous Magnetic Field

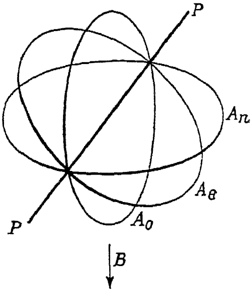

Fig. 19.—Rotation of a loop of wire in a homogeneous magnetic field

Consider a plane closed loop of wire (fig. 19) of any shape enclosing an area A and situated in a homogeneous magnetic field of induction B. Suppose that the plane of the loop is initially in the position $A_n$, i.e. normal to the magnetic lines of force. In this position the number of lines of force (magnetic flux) through the surface bounded by the loop is BA. Now let the loop be rotated about the axis PP through an angle $\theta$. In the new position $A_\theta$ the number of lines of force passing through the surface bounded by the loop is BA $\cos\theta$. After having been rotated through 90° the loop is in the position $A_0$ in which its plane is parallel to the magnetic lines of force. In this position the flux through the surface bounded by the loop is zero.

Thus, as the loop is rotated, the magnetic flux through its surface varies. An E.M.F. is therefore induced in the loop and an induced current flows round it (p. 372). We will determine the strength of this induced current both by measurement and by calculation.

**Measurement of the Induced Current.**—Current is supplied to the larger primary coil S (fig. 20) by a plug A (p. 375). At the ends of a diameter of this coil are mounted the bearings of an axis lying in the plane of the coil. To the middle of this axis is attached the flat coil of wire $q$ (secondary coil), which can therefore be rotated so that its plane remains vertical and makes any desired angle with the plane of the primary coil. This angle can be read off upon a large circular scale T. The ends of the coil $q$ are connected to the bearings at the ends of its axis of rotation. These bearings are provided with binding screws K, the upper one being visible in the figure, while the lower one is hidden by the scale T. The coil $q$ may be rotated conveniently by means of the arm Z, which also acts as pointer for indicating the angle on the scale T.

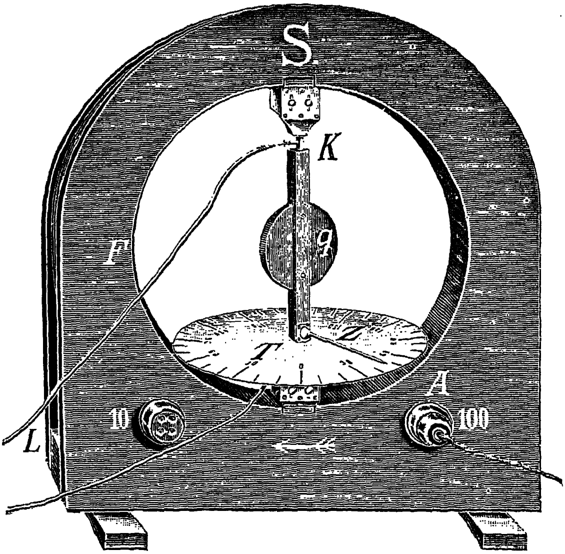

Fig. 20.—Grimsehl's apparatus for the investigation of induction by rotation

As the secondary coil is turned out of the position $q_n$ in which its plane is at right angles to the magnetic lines of force into the position in which its plane is parallel to these lines, the magnetic flux through the surface bounded by the coil varies from its maximum value $\Phi_n$ to the value zero. Hence a current is induced, the time-sum of which is given by $Q = \Phi_n/R$, where R is the resistance of the coil (see p. 386). This quantity of electricity can be measured with the help of a ballistic galvanometer connected between the binding screws K by the leads L.

Starting each time from the same normal position $q_n$, the secondary coil is now turned successively through a number of different angles $\theta$ smaller or greater than 90°. The corresponding throws of the ballistic galvanometer are then found to be proportional in each case to $(1 - \cos\theta)$.

When the velocity of rotation is constant, the curve obtained by plotting either the induced current strength or the induced E.M.F. as a function of the angle $\theta$ is of the type shown in fig. 21 (so-called sine curve). In this case the angle $\theta$ is proportional to the time and therefore the curves representing the induced current strength and the induced E.M.F. as functions of the time are also sine curves.

**Calculation of the Induced E.M.F.**—In fig. 22 let the arrows represent the direction of the homogeneous magnetic field of induction B, and let $AA_1$ be the loop of wire as seen from the side. Since the plane of the loop is at right angles to the plane of the figure, we see it as a straight line. Let O be the point of intersection of the axis of rotation with the plane of the figure, to which it is normal. The line MM represents the side view of a plane normal to the magnetic lines of force. Suppose that $AA_1$ is inclined to MM at the angle $\theta$.

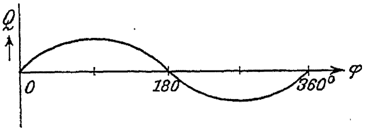

Fig. 21.—Curve representing either the induced E.M.F. or the induced current strength as a function of the angle through which the coil has been rotated out of its normal position

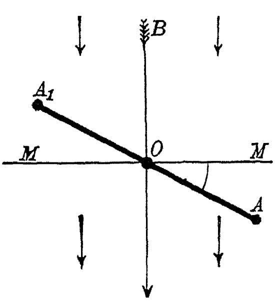

Fig. 22.—Rotation of a plane loop of wire in a magnetic field. (Angle MOA = $\theta$.)

If the plane of the coil were in the normal position MM, the magnetic flux $\Phi_n$ through the surface of area A bounded by the coil would be AB. In the position $AA_1$, i.e. at the inclination $\theta$, the flux is only

$$\Phi = \mathrm{AB}\cos\theta.$$

Now suppose the loop to be rotated about the axis O with a constant velocity such that the time taken to turn through the angle $2\pi$ (i.e. one complete revolution) is T. Let $t$ be the time taken to turn from the normal positive into the inclined position $AA_1$. Then we have

$$t : \mathrm{T} = \theta : 2\pi,$$

whence

$$\theta = \frac{2\pi}{\mathrm{T}} t.$$

Substitution in the above expression for $\Phi$ gives

$$\Phi = \mathrm{AB}\cos\left(\frac{2\pi}{\mathrm{T}} t\right).$$

Differentiating this equation with respect to $t$ we obtain

$$\frac{d\Phi}{dt} = -\frac{2\pi}{\mathrm{T}}\,\mathrm{AB}\sin\left(\frac{2\pi}{\mathrm{T}}t\right).$$

But the induced E.M.F. E is equal to $-\frac{d\Phi}{dt}$. Hence we obtain the following important formula for the induced E.M.F., viz.

$$\mathbf{E} = \frac{2\pi}{\mathbf{T}}\,\mathbf{AB}\sin\left(\frac{2\pi}{\mathbf{T}}t\right) = \frac{2\pi}{\mathbf{T}}\,\mathbf{AB}\sin\theta.$$

Introducing the constant angular velocity $\omega = 2\pi/\mathrm{T}$, this becomes

$$\mathbf{E} = \omega\,\mathbf{AB}\sin\omega t.$$

The maximum value of E is assumed when $\sin\theta = 1$, i.e. when $\theta = \frac{1}{2}\pi$. The plane of the loop is then parallel to the magnetic lines of force. Writing $\mathrm{E}_0$ for this maximum value of E, we have $\mathrm{E}_0 = \omega\mathrm{AB}$ and hence

$$\mathbf{E} = \mathbf{E}_0\sin\theta = \mathbf{E}_0\sin\left(\frac{2\pi}{\mathbf{T}}t\right) = \mathbf{E}_0\sin\omega t.$$

These expressions for the induced E.M.F. are of fundamental significance in all cases in which a closed conductor rotates in a homogeneous magnetic field. If the conductor does not consist of one single loop (as assumed so far) but of $n$ turns of wire, then the induced E.M.F. is $n$ times greater. This simply means that the factor $\mathrm{E}_0$ in the last formula is equal to $n\omega\mathrm{AB}$ instead of $\omega\mathrm{AB}$.

The formula shows that the induced E.M.F. in a closed conductor rotating with uniform velocity in a homogeneous magnetic field is proportional to the sine of the angle between its plane and a plane normal to the lines of force. This is in harmony with the result obtained by experiment.

It also follows from the formula that the induced current strength is proportional to the sine of the same angle.

**The Earth Inductor.**—The strength of a homogeneous magnetic field may be measured by rotating a plane coil of wire in it and observing the time-sum of the induced E.M.F. This method is used especially for measuring the strength of the earth's magnetic field.

According to p. 385 we have $\mathrm{E}\,dt = -n\,d\Phi$. Hence if the flux alters from the value $\Phi_1$ to the value $\Phi_2$ in the time $t$, the time-sum or time-integral of the induced E.M.F. is given by

$$\int \mathrm{E}\,dt = n(\Phi_1 - \Phi_2).$$

In the case in which the coil of wire is rotated about a vertical axis in the earth's magnetic field we have $\Phi = AB_h \cos\theta$, where $B_h$ is the magnitude of the horizontal component of the magnetic induction of the earth's field. Hence

$$\int E\,dt = nAB_h(\cos\theta_1 - \cos\theta_2).$$

Suppose that the plane of the coil is turned through 180°, starting from a position in which it is normal to the earth's field, i.e. normal to the magnetic meridian. In this case we have $\cos\theta_1 = \cos 0° = 1$ and $\cos\theta_2 = \cos 180° = -1$. The time-sum of the induced E.M.F. is therefore

$$\int \mathbf{E}\,\boldsymbol{dt} = \mathbf{2}\boldsymbol{n}\mathbf{AB}_h.$$

The quantity $\int E\,dt$ may be measured by means of a ballistic galvanometer; for it is equal to QR, where Q is the ballistically measured quantity of electricity and R is the resistance of the whole circuit. Thus we have

$$Q = \frac{2nAB_h}{R},$$

from which the value of $B_h$ may be obtained by calculation.

If now the same coil be mounted so as to turn about a horizontal axis lying in the magnetic meridian, the effective component of the earth's field is the vertical one $B_v$. As the coil is turned from an initial horizontal position through 180° into the next horizontal position, the quantity of electricity passing any point of the wire is

$$Q_1 = \frac{2nAB_v}{R}$$

Division of the last two equations gives

$$\frac{Q_1}{Q} = \frac{B_v}{B_h}.$$

The expression $Q_1/Q$ is simply the ratio of the throws of the ballistic galvanometer in the two cases, and can therefore be obtained by experiment. Thus we have a method of measuring the ratio $B_v/B_h$ of the vertical and horizontal components of the earth's field.

Referring to fig. 35, p. 131, we see that $B_v/B_h = \tan i$, where $i$ is the angle of inclination of the earth's magnetic field. Thus this angle can be determined directly from the ratio of the throws of the ballistic galvanometer. The earth inductor is therefore a very suitable instrument for investigations of the earth's field.

Fig. 23 is a diagram of the earth inductor employed by W. Weber. If it is desired to measure the horizontal component $B_h$, the axis M is adjusted so as to be exactly vertical, and the plane of the coil is rotated rapidly through 180° out of one east-west position (i.e. at right angles to the magnetic meridian) into the next. The throw of a ballistic galvanometer connected between the ends of the coil is then a measure of the quantity of electricity Q passing through it on account of the induced current. To determine the vertical component $B_v$, the outer frame of the instrument is rotated about the axis LL until the axis M is horizontal. Care must be taken to ensure that the axes M and LL both lie accurately in the same plane. The axis M is adjusted so as to coincide with the magnetic meridian. The coil is then rotated rapidly through 180° about the axis M from one horizontal position into the next, and the corresponding throw of the ballistic galvanometer is observed. The values of the components of the intensity of the earth's magnetic field obtained in this way agree with those previously given in § 5, p. 128 *et seq.*

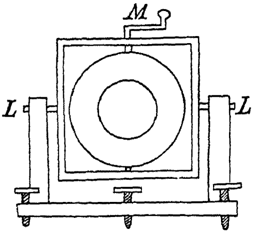

Fig. 23.—Diagram of an earth inductor

## 6. The Most General Form of the Law of Induction

**The Electric Vortex Field.**—The law of induction is capable of a much deeper interpretation than we have hitherto put upon it. We have seen that, when the magnetic flux through a closed conductor varies with time, a circuital E.M.F. is induced. This means that the line-integral of the electric field strength round the closed path formed by the conductor does not vanish (p. 30).

*The electric field associated with the induced current is therefore a vortex field.*

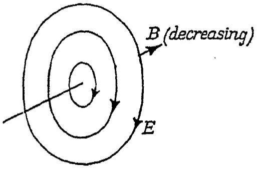

Fig. 24.—Electric vortex field

This vortex field is not occasioned, however, by the existence of the conductor. The induced circuital E.M.F. is obviously independent of the ohmic resistance of the circuit. Hence we may imagine this resistance to be increased indefinitely, until finally the induced current strength would be zero. In spite of this the circuital E.M.F. would still be induced. It would then give rise to an electrostatic field with closed lines of force encircling the magnetic flux. Such a field must be produced throughout the whole of the space surrounding the varying magnetic field (see fig. 24). This field does not represent a state of static equilibrium; it only persists as long as the magnetic flux which it surrounds continues to vary with time.

We must assume that this vortex field is also produced in the absence of dielectric, i.e. in a vacuum. It is completely analogous to the magnetic vortex field surrounding a conductor through which an electric current is flowing. The experimental proof of the existence of the induced electric vortex field in space devoid of matter has been provided by H. Hertz, whose experiments we shall discuss in § 2, p. 621 *et seq.*

Hence follows the most general statement of the law of induction, first given by James Clerk Maxwell * about 1870:

**A variable magnetic flux is associated with an electric vortex field, whose direction is everywhere at right angles to that of the magnetic field and whose circuital potential is equal to the rate of decrease of the magnetic flux.**

**Determination of the rate of decrease of magnetic flux in the case of moving bodies.**—In this form and in the form given on p. 385 the induction law has no definite meaning when applied to moving bodies, unless we have some principle to determine what magnetic flux at time $t + dt$ is to be compared with the flux as defined at time $t$. This principle is as follows: *The surface chosen for the calculation of the flux shall be determined throughout by the same material parts of the moving system.*

As an example of this we may take the experiment in which a current is induced by the motion of a solid metal disc in a magnetic field (p. 378). In discussing this experiment we explained the induced E.M.F. as due to the motion of the contiguous "spokes" of the solid wheel, i.e. we referred it to surfaces rigidly connected with the moving system. The principle stated above at once becomes clear when we remember that the carriers of electricity (electrons in metals and ions in electrolytes) share the motion of the moving system.

## 7. Mutual Induction and Self-induction

**Mutual Induction.**—A closed conductor carrying an electric current is surrounded by a magnetic field. Any variation of this field must induce an electric current in a second closed conductor situated in the neighbourhood of the first. Hence any variation of the current strength in the first circuit must induce a current in the second. We have already become acquainted with this phenomenon on pp. 372 and 387.

In fig. 25 let I and II represent the two neighbouring circuits. A current of any desired strength can be sent through the primary circuit I: in the secondary circuit II is connected a voltmeter or galvanometer.

When the circuit I is closed, a certain magnetic flux passes through the surface bounded by it. The magnitude $\Phi'$ of this flux is proportional to the primary current strength. A certain fraction of it, determined by the shapes of the two circuits and by their relative positions, will pass through the surface bounded by the circuit II. The flux $\Phi$ through the secondary circuit is also proportional to the primary current strength I, and we may write:

---

* See footnote, Vol. II, p. 47.

$$\Phi = \mathrm{MI}.$$

In this equation M is a factor depending only upon the geometrical shapes and the relative positions of the two circuits, and independent of the primary current strength I. It is called the **coefficient of mutual induction** or the **mutual inductance.**

Alteration of the current strength I by moving the contact K along the rheostat R causes the magnetic flux $\Phi$ through the secondary circuit to alter in the same ratio. An E.M.F. (or a current) is therefore indicated by the instrument in the circuit II. As we have seen above, the induced E.M.F. is equal to the rate of decrease of the magnetic flux. Thus when the variation of $\Phi$ is uniform it is given by $\mathrm{E} = -(\Phi_2 - \Phi_1)/t$. When the variation is not uniform, the above quotient must be replaced by the differential coefficient, thus:

$$\mathrm{E} = -\frac{d\Phi}{dt}.$$

Substituting $\Phi = \mathrm{MI}$ and remembering that M is a constant, we have

$$\mathrm{E} = -\mathrm{M}\frac{d\mathrm{I}}{dt}.$$

From this we see that, when $\frac{d\mathrm{I}}{dt} = 1$, i.e. when the current strength increases uniformly by one unit per second, the equation reduces to $\mathrm{E} = -\mathrm{M}$. Hence we obtain the new definition:

**The coefficient of mutual induction or the mutual inductance is numerically equal to the E.M.F. induced in the circuit II when the current strength in the circuit I varies at the rate of one unit per second.**

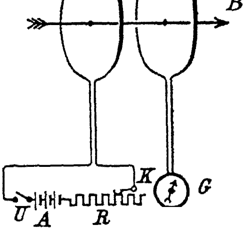

Fig. 25.—Mutual induction

In this definition the circuits I and II are interchangeable. The system possesses unit mutual inductance when a variation of current strength in one of the circuits at the rate of 1 ampere per second induces an E.M.F. of 1 volt in the other circuit. The unit of mutual inductance defined in this way is called **1 henry.***

In fig. 25 each of the two circuits was represented as consisting of only one turn of wire. If we replace the circuit I by a coil with $n_1$ turns, the magnetic flux produced by the primary current I will be $n_1$

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* In honour of the American physicist JOSEPH HENRY (1797–1878).

times greater than before. Similarly if the conductor II be replaced by a coil with $n_2$ turns, the flux $\Phi$ through one turn is to be reckoned $n_2$ times; for the induced E.M.F.s in all the turns add together owing to the fact that the turns are all connected together in series. Hence in calculating the mutual inductance we get the factors $n_1$ and $n_2$, and therefore:

*The mutual inductance of two circuits with* $n_1$ *and* $n_2$ *turns respectively is proportional to the product* $n_1 n_2$.

When the primary current I in the circuit I is switched off by opening the switch U (fig. 25), the flux decreases to the value zero in a very short time. Hence the induced E.M.F. in the other circuit may assume a very high value (p. 516).

**Self-induction.**—When only the circuit I is present, the whole of the magnetic flux produced by the primary current simply passes through the surface bounded by this circuit. Any alteration of the current strength I causes an alteration of this flux. If we look along the magnetic lines of force in their positive sense, the primary current will appear as clockwise. Suppose now that the current *increases.* This will cause the magnetic flux to increase, and consequently an E.M.F. and electric current will be induced in the anti-clockwise sense, i.e. in the opposite sense from the inducing current.

*An increase of the strength of the current flowing in a circuit gives rise to an induced current in the opposite sense in the same circuit. The effect of this induced current is therefore to retard the increase of the inducing current.*

Conversely, when the strength of the inducing current *decreases,* the magnetic flux through the coil also decreases. Consequently an E.M.F. and electric current are induced in the clockwise sense, as viewed along the positive sense of the magnetic lines of force. In this case the direction of the induced current is the same as that of the inducing current.

*A decrease of the strength of the current flowing in a circuit gives rise to an induced current in the same sense in the same circuit. The effect of this induced current is therefore to retard the decrease of the inducing current.*

If the circuit does not consist of a single loop of wire, but of a coil with many turns, the inductive effect produced by a variation of the current strength is more pronounced (see below); for the magnetic flux corresponding to the current in any particular turn passes not only through the surface bounded by that turn, but also through all the surfaces bounded by the other turns. The process whereby a variation of current strength in a circuit produces an E.M.F. in the same circuit is known as *self-induction.*

The magnitude of the induced E.M.F. is proportional to the rate of variation of the strength of the inducing current. A sudden interruption of this current may give rise to a very high self-induced E.M.F. Indeed the self-induced E.M.F. may far exceed the inducing E.M.F. (fig. 9, p. 613). The former is equal to the change of magnetic flux divided by the corresponding time. Now when the current is suddenly switched off, the time taken for the flux to fall to zero is very small. It is actually observed that the spark appearing at the point where the current is broken may be much longer than would correspond to the primary E.M.F.

*The E.M.F. induced on closing the circuit is in the opposite sense from the primary current: the E.M.F. induced on opening the circuit is in the same sense as the primary current.*

The occurrence of the so-called "extra currents" produced by these induced E.M.F.s was demonstrated by **Faraday** in the following manner.

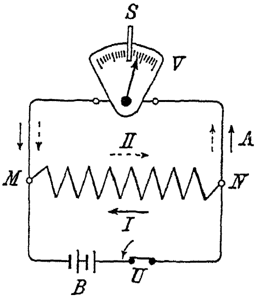

Fig. 26*a*.—Demonstration of the momentary induced current on closing circuit

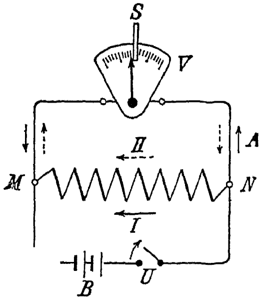

Fig. 26*b*.—Momentary induced current on opening circuit

**1. E.M.F. induced on closing the circuit.**—Between the binding screws M and N (fig. 26 *a*) are connected in parallel a galvanometer V, a coil of low ohmic resistance, and a battery B with switch U. When the current is switched on by closing U, the pointer of the galvanometer assumes (after a time) a certain constant position. It is then held in this position by some object (e.g. a small rod S) which prevents it from returning to zero when the current is switched off again, but which does not hinder it from moving in the opposite direction. When now the current is switched off and then switched on again, the pointer of the galvanometer moves farther away from the zero position. This would be impossible if the E.M.F. at the moment after switching on the current were the same as that during the subsequent steady state. The explanation is as follows. At the moment of switching on the current I (see figure) an E.M.F. is induced in the coil in the direction of the dotted arrow II, i.e. in the opposite sense. In the branch of the system containing the galvanometer, however, this induced E.M.F. causes an instantaneous current in the *same* sense as the primary current. It will be seen from the figure that, though the primary and induced currents flow in the same sense through the galvanometer, they flow in opposite senses through the coil. The induced current is therefore such as to retard the increase of current strength through the coil.

**2. The E.M.F. induced on opening the circuit.**—This may be demonstrated with the arrangement shown in fig. 26 *b* (method due to **Faraday**). The only difference from fig. 26 *a* is that the pointer of the galvanometer is prevented by the small rod S from moving out of its zero position in the direction corresponding to the primary current. It is free, however, to move in the opposite direction. The switch U is first closed and is then opened again. On opening the switch the pointer of the galvanometer is deflected violently in the direction opposite from that in which it would (if free) have been deflected by the original current. It will be seen from the figure that the induced "extra current" II flows in the same sense as the primary current I through the coil, but in the opposite sense from the primary current in the branch of the system containing the galvanometer (e.g. at A).

The magnetic flux $\Phi$ is always proportional to the strength I of the current producing it (p. 183). If the circuit consists of a coil of wire with $n$ turns, each of the magnetic lines of force threads through all of the turns. The total flux is therefore $n$ times the flux $\Phi$ through each turn, and we may write

$$n\Phi = \mathrm{LI}.$$

In this equation L is a factor depending only upon the geometrical shape of the conductor in which the self-induction occurs. It is called the **coefficient of self-induction** or the **self-inductance.** From the above equation we see that:

*The self-inductance of a circuit is equal to the magnetic flux through the circuit produced by unit current strength.*

Suppose that an alteration of the current strength from $I_1$ to $I_2$ causes the flux through each turn to change from $\Phi_1$ to $\Phi_2$. Then $n\Phi_1 = LI_1$ and $n\Phi_2 = LI_2$. According to p. 385 the induced E.M.F. is given by $E = n(\Phi_1 - \Phi_2)/t$. Substituting the values of $n\Phi_1$ and $n\Phi_2$ just obtained, we have

$$E = L\frac{I_1 - I_2}{t}.$$

If the alteration of the current strength does not take place at a uniform rate, we must write instead

$$\mathbf{E} = -\,\mathbf{L}\frac{d\mathbf{I}}{dt}.$$

The negative sign corresponds to the fact that the E.M.F. induced when the primary current strength increases (i.e. when $\frac{dI}{dt}$ is positive) is in the opposite sense from the E.M.F. of the inducing current.

In the special case in which the current strength varies uniformly by one unit per second $\left(\text{i.e. } \frac{dI}{dt} = 1\right)$ the induced E.M.F. becomes numerically equal to L. Hence the new definition:

*The self-inductance of a circuit is numerically equal to the E.M.F. induced in it when the current strength varies at the rate of one unit per second.*

The unit of self-inductance is determined by the choice of the units of current strength and E.M.F. If the current strength be measured in amperes and the E.M.F. in volts, the unit of self-inductance is **1 henry.** Thus:

**A circuit possesses a self-inductance of 1 henry when a variation of the current strength at the rate of 1 ampere per second induces in it an E.M.F. of 1 volt.**

**The Self-inductance of a Ring-shaped Solenoid.**—Suppose that the solenoid has a length $l$ and that it consists of $n$ turns each of cross-section A. The magnetic flux through each turn is then

$$\Phi = \mathrm{BA} = \mu_0 \mathrm{HA} = \mu_0 \frac{n\mathrm{I}}{l}\mathrm{A},$$

where B and H are respectively the magnetic induction and the field strength of the internal field, and I is the strength of the current flowing through the solenoid (compare p. 178). The total flux is $n$ times the flux through each turn. Hence the induced E.M.F. corresponding to a uniform variation of the current strength from $\mathrm{I}_1$ to $\mathrm{I}_2$ during the time interval $t$ is given by

$$\mathrm{E} = \frac{n(\Phi_1 - \Phi_2)}{t} = \frac{\mu_0 n^2 \mathrm{A}}{l}\,\frac{(\mathrm{I}_1 - \mathrm{I}_2)}{t} = \mathrm{L}\frac{(\mathrm{I}_1 - \mathrm{I}_2)}{t}.$$

Thus the self-inductance L of the solenoid is

$$\mathrm{L} = \mu_0 n^2 \mathrm{A}/l.$$

This equation may be brought into another very useful form. If $r$ is the radius of each turn, we have $\mathrm{A} = \pi r^2$ and hence $\mathrm{L} = \mu_0 \pi r^2 n^2/l$. Now $2\pi r n$ is the total length (D say) of the wire. The equation therefore transforms (p. 379) to

$$\mathrm{L} = \frac{\mu_0}{4\pi}\,\frac{\mathrm{D}^2}{l} = 10^{-9}\frac{\mathrm{D}^2}{l}.$$

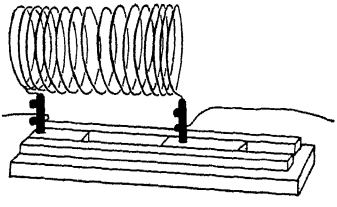

Fig. 27.—Variable self-inductance

The above formulæ may be used as rough approximations in the case of closely wound straight solenoids. For circuits of more complicated shapes empirical formulæ must be employed.

**The Construction of known Self-inductances.**—Self-inductances of 1 henry or of a fraction of 1 henry for measuring purposes are constructed in the form of coils carefully wound upon marble or serpentine.

*Variable Self-inductance.*—A convenient type of variable self-inductance is illustrated in fig. 27. It consists of a coil of wire with a number of turns which may be pulled out like a spiral spring. As the distance between the turns is increased, the fraction of the magnetic flux of each turn which passes through the other turns becomes smaller, and therefore the self-inductance is diminished without the ohmic resistance being affected.

The type of variable self-inductance most frequently used consists of a system of two coils connected in series (*variometer*). One of the two coils is smaller than the other and may be rotated within it. The self-inductance of the combination is a maximum when the planes of the two coils are parallel and when the sense of the current through both coils is the same. The minimum self-inductance corresponds to the position in which the planes of the coils are parallel, but in which the current passes through the coils in opposite senses. Any desired intermediate self-inductance can be obtained rapidly by suitable rotation of the inner coil.

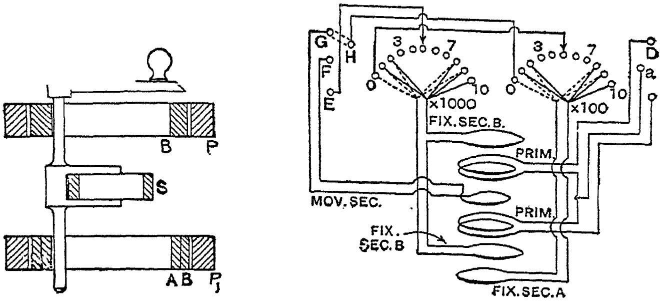

Fig. 28 and Fig. 29.—The Campbell variable mutual inductance
[Cambridge Instrument Co., Ltd., London.]

Figs. 28 and 29 show the construction and internal connexions of the Campbell Variable Mutual Inductance. P and $P_1$ (fig. 28) are two equal coaxial coils forming the primary circuit. They are wound in the same direction, connected in series, and their ends and middle point brought out to terminals D, *a* (fig. 29) on a dial. The secondary consists of three circuits, one of which (S) is movable and is connected with a pointer and scale; the others (A and B) are fixed, and connected in inductive series between the terminals E and H, while the movable secondary is connected between F and G.

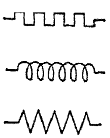

Fig. 31.—Symbols used in diagrams to represent resistances and self-inductances. (The top symbol denotes a resistance without self-inductance; the middle symbol, a self-inductance, practically without resistance; and the bottom symbol, a resistance generally.)

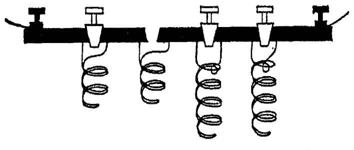

Fig. 30.—Bifilar non-inductive winding of resistances

*Non-inductive Winding.*—For many purposes it is desirable to coils with as low a self-inductance as possible. The coils of resistance wire in resistance boxes (p. 224), for example, must be wound so as to be free from self-inductance. For this reason *bifilar windings* of the type shown in fig. 30 are employed (so-called *non-inductive winding*). These may be regarded as made up of two opposing coils such that the flux through the one compensates the flux through the other. The residual flux is therefore zero, or at any rate so small that the self-inductance L may be neglected.

In diagrammatic representations of electric circuits we shall employ the symbols for resistances and self-inductances given in fig. 31.

*Relation between Self-inductance and Mutual Inductance.*—For the case of two coils arranged in such a way that the magnetic flux due to each passes completely and in the same direction through both, it may easily be deduced from the formulæ given above that the mutual inductance M is given by

$$M = \sqrt{L_1 L_2},$$

where $L_1$ and $L_2$ are the self-inductances of the two coils.

*The lines of force due to both coils being in the same sense, the mutual inductance is equal to the square root of the product of the individual self-inductances.*

If the flux due to each coil does not pass completely through the other (fig. 25, p. 394), the actual value of M is smaller than that corresponding to the above formula. Account may be taken of this by the introduction of a factor $k < 1$, viz. $M = k\sqrt{L_1 L_2}$. This factor $k$ is known as the *coupling factor.*

According as $k$ is large or small, the coupling is said to be tight or loose. The tightest coupling is when $k = 1$; the whole of the flux due to each coil then passes through the other.

## CHAPTER XIII

# Magnetic Fields in Space containing Matter

### 1. Fundamental Phenomena

We have already discussed on p. 206 the phenomenon that the *magnetic field in the space surrounding a solenoid* is considerably intensified by the introduction of an iron core. We explained this effect as due to circulatory molecular currents in the interior of the iron. The *magnetic flux* through the solenoid is also altered by the introduction of any material substance (especially iron).

Let us consider this further with the help of the arrangement shown in fig. 1. In order that the field may be well defined, we employ a ring-shaped coil with a narrow gap at the place where the leads pass to the battery. As has been explained earlier, the magnetic *field lines* in the interior of the coil (supposed devoid of matter) are all closed. With the help of a test coil connected to a voltmeter (see figure) we can show that the same is true of the magnetic *lines of force*. (The magnetic lines of force are related to the induction **B** in the same way as are the magnetic field lines to the field strength **H**.)

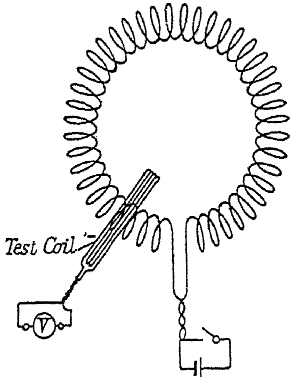

Fig. 1.—Field of force of a ring-shaped coil with narrow gap

When the test coil is brought near to the ring-shaped coil or rotated in its neighbourhood, no deflection of the voltmeter is observed. Hence there are no lines of force outside the ring-shaped coil. But if we introduce the test coil radially into the gap of the ring-shaped coil, we obtain a deflection of the voltmeter such that the time-sum of the induced E.M.F. corresponds to an increase in the number of lines of force passing through the test coil from zero to the total number in the interior of the ring-shaped coil. Provided that the diameter of the test coil be sufficiently large, we may arrange that it shall encircle the ring-shaped coil, as indicated in the figure. We may then move the test coil so as to pass right round the ring-shaped coil, encircling it the whole time. The leads to the ring-shaped coil must be twisted together, so as to have no magnetic effect. No induced E.M.F. is observed in the test coil—a proof that the magnetic flux through this coil must remain constant. The magnetic lines of force must therefore be closed.

If H is the magnitude of the magnetic field strength in the interior of the ring-shaped coil (supposed devoid of matter), then the magnitude $B_0$ of the magnetic induction is $\mu_0 H$ and the magnetic flux is $B_0 A$, where A is the area concerned. Suppose now that the interior of the coil is filled with a ring of iron with a narrow gap in it. All the lines of force still run in the interior; but on introducing the test coil radially into the slit, a deflection of the voltmeter is obtained corresponding to a time-sum of the induced E.M.F. about a thousand times greater than before. The number of lines of force in the interior has therefore been increased about a thousandfold by the introduction of the iron core.

When a ring of nickel or cobalt is used, we obtain about a hundredfold increase of the number of lines of force. With other substances such as aluminium, copper or wood, there is no observable effect. In the case of bismuth very exact measurements show that there is a small decrease in the number of lines of force. Substances which increase the number of lines of force in the interior of the ring-shaped coil are said to be **paramagnetic,** those which cause a decrease are said to be **diamagnetic.***

Though for most paramagnetic substances the increase in the number of lines of force is only about one part in a hundred thousand, with iron, nickel and cobalt it is of the order of magnitude of 100 to 5000 times. Substances which produce such a large increase are said to be **ferromagnetic.**†

## 2. Description of Magnetic Fields in Space containing Matter

**Field Strength (or Intensity) and Magnetic Induction (or Flux Density).**—The characterization of a magnetic field by the field strength (or intensity) **H** (measured in ampere-turns per cm. or amperes per cm.) and by the magnetic induction (or flux density) **B** (measured in volt-seconds per cm.$^2$) permits of a simple extension of the results for fields in space devoid of matter to fields in space containing matter. The method is similar to that with which we have already become acquainted in the description of electrostatic fields by means of the vectors **E** and **D**.

---

* Gr., *diá*, through, away; *pará*, near, hither. The terms refer to the forces acting on the respective substances in an unhomogeneous field; paramagnetic bodies tend to move towards regions of greater field strength, diamagnetic bodies towards regions of lesser field strength. (For further details, see p. 410.)

† Lat., *ferrum*, iron.

In the case of electrostatic fields we found that the transition of the field from vacuum to the dielectric was vortex-free as described by the vector **E**, and source-free as described by the vector **D**. In the same way the transition of the vector **H** from vacuum to a material substance is vortex-free (or *irrotational*) and the corresponding transition of the vector **B** is source-free (or *solenoidal*). At the surface forming the boundary between the two media, the *tangential* component of **H** (like that of **E**) is continuous, and likewise the *normal* component of **B** (like that of **D**).

Referring to fig. 109, p. 89, for example, we see that the vector **D** extends into the dielectric without discontinuity. Indeed the object of its definition was to enable us to describe the field without taking account of the matter present. The transition of the vector **D** from vacuum to dielectric is thus source-free.

The same is true of the vector **B** in the case of magnetic fields. Consider, for example, a ring-shaped coil with an iron core in which is a narrow gap. Suppose that the lines of force within the core are uniformly distributed over the cross-section. Then, neglecting the spreading of the lines of force at the gap (see fig. 33, p. 423), the number of these lines crossing the air gap is the same as that in the core and their distribution is also the same; for according to p. 116 the lines of force are all closed. As the above experiment shows, the magnetic flux through a cross-section of the air gap is the same as that through a cross-section of the iron.

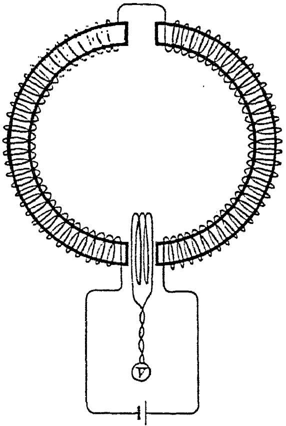

Fig. 2.—Proof of the equality of the numbers of lines of force in the iron core and in the air gap

This can also be demonstrated with the help of an iron core made in two pieces as in fig. 2. When the test coil is introduced into one of the gaps, an induced E.M.F. of short duration is observed, whose magnitude corresponds to the number of lines of force crossing the gap. The two halves of the iron core are now pushed together so as to close both gaps. This produces no effect in the test coil, showing that the magnetic flux has remained unaltered. Hence the number of lines of force in the air gap must be the same as in the iron.

*The transition of the vector* B *from the air into the iron is therefore source-free.*

We will now consider the vector **H**. The purpose of this vector is to describe the fact that the field strength produced within the above ring-shaped coil by a given constant current flowing in it is independent of the nature of the material of which the core is made. One and the same number of ampere-turns can, however, produce *different* numbers of lines of force in the interior, since the lines of force correspond, not to **H**, but to **B**.

Just as in the case of electrostatic fields the vectors **E** and **D** were connected by means of the dielectric constant, so in the present case the relation between **B** and **H** is expressed by a similar equation, viz.

$$\mathbf{B} = \mu \mu_0 \mathbf{H}.$$

The factor $\mu$ is called the **relative permeability** (p. 207). It is a measure of the increase of the magnetic flux by the lines of force of the molecular currents of the material medium.

The above equation may also be written in the form $\mathbf{B} = \mu_{\text{abs.}}\, \mathbf{H}$. Here $\mu_{\text{abs.}} = \mu\mu_0$ is known as the **absolute permeability.** In what follows we shall always understand by the term *permeability* the purely numerical quantity $\mu$.

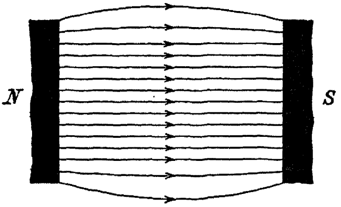

Fig. 3.—Homogeneous magnetic field in non-magnetic medium (diagram of lines of force)

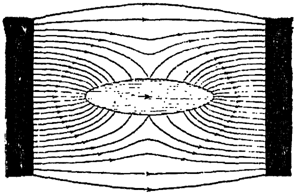

Fig. 4.—Diagram of lines of force with iron in the field

We now proceed to consider **H** in the interior of the material medium, following the method previously adopted for the vector **E** in the analogous electrostatic case (p. 89).

Into a homogeneous magnetic field (fig. 3) we introduce a piece of soft iron (fig. 4). The diagram of the lines of force (corresponding to **B**) is thereby changed in a manner corresponding to that of fig. 109, p. 89, where a body of higher dielectric constant was introduced into an electric field.

In the case of a magnetic field, however, the phenomena are much more marked. The piece of iron becomes a magnet, with a field of its own (see fig. 5). It will be noticed that the lines of force of this induced field are to some extent opposed to those of the original field. The field shown in fig. 4 is obtained by the addition of the fields shown in figs. 3 and 5. Thus the fact that the lines of force of the originally homogeneous field are as it were "sucked" into the iron is due to the superposition of the fields of the molecular magnets.

Let us now trace the course of the *field lines* (corresponding to **H**) from the external space into the iron. For this purpose we imagine a thin tube bored through the body in the direction of the field lines (see fig. 6, in which the superimposed homogeneous field is not shown).

This tube is to be imagined as devoid of matter. We may now suppose the field strength H in the tube to be measured by any of the methods previously described. It will be *smaller* than it would be if the iron were not present; for a certain number of the field lines of the directed molecular magnets are opposed to the field lines of the original field.

*Note.*—The distribution of the resultant field strength $\mathbf{H}_i$ in the interior of the iron is in general dependent upon the shape of the body and upon the strength $\mathbf{H}_a$ of the field in the space devoid of matter. Only in the case of an ellipsoid of revolution is the internal field strength $\mathbf{H}_i$ proportional to $\mathbf{H}_a$ and homogeneous when the original field is homogeneous.

It is very instructive to consider the application of the above to the case of a bar magnet. We form the line-integral of H round a closed path, starting from the north pole and passing along a field line through the external space to the south pole and then back to the north pole through a narrow hole imagined bored through the magnet. It is self-evident that the value of the closed line-integral so obtained must be *zero*; for no current flows through the surface bounded by the closed path.

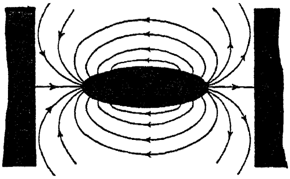

Fig. 5.—Lines of force due to the induced magnetization of the soft iron

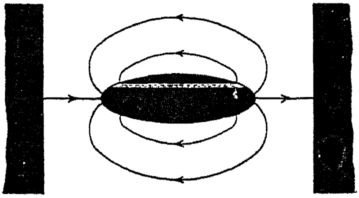

Fig. 6.—Field strength in the interior of the iron (diagram of field lines)

Take, for example, the case of an iron ring magnet (toroid) with a narrow air gap (fig. 7). Here we may regard both $H_i$ and $H_a$ as constant. Hence the line-integral of H round the closed path shown in the figure (which is to be imagined as passing through the magnet within a circular tube containing only air) will be given by

$$H_i l_i + H_a l_a = 0,$$

where $l_i$ and $l_a$ are the lengths of the path in the iron and in the air respectively. From this equation we have

$$H_i = -\frac{l_a}{l_i} H_a.$$

The field strength in the interior is therefore very small in comparison with that in the gap, and is in the opposite sense.

The transition of H from the external space into the matter is therefore accompanied by a discontinuity.

*The transition is vortex-free,* however; for the line-integral round a closed path vanishes, as shown above.

We will now consider the *relation between magnetic field strength and magnetic induction.* As we have seen above, the definition of **B** in the interior of a material medium is similar to that of **D** in the analogous electrostatic case (see fig. 108, p. 89). Thus we imagine a very thin layer cut out of the body at right angles to its magnetization (fig. 8). This corresponds roughly to the air gap of the toroid in fig. 7. The magnetic flux through this layer, which is supposed devoid of matter, is $\mu$ times greater than it would be in the absence of the body; for the lines of force from all the directed molecular magnets pass through it. The magnetic induction B is $\mu$ times the *effective* or *resultant* field strength $H_i$, which is equal to the original field strength diminished by the strength of the fields of the directed molecular magnets.

If we form the line-integral of the magnetic induction around a closed path through a permanent magnet (e.g. a toroidal magnet like that in fig. 9), we shall obtain a value not equal to zero. The value obtained by passing a number of times round the path is proportional to that number. This is again self-evident; for within the magnet the integration path threads through the molecular circulatory currents, whose planes are normal to the lines of force.

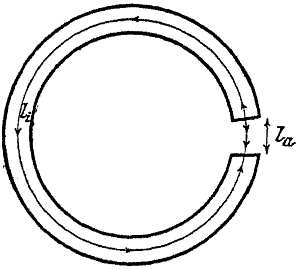

Fig. 7.—Line-integral of the field strength H round a closed path through a toroid with narrow gap.

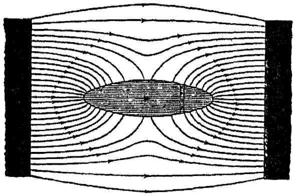

Fig. 8.—Magnetic induction in the interior

**Refraction of Magnetic Lines of Force.**—The vector **H** may be defined in the interior of a material body as the *vortex-free* continuation of the external field. The line-integral of the field strength round a closed path passing through a magnet is zero. On the other hand we see from figs. 6 and 7 above that there is a change in the direction of the field lines at the transition from vacuum (or air) to the material medium; the polar regions act as sources or sinks. In contrast to this the transition of the vector **B** (i.e. the lines of force or lines of induction) is *source-free*; for the lines of force are always closed. Just as in the case of the field lines and lines of force of an electrostatic field (p. 92), we have a *refraction of the magnetic lines of force*. In the source-free transition the normal component of the vector **B** shows no discontinuity: in the vortex-free transition the tangential component of the vector **H** shows no discontinuity. (Compare p. 91.) Hence writing the subscripts *a* and *i* to denote vacuum (or air) and the interior of the material medium respectively, and the subscripts $n$ and $t$ to denote normal and tangential components respectively, we have

$$B_{an} = B_{in};\ \frac{B_{at}}{B_{it}} = \frac{\mu_a}{\mu_i};\ \frac{H_{an}}{H_{in}} = \frac{\mu_i}{\mu_a};\ H_{at} = H_{it};$$

whence (see fig. 10)

$$\frac{\tan \alpha_a}{\tan \alpha_i} = \frac{\mu_a}{\mu_i}.$$

In the case of ferromagnetic substances (e.g. iron) $\mu_i$ is very large as compared with the value $\mu_a = 1$ for air. Hence the lines of force are practically always normal to the surfaces between such materials and the air.

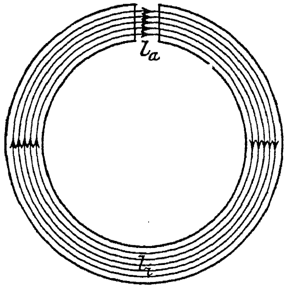

Fig. 9.—Line-integral of the magnetic induction B round a closed path through a toroid with narrow gap.

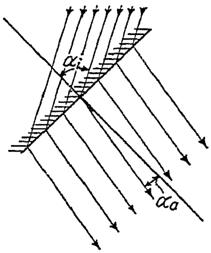

Fig. 10.—Refraction of magnetic lines of force

**Susceptibility.**—Since in very many cases the permeability is very little different from unity, another quantity is frequently used to characterize the magnetic properties of substances.

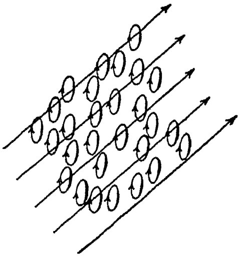

Fig. 11.—Diagram of the directed molecular currents in a magnetic field.

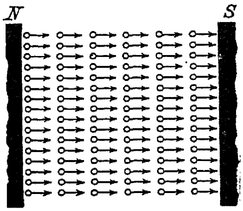

Fig. 12.—Induced magnetization

We have seen above that sources or sinks of the vector **H** occur at the interface between a material body and vacuum or, more generally, between any two material bodies. We may interpret this as follows:

When a magnetizable substance is introduced into a homogeneous magnetic field (fig. 11), the molecular circulatory electric currents are orientated so that their planes are normal to the field. This gives the state depicted in the figure. Replacing the molecular currents by molecular magnets (as in fig. 12), we see that the head to tail arrangement produces a compensation of northern and southern polarity in the interior of the substance, but that a "magnetic layer" (i.e. a layer of uncompensated molecular north or south poles) appears at the surfaces.

In the external space, therefore, the magnetic induction $B_0 = \mu_0 H$ corresponding to the original field strength H is increased by an amount equal to the magnetic flux density of this "magnetic layer".

The state of affairs is analogous to that treated on p. 88 in connexion with electric fields. Just as there we introduced a vector **I** (the electric polarization, p. 99), so here we introduce an analogous vector $\mathbf{I}_m$ to characterize the "magnetic layer". In analogy with the electric case, the vector $\mathbf{I}_m$ gives the magnetic moment per unit volume for the corresponding field strength **H**.

The magnetic induction $\mathbf{I}_m$ of the "magnetic layer" referred to above is put equal to $\kappa\mu_0\mathbf{H}$ and the factor $\kappa$ is called the *susceptibility*. Thus $\kappa\mu_0$ corresponds to the constant $\epsilon$ of p. 101. The actual magnetic induction **B** is therefore given by

$$\mathbf{B} = \mu_0\mathbf{H} + \mathbf{I}_m = \mu_0\mathbf{H}(1+\kappa) = \mu\mu_0\mathbf{H}.$$

Hence it follows that $$\kappa = \mu - 1.$$

It was mentioned above that the quantity $\kappa$ refers to unit volume. In scientific literature the electromagnetic so-called C.G.S. units are taken as basis, and the quantity $(\mu - 1)/(4\pi)$ is always defined as the susceptibility. We will designate this quantity by $\kappa'$. We then have the relationships:

$$\kappa = 4\pi\kappa' \quad \text{and} \quad \mu = 1 + 4\pi\kappa'.$$

The value of $\kappa$ is positive for paramagnetic substances and negative for diamagnetic substances.

**Force Effects in an Unhomogeneous Magnetic Field.**—Just as a body with a dielectric constant different from that of the surrounding medium is acted upon by forces when placed in an electric field, so also for a body with a different permeability from that of its surroundings and situated in a magnetic field.

A system possessing a dipole field (e.g. a bar magnet, a solenoid, a circulatory electric current) can be acted upon only by a turning moment when placed in a homogeneous field. In consequence of the induced magnetization (or, otherwise expressed, in consequence of the refraction of the lines of force) a long bar of iron is also acted upon by a turning moment. The stable position of the bar is that in which its longitudinal axis is in the direction of the magnetic field. In the case of substances whose permeability is not very different from unity (whether paramagnetic or diamagnetic) a cylindrical piece has no specially stable position. (This is connected with the law of refraction of the lines of force, p. 407.) Not until we come to consider un-